Week 1 Notes:

§8.1 Periodic Functions

If y = f(x) is a function and "p" is a nonzero constant such that f(x) = f(x + p) for every "x" in the domain of f, then f is called a **periodic function.** The smallest positive constant "p" is called the **period** of the function, f.

***Note**: If a periodic function is shifted "p" units right or left the resulting graph will be the same.*

Finding Period
The distance from low to low, high to high, midline to midline (in terms of the independent).

Class Work: p. 503 #10, 12 & 14 & #21-24 do g(x + 3)*

**Discuss the period of the function g(x) and notice what happened when x was translated 3 units left.*

The **frequency** of a periodic function gives the portion of a complete cycle completed in one unit of time. It is the reciprocal of the period and likewise the period is the reciprocal of the frequency.

Frequency: $f = {}^{1}/_{P}$ & Period: $P = {}^{1}/_{f}$

Class Work: In the exercises p. 503 #10, 12 & 14 give the frequency

The **midline/centerline** of a periodic function is halfway between the high and the low of the function.

Midline: $y = \dfrac{\text{max dependent} + \text{min dependent}}{2}$

Class Work: In the exercises p. 503 #10, 12 & 14 recognize the midline & mathematically investigate
In exercise #21-24 investigate g(x) + 2 with a graph

The **amplitude** of a periodic function is the highest distance attained above/below the midline. Because it is a distance it is the absolute value.

Amplitude: A = max dependent – midline **or** A = midline – min dependent
or
$A = \dfrac{\text{max dependent} - \text{min dependent}}{2}$

Class Work: In the exercises p. 503 #10, 12 & 14 recognize the amplitude & investigate
In exercise #21-24 investigate 2g(x) with a table & graph

Application of Patterns in Period Functions
Recognizing the period (sometime frequency to get period), midline/centerline, and amplitude give very important context clues about the behavior and therefore graphs & tabular values of a periodic f(n).

Class Work: In the exercises p. 505 #26, 28 & 37

§8.2 Angle Measure

An **arc length** is the curved segment of a circle "swept out" by rotating a ray of length "r" through some angle from a fixed position (usually called the initial side) to an ending position (usually called the terminal side). If the ray is rotated 2π or 360º it creates a circle, so in rotating only a portion of the circle we are rotating a portion of the entire circumference of the circle.

Arc Length as F(n) of Radius $\quad s = \text{portion of circumference} \cdot r$

$\text{Portion of Circumference} = \text{portion} \cdot 2\pi$

Arc Length as F(n) of Angle $\quad s = \theta r \quad$ where θ is in radians → $\theta^\circ \cdot {}^{\pi}/_{180}$

Want to find portion?

$$\text{Portion} = \frac{\text{Measure of angle in radians} \cdot r}{2\pi r} = \frac{\theta \text{ in radians}}{2\pi}$$

Want to find arc length?

$s = C = 2\pi r \cdot \text{portion of entire circle} = \theta r \quad$ since θ in radians is ${}^{\theta^\circ}/_{360^\circ} \cdot 2\pi$ which is the same as $\theta^\circ \cdot {}^{\pi}/_{180}$

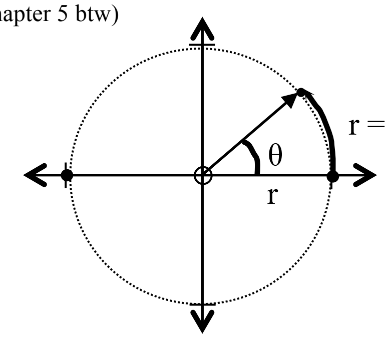
Why Radians?

∠'s are not Real Numbers and radians are, so with radian measure the trig function has a domain with a Real Number.

What is a Radian?

1 radian is an ∠ θ that has an arc length, s, equal to the radius of the circle, r, so $\theta = {}^{s}/_{r}$

If an ∠θ is drawn in standard position with a radius, r, of 1, then the arc, s, subtended by the rotation of the ray will measure 1 radian (radian measure was the ***t*** measure that we used in Chapter 5 btw)



This means that for any angle, θ, $\theta = {}^{s}/_{r}$

Note: *A radian is approximately equal to 57.296º and 1º ≈ 0.01745 radians. Don't use approximations to do conversions!*
Note2*: When r = 1, θ = s*

Converting

Because the distance around an entire circle, C= $2\pi r$ and C is the arc length of the circle this means the ∠θ corresponding to the ∠ swept out by a circle is equivalent to 2π times the radius:

$$360^\circ = 2\pi(\text{radius}) \text{ thus } \pi = 180r \text{ and when } r = 1 \text{ as in the unit circle} \rightarrow \quad \pi = 180$$

$$\text{So, } 1^\circ = {}^{\pi}/_{180} \text{ or } 1 \text{ radian} = {}^{180}/_{\pi}$$

Class Work: p. 521 # 4, 14, 15, 20, 24, 26 & 30

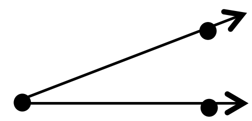
<u>Appendix 6.1: Angle Measure</u> (from Stewart's PreCalculus text)

You should note that his section re-covers some of the same topics covered in our text in §8.2 but with a little more clarity in my opinion. Our text does provide more exercises on basic conversions, however. This section picks up where our text left off and further discusses other important facts about angles in a unit circle and extends the information into more application problems.

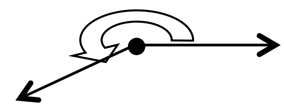
Initial Side – The ray that begins the rotation to create an angle

Terminal Side – The ray that represents where the rotation of the initial side stopped

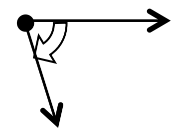
Angle – Two rays with a common endpoint $\angle ABC$



Positive – An angle created by the initial side rotating counterclockwise



Negative – An angle created by the initial side rotating clockwise



Coterminal Angles – Angles that differ by a measure of 2π or 360º. Find by $\theta + n\bullet 360^{\circ}$ or $s + n\bullet 2\pi$. Coterminal angles can be positive or negative, and can be found by using $n \in I$.

Example: Find an angle that is between 0º and 360º that is coterminal with the one given. ***Note:*** *Another way of saying this is,* "Find the measure of the least possible positive measure coterminal angle."
(Hint: "+" divide ∠ by 360/2π & use integer multiple of 360/2π -- ∠ minus integer multiple)
"–" less than -360º/-2π -- 360º/2π plus angle)
"–" greater than -360º/-2π find coterminal between 0º & -360º -- 360º/2π minus coterminal)

a) 361º #40 p. 440

b) -100º #42 p. 440

c) $^{51\pi}/_{2}$ #50 p. 440

d) $^{-7\pi}/_{3}$ #46 p. 440

Example: Give 2 positive & 2 negative angles that are coterminal with 135º #28 p. 440
(Hint: $\theta \pm n\bullet 360^{\circ}$)

Example: Give 2 positive & 2 negative angles that are coterminal with $^{11\pi}/_{6}$ #30 p. 440
(Hint: $\theta \pm n\bullet 2\pi$)

More Practice with Arc Length, θ, and Radius

$s = \theta r$ where θ is in radians → $\theta^o \cdot {}^{\pi}/_{180}$

Classwork: p. 440 #54, 56 & 58

Area of a Sector of a Circle

Since the area of a circle is πr^2 and the portion (sector) of a circle makes up ${}^{\theta}/_{2\pi}$ then:

$$\text{Area} = A = ({}^{\theta}/_{2\pi})(\pi r^2) \text{ or } \frac{ç\, r^2}{2} \quad \text{(where } \theta \text{ is in radians)}$$

Example: #62 p. 441

Linear Speed

Linear Speed = v
Distance = s
Time = t
Since D = rt and r is the velocity and distance is the arc length, s

$$\mathbf{v = {}^{s}/_{t} = {}^{r\theta}/_{t}}$$

Linear speed is how fast a point is moving around the circumference of a circle (how fast it's position is changing. Important in Calculus and Physics.).

Angular Speed

Angular Speed = ω (read as omega; units are radians per unit time)
Angle with relation to terminal side and ray $\overrightarrow{OP}$ = θ **rad**
Time = t

$$\boldsymbol{\omega = {}^{\theta}/_{t}}$$

Angular speed is how fast the angle formed by the movement of point P around the circumference is changing (how fast an angle is formed).

Now we can make some substitutions into these equations based on Section 6.1's definition of an arc and get equivalent statements.

Since $s = \theta r$ we can substitute into $v = {}^{s}/_{t}$ and find $v = {}^{r\theta}/_{t}$ but ${}^{\theta}/_{t} = w$ so

$$\mathbf{v = r\,\boldsymbol{\omega}}$$

Class Work: p. 441 #69 & 70 Remember that how far something circular will travel is a result of the number of revolutions that it is put through & the circumference of the circular object. Unit conversions are usually present so you may want to review those.

Example: Suppose that P is on a circle with a radius of 15 in. and a ray $\overrightarrow{OP}$ is rotated with angular speed $^{\pi}/_{2}$ $^{rad}/_{sec}$. **Trigonometry,* 9th ed., Lial, Hornsby & Schneider

a) Find the angle generated by P in 10 second.

Step 1: List info that you have & what you need to find

Step 2: Decide on the formula needed to find

Step 3: Solve

b) Find the distance traveled by P along circle in 10 seconds.

c) Find linear speed of P in $^{in}/_{sec}$.

Example: Find the linear speed of a point on a fly wheel of radius 7 cm if the fly wheel is rotating 90 times per second.
**Trigonometry,* 9th ed., Lial, Hornsby & Schneider

Step 1: Find the angular speed. You know the revolutions per second, and you know how many radians per revolution, so multiplying these facts will give the angular speed in radians per second.

Step 2: Use the appropriate formula to find linear speed.

Example: Find the linear speed of a person riding a Ferris wheel in $^{mi}/_{hr}$ whose radius is 25 feet if it takes 30 seconds to turn $^{5\pi}/_{6}$ radians.
**Trigonometry,* 9th ed., Lial, Hornsby & Schneider

Step 1: List info that you have & what you need to find

Step 2: Decide on the formula needed to find

Step 3: Solve

Step 4: Convert to mph from $^{ft}/_{sec}$

Class Work: p. 442 #78 & 83