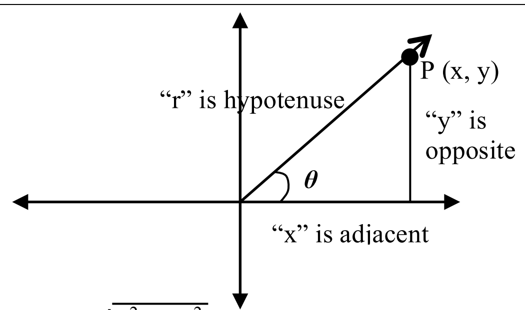
§9.1 Triangle Trigonometry & Applications

I have already discussed all the trigonometric functions and their relationship to the sine and cosine functions and the x and y coordinates on the unit circle, but let's review.

Next we will define the trigonometric functions, review some basic geometry and make the connection to the special triangles that I asked you to memorize in the last section.

Standard Position of *t*



$r = \sqrt{x^2 + y^2}$ (Pythagorean Theorem)
$r > 0$ since it is a distance
(an <u>undirected vector</u> meaning it has no direction)

Based on the ***θ*** in Standard Position with a point on the terminal side defined as (x, y). The 6 trigonometric functions can be defined as seen below. The names of the 6 functions are sine, cosine, tangent, cotangent, secant and cosecant.

$\sin\theta = \dfrac{opp}{hyp} = \dfrac{y}{r}$ ***Note:*** When $r = 1$, $\sin\theta = y$

$\cos\theta = \dfrac{adj}{hyp} = \dfrac{x}{r}$ ***Note:*** When $r = 1$, $\cos\theta = x$

$\tan\theta = \dfrac{opp}{adj} = \dfrac{\sin\theta}{\cos\theta} = \dfrac{y}{x} \qquad x \neq 0$

$\cot\theta = \dfrac{adj}{opp} = \dfrac{1}{\tan\theta} = \dfrac{\cos\theta}{\sin\theta} = \dfrac{x}{y} \qquad y \neq 0$

$\sec\theta = \dfrac{hyp}{adj} = \dfrac{1}{\cos\theta} = \dfrac{r}{x} \qquad x \neq 0$

$\csc\theta = \dfrac{hyp}{opp} = \dfrac{1}{\sin\theta} = \dfrac{r}{y} \qquad y \neq 0$

Note: *These are the **exact** values for the 6 trig f(n). A calculator will yield only the approximate values of the functions.*

Our main focus will be on application problems:

Example: p. 625 #38

A report describing a firefighter training accident stated that a 24-ft extension ladder was placed against a wall at a 75º angle with the ground. At the time of the accident, the top of the ladder was at a height of 20 ft. 1 inch.

a) Was the ladder at its full 24-foot length? If so, explain how you know. If not, how long was the ladder?

b) The report states the ladder "was positioned at an angle of about 75º (4 to 1 ratio)." Use trig to confirm or refute the claim that there was a 4 to 1 ratio. What do you think was meant by this?

Example: p. 625 #40

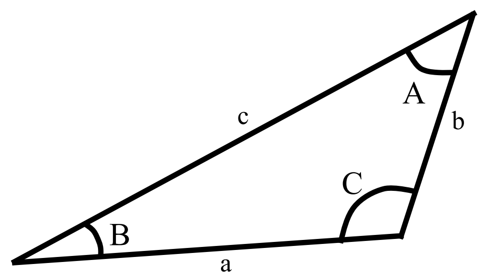
The Leaning Tower of Pisa is leaning because of unstable ground upon which it was built. According to the following statement. "The inclination is c. 5.5º towards the south; this means that the seventh cornice protrudes about 4.5m over the first cornice."

The horizontal protrusions at each level are referred to as cornices. What is the vertical distance between the first and the seventh cornice?

§9.2 The Law of Cosines

First we need the definition for an **oblique triangle**. This is nothing but a triangle that is not a right triangle. In other words, all angles in the triangle are not of a measure of 90º. These are the type of triangle that are of interest to us in the application of the **Law of Sines** and **Law of Cosines**.

Next, let's review the naming convention of triangles (from your study of Geometry, hopefully). The angles are name with capital letters. The sides opposite an angle are named with the comparable lower case letter.



Now, we need to refresh our memory of some axioms from Geometry.

Congruency Axioms (Methods to establish congruency.)

SAS: Side Angle Side — 2 sides known w/ included angle

ASA: Angle Side Angle — 2 angles known w/ included or non-included side (AAS non-included side)

SSS: Side Side Side — 3 sides known

Note: *We will always need at least 1 side to have congruency.*

It is with these axioms that we will use the **Law of Sines** and **Law of Cosines** that are the methods for solving oblique triangles.

There are a total of 4 cases for using the **Law of Sines** or **Law of Cosines**. They are as follows:

***CASE 1:** When we know one side and 2 angles. (SAA and ASA)

***CASE 2:** When two sides and one angle are known. (SSA—Ambiguous Case and SAS)

CASE 3: When two sides and 1 included angle are known. (SAS)

CASE 4: When three sides are known. (SSS)

*We will see these cases in the next section using the Law of Sines.

The Law of Cosines is used to solve a triangle under the last 2 cases we studied in the last section.

***CASE 3:** When two sides and 1 included angle are known. (SAS)

***CASE 4:** When three sides are known. (SSS)

An important fact to remember is that the sum of 2 sides is never greater than the 3rd side. If any of the following hold then a Δ DNE:

$a + b > c \qquad a + c > b \qquad b + c > a$

Law of Cosines

$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$b^2 = a^2 + c^2 - 2ac \cos B$$
$$c^2 = a^2 + b^2 - 2ab \cos C$$

Note: *The side on the left side is the ∠ for which you take the cosine & the other 2 sides are multipliers of twice the cosine.*

If you wish to see the Law of Cosines proven please refer to p. 629-630 of our Stewart, Wilson, Cox & Adamson text. The long and short of it is that we place the oblique triangle in the coordinate system and use the distance formula and trig to get the coordinates of the terminal point in the coordinate system.

SAS Case

1) Plug into Law of Cosines appropriately to find the 3rd side
2) Find the missing ∠ using the Law of Sines (Our Wilson, Cox, Adamson & Stewart text will use Law of Cosines on all)

Example: Solve the ΔABC if B = 73.5º, a = 28.2ft. & c = 46.7 ft.

Step 1: Use Law of Cosines to get b
Step 2: Use Law of Sines to get ∠A&C
Step 3: Subtract to get other ∠

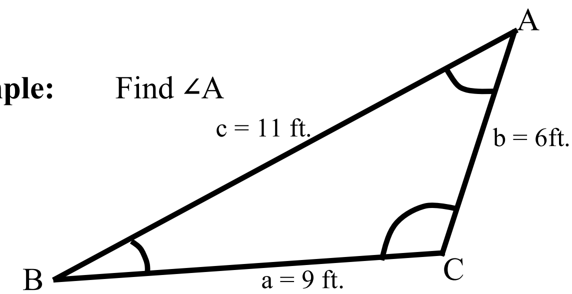
SSS Case

1) Use Law of Cosines to find 1 angle
2) Use Law of Cosines or Law of Sines to find 2nd angle
3) Subtract to get 3rd angle

Example: Solve ABC if a = 25.4 ft, b = 42.8 ft. & c = 59.3ft.

Note: *The Law of Sines does lead to the ambiguous case 61.4º, but we know it can't be, because sin C is opposite the larger side and that would make side b the largest side since it would be the largest angle.*

Example: Find ∠A



Example: Two boats leave a harbor at the same time, traveling on courses that make an angle of $82\ ^1/_3{}^o$ between them. When the slower has traveled 62.5 km, the faster one has traveled 79.4 km. What's the distance between the boats?