

§7.5 More Trig Equations

Example: Solve $\cos 2x = \sin x$
on $[0, 2\pi)$

Double Angle Identities

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$\cos 2A = 1 - 2 \sin^2 A$$

$$\cos 2A = 2 \cos^2 A - 1$$

$$\sin 2A = 2 \sin A \cos A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Note: Did you get 3 solutions? Don't forget about reference angles.

Example: Solve $2 \cos^2 \theta - 2 \sin^2 \theta + 1 = 0$ on $[0, 360^\circ)$

Note: Because we end up solving for $2A$, we need to make sure we answer for A by dividing all by 2!

Example: Solve $\cos 2x + \cos x = 0$

Example: Solve #24 from Stewart p. 528 $\tan(x/4) + \sqrt{3} = 0$

Sine of a Sum/Difference

$$\sin (A + B) = \sin A \cos B + \cos A \sin B$$

$$\sin (A - B) = \sin A \cos B - \cos A \sin B$$

Cosine of a Sum/Difference

$$\cos (A + B) = \cos A \cos B - \sin A \sin B$$

$$\cos (A - B) = \cos A \cos B + \sin A \sin B$$

Tangent of a Sum/Difference

$$\tan (A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan (A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

Example: #40 & #42 p. 529

Double Angle Identities

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$\cos 2A = 1 - 2 \sin^2 A$$

$$\cos 2A = 2 \cos^2 A - 1$$

$$\sin 2A = 2 \sin A \cos A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Half Angle Identities

$$\cos \frac{A}{2} = \pm \sqrt{\frac{1 + \cos A}{2}}$$

$$\sin \frac{A}{2} = \pm \sqrt{\frac{1 - \cos A}{2}}$$

$$\tan \frac{A}{2} = \pm \sqrt{\frac{1 - \cos A}{1 + \cos A}}$$

$$\tan \frac{A}{2} = \frac{\sin A}{1 + \cos A}$$

$$\tan \frac{A}{2} = \frac{1 - \cos A}{\sin A}$$

* + or - depends on quadrant of $A/2$

Example: #44 p. 529 of Stewart

Sum to Product

$$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right)$$

$$\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right) \sin\left(\frac{A-B}{2}\right)$$

$$\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right)$$

$$\cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right) \sin\left(\frac{A-B}{2}\right)$$

Example: #54 p. 529 Stewart