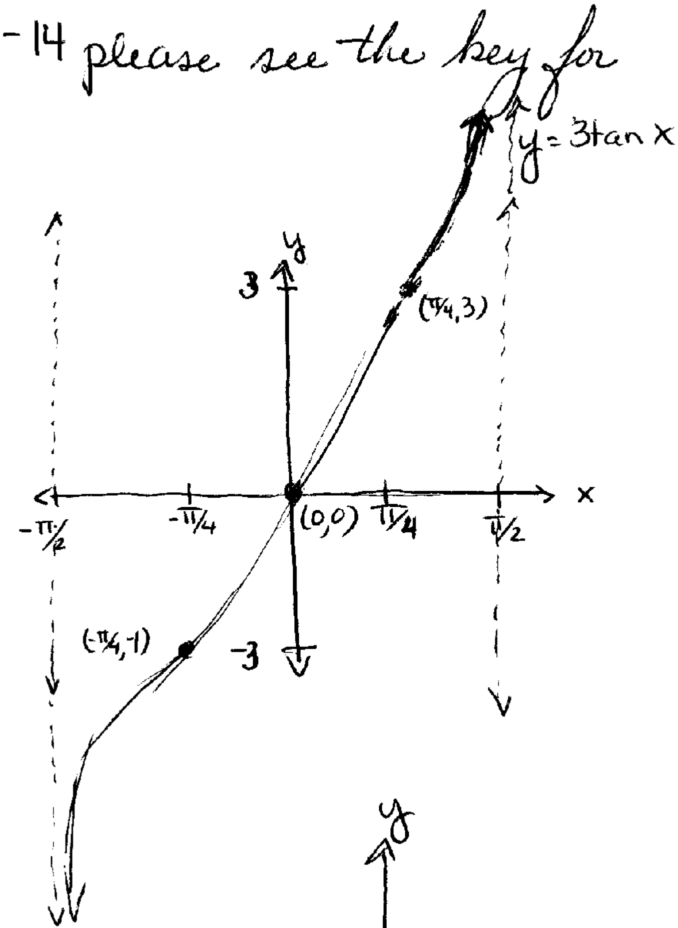
Ch. 4 Form A

For solutions to #1 - 14 please see the key for Practice Midterm #2

#15 Will not be tested

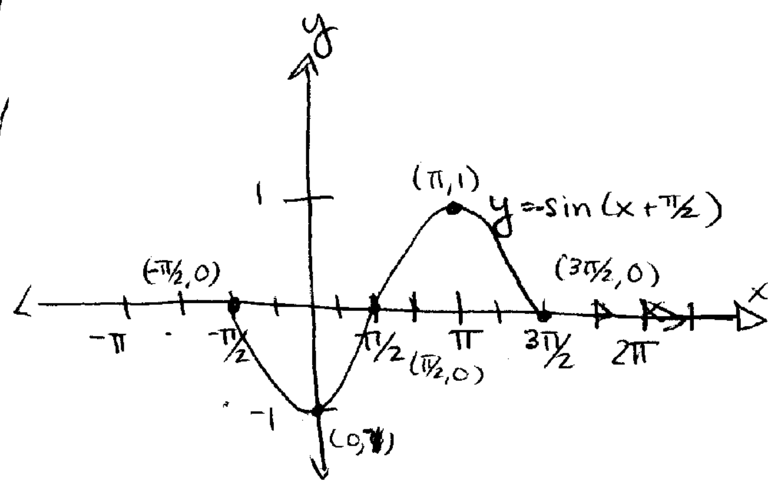
#16 $y = 3\tan x$

x	y	y'
$-\pi/2$	undef	undef.
$-\pi/4$	-1	-3
0	0	0
$\pi/4$	1	3
$\pi/4$	undef	undef.



#17 $y = -\sin\left(x + \pi/2\right)$ $\quad - (-\pi/2)$

x'	x	y	y'
$-\pi/2$	0	0	0
0	$\pi/2$	1	-1
$\pi/2$	π	0	0
π	$3\pi/2$	-1	1
$3\pi/2$	2π	0	0



#18 Not tested

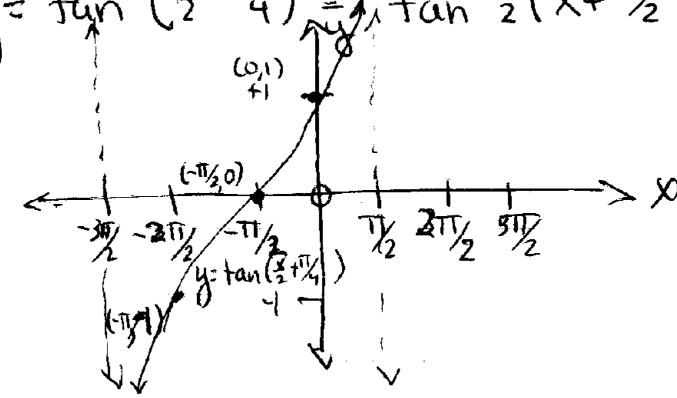
#19 $y = \tan\left(\frac{x}{2} + \frac{\pi}{4}\right) = \tan \frac{1}{2}\left(x + \pi/2\right)$ $\quad - (-\pi/2)$

Period 2π

$2 \cdot -\pi/2 < 2x < 2\pi/2$

$-\pi < x < \pi$

x''	x'	x	y
$-3\pi/2$	$-\pi$	$-\pi/2$	undef.
$-\pi$	$-\pi/2$	$-\pi/4$	-1
$-\pi/2$	0	0	0
0	$\pi/2$	$\pi/4$	1
$\pi/2$	π	$\pi/2$	undef



Ch. 4 con'd

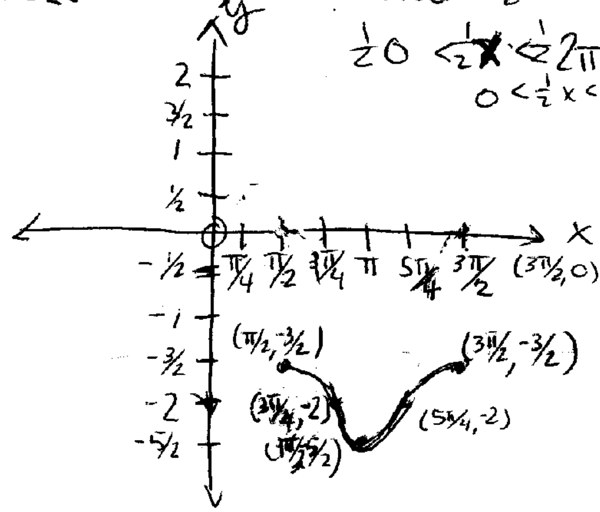
#20 $y = -2 + \frac{1}{2}\cos(2x - \pi) = -2 + \frac{1}{2}\cos 2(x - \pi/2)$

Period $\frac{2\pi}{2} = \pi$

$\frac{1}{2}0 < \frac{1}{2}x < \frac{1}{2}2\pi$

$0 < \frac{1}{2}x < \pi$

x''	x'	x	y	y'	y''
$\pi/2$	0	0	1	$1/2$	$-3/2$
$3\pi/4$	$\pi/4$	$\pi/2$	0	0	-2
π	$\pi/2$	π	-1	$-1/2$	$-5/2$
$5\pi/4$	$3\pi/4$	$3\pi/2$	0	0	-2
$3\pi/2$	π	2π	1	$1/2$	$-3/2$



Ch. 5 Form B

Alternate according to answer book

(17) RHS: $\dfrac{\cot^2\beta - \tan^2\beta}{\cot\beta - \tan\beta} = \dfrac{\frac{\cos^2\beta}{\sin^2\beta} - \frac{\sin^2\beta}{\cos^2\beta}}{\frac{\cos\beta}{\sin\beta} - \frac{\sin\beta}{\cos\beta}} = \dfrac{\frac{\cos^4\beta - \sin^4\beta}{\sin^2\beta\cos^2\beta}}{\frac{\cos^2\beta - \sin^2\beta}{\sin\beta\cos\beta}} = \dfrac{(\cos^2\beta + \sin^2\beta)(\cos^2\beta - \sin^2\beta)}{\sin^2\beta\cos^2\beta}$

$\cdot \dfrac{\sin\beta\cos\beta}{\cos^2\beta - \sin^2\beta} = \dfrac{\cos^2\beta + \sin^2\beta}{\sin\beta\cos\beta} = \dfrac{1}{\sin\beta\cos\beta}$

LHS: $\dfrac{\sec^2\beta}{\tan\beta} = \dfrac{1}{\cos^2\beta} \cdot \dfrac{\cos\beta}{\sin\beta} = \dfrac{1}{\cos\beta\sin\beta}$

(18) RHS: $\dfrac{2}{\csc^2\frac{x}{2}} = 2\sin^2\frac{x}{2} = 2\left(\dfrac{1-\cos x}{2}\right) = 1 - \cos x = \text{LHS}$

(19) RHS: $2\sin^3\beta\cos\beta + 2\sin\beta\cos^3\beta = 2\sin\beta\cos\beta(\sin^2\beta + \cos^2\beta)$

$= 2\sin\beta\cos\beta = \sin 2\beta = \text{LHS}$

(20) LHS: $= 2\tan\alpha\sin\alpha\sec\alpha = 2\tan\alpha\sin\alpha \cdot \dfrac{1}{\cos\alpha} = 2\tan\alpha \cdot \tan\alpha$

$= 2\tan^2\alpha = 2(\sec^2\alpha - 1) = 2\sec^2\alpha - 2 = \text{RHS}$

Ch.5 Form B

(1) $\cos x = -3/4$, x in QII

$\sin^2 x = 1 - 9/16 = \frac{16}{16} - \frac{9}{16} = \frac{7}{16} \Rightarrow \boxed{\sin x = +\frac{\sqrt{7}}{4}}$

$\boxed{\tan x = \frac{\sin x}{\cos x} = \frac{\sqrt{7}/4}{-3/4} \Rightarrow -\frac{\sqrt{7}}{3}}$ $\boxed{\cot = -\frac{3\sqrt{7}}{7}}$ $\boxed{\sec = -4/3}$ $\boxed{\csc = \frac{4\sqrt{7}}{7}}$

(2) $\sin x = -1/2$ in $180° < x < 270°$ (QIII)

(2) $\cot x = ?$

$\cos^2 x = 1 - \frac{1}{4} = \frac{3}{4} \Rightarrow \cos = -\frac{\sqrt{3}}{2}$

$\boxed{\cot x = \frac{\cos x}{\sin x} = \frac{-\sqrt{3}/2}{-1/2} = \sqrt{3}}$

(3) $\boxed{\cos x = -\sqrt{3}/2}$ see #2 for work

(4) $\sin s = -2/3$ in QIII & $\cos t = -1/3$ in QII

(4) $\cos(s-t)$

$\cos^2 s = 1 - 4/9 = \frac{9}{9} - \frac{4}{9} = \frac{5}{9} \Rightarrow \cos s = -\frac{\sqrt{5}}{3}$

$\sin^2 t = 1 - \frac{1}{9} = \frac{9}{9} - \frac{1}{9} = \frac{8}{9} \Rightarrow \sin t = +\frac{2\sqrt{2}}{3}$

$\cos(s-t) = \cos s \cos t + \sin s \sin t$

$-\frac{\sqrt{5}}{3} \cdot -\frac{1}{3} + -\frac{2}{3} \cdot \frac{+2\sqrt{2}}{3} = \frac{\sqrt{5}}{9} - \frac{4\sqrt{2}}{9}$

$= \frac{\sqrt{5} - 4\sqrt{2}}{9}$

(5) $\sin(s+t) = \sin s \cos t + \sin t \cos s$

$= -\frac{2}{3} \cdot -\frac{1}{3} + \frac{+2\sqrt{2}}{3} \cdot -\frac{\sqrt{5}}{3}$

$= \frac{2}{9} - \frac{2\sqrt{10}}{9}$

$= \frac{2(1 - \sqrt{10})}{9}$

(6) $\sin 2s = 2 \sin s \cos s$

$= 2(-2/3)(-\frac{\sqrt{5}}{3}) = \frac{4\sqrt{5}}{9}$

(7) $\tan \frac{t}{2} = \pm\sqrt{\frac{1-\cos t}{1+\cos t}} = \pm\sqrt{\frac{1+1/3}{1-1/3}}$

$= \pm\sqrt{\frac{4/3}{2/3}} = \pm\sqrt{2} =$

(8) If $x = \frac{5\pi}{12}$ find $\sin x$ by sum id $\Rightarrow \sin\left(\frac{3\pi}{12} + \frac{2\pi}{12}\right) = \sin 45 \cos 30 + \sin 30 \cos 45$

$\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{1}{2} \cdot \frac{\sqrt{2}}{2} = \boxed{\frac{\sqrt{6}+\sqrt{2}}{4}}$

(9) cos w/ ½ angle id $\Rightarrow$ $5\pi/12 = 75°$ so $\cos(2 \cdot 5\pi/12) = \cos(10\pi/12) = \cos(\frac{1}{2} \cdot 10\pi/12)$ & $10\pi/12$ is ref $\angle$ to $30°$ in QII $\Rightarrow \cos(\frac{1}{2} \cdot \frac{\pi}{6}) = \pm\sqrt{\frac{1+\cos \pi/6}{2}} = +\sqrt{\frac{1 - \sqrt{3}/2}{2}} = \sqrt{\frac{2-\sqrt{3}}{4}} = \frac{\sqrt{2-\sqrt{3}}}{2}$

(10) $\sin 47° = 2 \sin 94° \cos 94°$ False. $\sin 2A = 2 \sin A \cos A$ so $A = \frac{47°}{2} \neq 94°$

(11) $\cos(-19°) = -\cos 19°$ False. $\cos(-A) = \cos A$ Neg. $\angle$ identities

(12) $\cos 84° = \cos 51° \cos 33° - \sin 51 \sin 33$ True $\cos(51+33) = \cos 51 \cos 33 - \sin 51 \sin 33$

(13) $\sin \overset{2(19)}{38°} = 1 - 2\sin^2 19°$ False $\underline{\cos}\, 2 \cdot 19° = 1 - 2\sin^2 19$

(14) Derive $\sin(A-B+C) = \sin[(A-B)+C] = \sin(A-B)\cos C + \cos(A-B)\sin C$

$= (\sin A \cos B - \cos A \sin B)\cos C + (\cos A \cos B + \sin A \sin B)\sin C = (\sin A \sin B \sin C - \cos A \cos B \cos C) + (\sin A \cos B \cos C + \cos A \cos B \sin C)$

Ch. 5 Form B Cond

(15) Express in terms of $\sin\theta$ & $\cos\theta$

$$\sec\theta + \sin\theta = \left(\frac{1}{\cos\theta} + (\sin\theta)\frac{\cos\theta}{\cos\theta}\right) = \boxed{\frac{1+\sin\theta\cos\theta}{\cos\theta}}$$

(16)
$$\frac{\sec\theta\csc\theta}{\tan\theta\cot\theta} = \frac{\frac{1}{\cos\theta}\cdot\frac{1}{\sin\theta}}{\frac{1}{\tan\theta}\cdot\tan\theta} = \frac{\frac{1}{\cos\theta\sin\theta}}{1} = \boxed{\frac{1}{\cos\theta\sin\theta}}$$

(17) LHS:
$$\frac{\sec^2\beta}{\tan\beta} = \frac{\tan^2\beta+1}{\tan\beta} = \frac{\tan^2\beta}{\tan\beta} + \frac{1}{\tan\beta} = \tan\beta + \cot\beta$$

$$= \frac{\tan\beta + \cot\beta}{1}\cdot\frac{\cot\beta - \tan\beta}{\cot\beta - \tan\beta} = \frac{\cot^2\beta - \tan^2\beta}{\cot\beta - \tan\beta} = \text{RHS}$$

(18) RHS:
$$\frac{2}{\frac{1}{\sin^2\frac{x}{2}}} = 2\sin^2\frac{x}{2}$$

Both Sides:

$$1 - \cos x = 2\sin^2\frac{x}{2}$$
$$-\cos x = -1 + 2\sin^2\frac{x}{2}$$
$$\cos x = 1 - 2\sin^2\frac{x}{2}$$

Let $x = 2A$

$$\cos 2A = 1 - 2\sin^2\frac{2A}{2}$$
$$\cos 2A = 1 - 2\sin^2 A$$

(19)
$$\sin 2\beta = 2\sin^3\beta\cos\beta + 2\sin\beta\cos^3\beta$$
$$= 2\sin\beta\cos\beta(\sin^2\beta + \cos^2\beta)$$
$$= 2\sin\beta\cos\beta$$

(20)
$$2\tan\alpha\sin\alpha\sec\alpha = 2\sec^2\alpha - 2$$
$$2\,\frac{\sin\alpha}{\cos\alpha}\cdot\sin\alpha\cdot\frac{1}{\cos\alpha} = 2(\sec^2\alpha - 1)$$
$$2\,\frac{\sin^2\alpha}{\cos^2\alpha} = 2\tan^2\alpha$$
$$2\tan^2\alpha = 2\tan^2\alpha$$

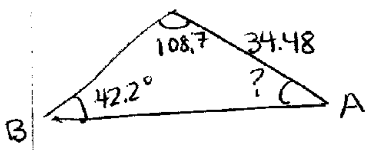
Ch.7 Form A

① $b=30$ & $c=42$ in $\triangle ABC$ which is impossible?

ⓐ $a=80$ ⓑ $a=63$ ⓒ $a=29$ ⓓ $a=15$

$b+c=30+42=72$ ∴ $\boxed{ⓐ\ 80}$ isn't possible since the sum of any 2 sides must be > than 3rd side. (p.319)

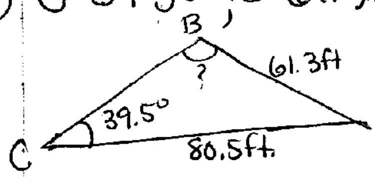
② $B=42.2°$, $C=108.7°$, $b=34.48\text{m}$ find A



SAA $\quad \dfrac{\sin B}{b} = \dfrac{\sin A}{a} = \dfrac{\sin C}{c}$

$\angle A = 180-108.7-42.2 = \boxed{29.1°}$

③ $C=39°30'$, $c=61.3\text{ft}$, $b=80.5\text{ft}$. find B



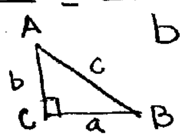
ASS $\quad \dfrac{\sin B}{80.5} = \dfrac{\sin 39.5°}{61.3} \Rightarrow \sin B = \dfrac{80.5 \sin 39.5}{61.3}$

$B = \sin^{-1}(0.8353066351) \doteq 56.64778065$ or $56°38.8668'$

or $B = 180-56.64778065 = 123.3522194$ or $123°21.1336'$

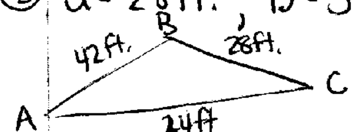
$\boxed{B \doteq 56.6° \text{ or } 123° \quad \text{or } B \doteq 56°39' \text{ or } 123°21'}$

④ $\dfrac{\sin 90°}{c} = \dfrac{\sin B}{b} \Rightarrow \dfrac{1}{c} = \dfrac{\sin B}{b} \Rightarrow b = c\sin B \Rightarrow \sin B = \dfrac{b}{c}$



which is the trig def of sine

⑤ $a=28\text{ft.}$, $b=34\text{ft.}$, $c=42\text{ft.}$ 3 sides so Law of Cosines



$28^2 = 42^2+34^2-2(42)(34)\cos A$

$A = \cos^{-1}\left[\dfrac{28^2-42^2-34^2}{-2(42)(34)}\right] \Rightarrow A = \cos^{-1}(0.7478991597)$

$= 41.59127714 \approx 42°$

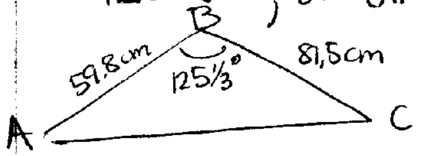
* $B = \cos^{-1}\left[\dfrac{34^2-28^2-42^2}{-2(28)(42)}\right] = \cos^{-1}(0.5918367347)$

$\doteq 53.7125429 \doteq 54°$

* $C = 180-54-42 = 84°$

Ch.7 Form A cont'd

(6) $B = 125°20'$, $a = 81.5\text{cm}$, $c = 59.8\text{cm}$



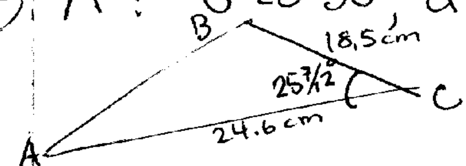
ASA

$$b = \sqrt{(59.8)^2 + (81.5)^2 - 2(59.8)(81.5)\cos(\tfrac{376}{3})}$$
$$\doteq 125.9187302 \approx \boxed{125.9\text{cm}}$$

$$C = \cos^{-1}\left[\frac{(59.8)^2 - (81.5)^2 - (125.9187302)^2}{-2(81.5)(125.9187302)}\right]$$
$$= \cos^{-1}(0.9218983826) = 22.99479038$$
$$\approx \boxed{22.8° \text{ or } 22°50'}$$

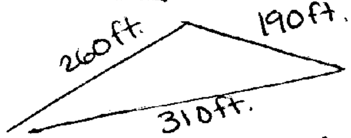
$$\angle A = 180 - 125\tfrac{1}{3}° - 22\tfrac{4}{5}° = 31\tfrac{13}{15}° \text{ or } \boxed{31.9° \text{ or } 31°50'}$$

(7) $A = ?$ $C = 25°35'$, $a = 18.5\text{cm}$ & $b = 24.6\text{ cm}$



$$A = \tfrac{1}{2} ba \sin C$$
$$= \tfrac{1}{2}(24.6)(18.5)\sin(25\tfrac{7}{12})$$
$$\doteq 98.26141423 \approx \boxed{98.3\text{ cm}^2}$$

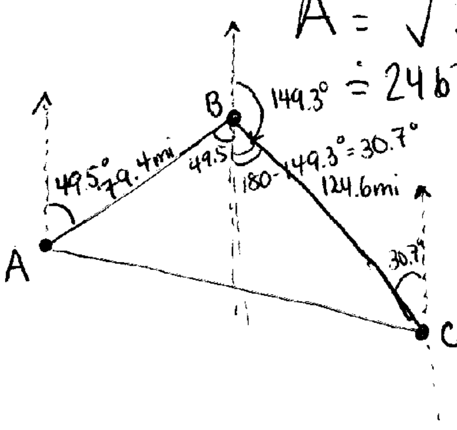
(8) The △ lot sides measure 260ft, 190ft & 310ft. Find the area.



$$S = \frac{260+190+310}{2} = \frac{760}{2} = 380$$

$$A = \sqrt{380(\overset{120}{380-260})(\overset{70}{380-310})(\overset{190}{380-190})} =$$
$$\doteq 24626.81465 \approx \boxed{25000\text{ ft}^2}$$

(9)



$\overline{AC} = ?$

$\angle B = 80.2°$

$$\overline{AC} = \sqrt{79.4^2 + 124.6^2 - 2(79.4)(124.6)\cos 80.2°}$$
$$\doteq 135.8737397 \approx \boxed{136\text{ mi.}}$$