SKILLS

5–10 ■ The graph of a function f is given. Determine whether f is one-to-one.

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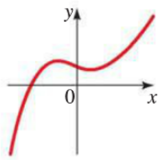

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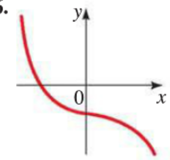

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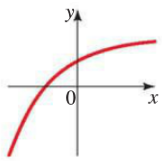

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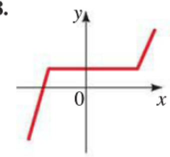

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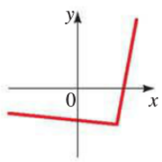

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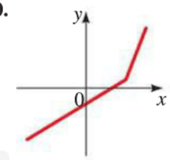


11–20 ■ Determine whether the function is one-to-one.

11. $f(x) = -2x + 4$ **12.** $f(x) = 3x - 2$

13. $g(x) = \sqrt{x}$ **14.** $g(x) = |x|$

15. $h(x) = x^2 - 2x$ **16.** $h(x) = x^3 + 8$

17. $f(x) = x^4 + 5$

18. $f(x) = x^4 + 5, \quad 0 \le x \le 2$

19. $f(x) = \dfrac{1}{x^2}$ **20.** $f(x) = \dfrac{1}{x}$

21–22 ■ Assume that f is a one-to-one function.

21. **(a)** If $f(2) = 7$, find $f^{-1}(7)$.
(b) If $f^{-1}(3) = -1$, find $f(-1)$.

22. **(a)** If $f(5) = 18$, find $f^{-1}(18)$.
(b) If $f^{-1}(4) = 2$, find $f(2)$.

23. If $f(x) = 5 - 2x$, find $f^{-1}(3)$.

24. If $g(x) = x^2 + 4x$ with $x \ge -2$, find $g^{-1}(5)$.

25–36 ■ Use the Inverse Function Property to show that f and g are inverses of each other.

25. $f(x) = x - 6; \quad g(x) = x + 6$

26. $f(x) = 3x; \quad g(x) = \dfrac{x}{3}$

27. $f(x) = 2x - 5; \quad g(x) = \dfrac{x + 5}{2}$

28. $f(x) = \dfrac{3 - x}{4}; \quad g(x) = 3 - 4x$

29. $f(x) = \dfrac{1}{x}; \quad g(x) = \dfrac{1}{x}$

30. $f(x) = x^5; \quad g(x) = \sqrt[5]{x}$

31. $f(x) = x^2 - 4, \quad x \ge 0;$
$g(x) = \sqrt{x + 4}, \quad x \ge -4$

32. $f(x) = x^3 + 1; \quad g(x) = (x - 1)^{1/3}$

33. $f(x) = \dfrac{1}{x - 1}, \quad x \ne 1; \quad g(x) = \dfrac{1}{x} + 1, \quad x \ne 0$

34. $f(x) = \sqrt{4 - x^2}, \quad 0 \le x \le 2;$
$g(x) = \sqrt{4 - x^2}, \quad 0 \le x \le 2$

35. $f(x) = \dfrac{x + 2}{x - 2}; \quad g(x) = \dfrac{2x + 2}{x - 1}$

36. $f(x) = \dfrac{x - 5}{3x + 4}; \quad g(x) = \dfrac{5 + 4x}{1 - 3x}$

37–60 ■ Find the inverse function of f.

37. $f(x) = 2x + 1$ **38.** $f(x) = 6 - x$

39. $f(x) = 4x + 7$ **40.** $f(x) = 3 - 5x$

41. $f(x) = 5 - 4x^3$ **42.** $f(x) = \dfrac{1}{x^2}, \quad x > 0$

43. $f(x) = \dfrac{1}{x + 2}$ **44.** $f(x) = \dfrac{x - 2}{x + 2}$

45. $f(x) = \dfrac{x}{x + 4}$ **46.** $f(x) = \dfrac{3x}{x - 2}$

47. $f(x) = \dfrac{2x + 5}{x - 7}$ **48.** $f(x) = \dfrac{4x - 2}{3x + 1}$

49. $f(x) = \dfrac{1 + 3x}{5 - 2x}$ **50.** $f(x) = \dfrac{2x - 1}{x - 3}$

51. $f(x) = \sqrt{2 + 5x}$ **52.** $f(x) = x^2 + x, \quad x \ge -\frac{1}{2}$

53. $f(x) = 4 - x^2, \quad x \ge 0$ **54.** $f(x) = \sqrt{2x - 1}$

55. $f(x) = 4 + \sqrt[3]{x}$ **56.** $f(x) = (2 - x^3)^5$

57. $f(x) = 1 + \sqrt{1 + x}$

58. $f(x) = \sqrt{9 - x^2}, \quad 0 \le x \le 3$

59. $f(x) = x^4, \quad x \ge 0$ **60.** $f(x) = 1 - x^3$

61–64 ■ A function f is given. **(a)** Sketch the graph of f. **(b)** Use the graph of f to sketch the graph of f^{-1}. **(c)** Find f^{-1}.

61. $f(x) = 3x - 6$ **62.** $f(x) = 16 - x^2, \quad x \ge 0$

63. $f(x) = \sqrt{x + 1}$ **64.** $f(x) = x^3 - 1$

65–70 ■ Draw the graph of f and use it to determine whether the function is one-to-one.

65. $f(x) = x^3 - x$ **66.** $f(x) = x^3 + x$

67. $f(x) = \dfrac{x + 12}{x - 6}$ **68.** $f(x) = \sqrt{x^3 - 4x + 1}$

69. $f(x) = |x| - |x - 6|$ **70.** $f(x) = x \cdot |x|$

71–74 ■ A one-to-one function is given. **(a)** Find the inverse of the function. **(b)** Graph both the function and its inverse on the same screen to verify that the graphs are reflections of each other in the line $y = x$.

71. $f(x) = 2 + x$

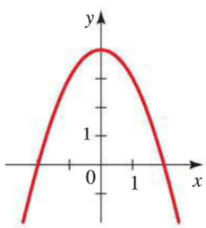
72. $f(x) = 2 - \frac{1}{2}x$

73. $g(x) = \sqrt{x + 3}$

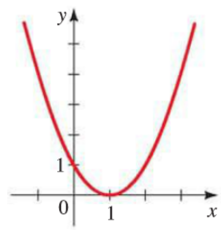
74. $g(x) = x^2 + 1, \quad x \geq 0$

75–78 ■ The given function is not one-to-one. Restrict its domain so that the resulting function *is* one-to-one. Find the inverse of the function with the restricted domain. (There is more than one correct answer.)

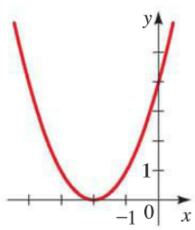
75. $f(x) = 4 - x^2$



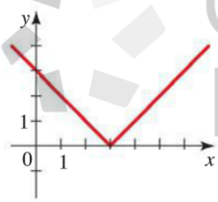
76. $g(x) = (x - 1)^2$



77. $h(x) = (x + 2)^2$

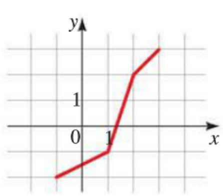


78. $k(x) = |x - 3|$



79–80 ■ Use the graph of f to sketch the graph of f^{-1}.

79.



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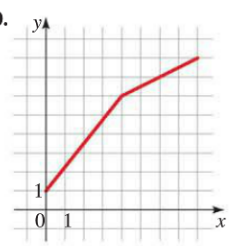


APPLICATIONS

81. Fee for Service For his services, a private investigator requires a \$500 retention fee plus \$80 per hour. Let x represent the number of hours the investigator spends working on a case.

(a) Find a function f that models the investigator's fee as a function of x.
(b) Find f^{-1}. What does f^{-1} represent?
(c) Find $f^{-1}(1220)$. What does your answer represent?

82. Toricelli's Law A tank holds 100 gallons of water, which drains from a leak at the bottom, causing the tank to empty in 40 minutes. Toricelli's Law gives the volume of water remaining in the tank after t minutes as

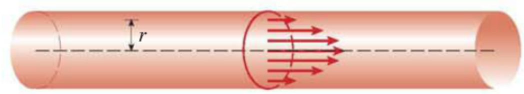
$$V(t) = 100\left(1 - \frac{t}{40}\right)^2$$

(a) Find V^{-1}. What does V^{-1} represent?
(b) Find $V^{-1}(15)$. What does your answer represent?

83. Blood Flow As blood moves through a vein or artery, its velocity v is greatest along the central axis and decreases as the distance r from the central axis increases (see the figure below). For an artery with radius 0.5 cm, v (in cm/s) is given as a function of r (in cm) by

$$v(r) = 18{,}500(0.25 - r^2)$$

(a) Find v^{-1}. What does v^{-1} represent?
(b) Find $v^{-1}(30)$. What does your answer represent?



84. Demand Function The amount of a commodity that is sold is called the *demand* for the commodity. The demand D for a certain commodity is a function of the price given by

$$D(p) = -3p + 150$$

(a) Find D^{-1}. What does D^{-1} represent?
(b) Find $D^{-1}(30)$. What does your answer represent?

85. Temperature Scales The relationship between the Fahrenheit (F) and Celsius (C) scales is given by

$$F(C) = \tfrac{9}{5}C + 32$$

(a) Find F^{-1}. What does F^{-1} represent?
(b) Find $F^{-1}(86)$. What does your answer represent?

86. Exchange Rates The relative value of currencies fluctuates every day. When this problem was written, one Canadian dollar was worth 1.0573 U.S. dollar.

(a) Find a function f that gives the U.S. dollar value $f(x)$ of x Canadian dollars.
(b) Find f^{-1}. What does f^{-1} represent?
(c) How much Canadian money would \$12,250 in U.S. currency be worth?

87. Income Tax In a certain country, the tax on incomes less than or equal to €20,000 is 10%. For incomes that are more than €20,000, the tax is €2000 plus 20% of the amount over €20,000.

(a) Find a function f that gives the income tax on an income x. Express f as a piecewise defined function.
(b) Find f^{-1}. What does f^{-1} represent?
(c) How much income would require paying a tax of €10,000?

88. Multiple Discounts A car dealership advertises a 15% discount on all its new cars. In addition, the manufacturer offers a \$1000 rebate on the purchase of a new car. Let x represent the sticker price of the car.

(a) Suppose only the 15% discount applies. Find a function f that models the purchase price of the car as a function of the sticker price x.

(b) Suppose only the \$1000 rebate applies. Find a function g that models the purchase price of the car as a function of the sticker price x.

(c) Find a formula for $H = f \circ g$.

(d) Find H^{-1}. What does H^{-1} represent?

(e) Find $H^{-1}(13{,}000)$. What does your answer represent?

89. Pizza Cost Marcello's Pizza charges a base price of \$7 for a large pizza plus \$2 for each topping. Thus, if you order a large pizza with x toppings, the price of your pizza is given by the function $f(x) = 7 + 2x$. Find f^{-1}. What does the function f^{-1} represent?

DISCOVERY ■ DISCUSSION ■ WRITING

90. Determining When a Linear Function Has an Inverse For the linear function $f(x) = mx + b$ to be one-to-one, what must be true about its slope? If it is one-to-one, find its inverse. Is the inverse linear? If so, what is its slope?

91. Finding an Inverse "in Your Head" In the margin notes in this section we pointed out that the inverse of a function can be found by simply reversing the operations that make up the function. For instance, in Example 6 we saw that the inverse of

$$f(x) = 3x - 2 \quad \text{is} \quad f^{-1}(x) = \frac{x+2}{3}$$

because the "reverse" of "Multiply by 3 and subtract 2" is "Add 2 and divide by 3." Use the same procedure to find the inverse of the following functions.

(a) $f(x) = \dfrac{2x+1}{5}$ **(b)** $f(x) = 3 - \dfrac{1}{x}$

(c) $f(x) = \sqrt{x^3 + 2}$ **(d)** $f(x) = (2x - 5)^3$

Now consider another function:

$$f(x) = x^3 + 2x + 6$$

Is it possible to use the same sort of simple reversal of operations to find the inverse of this function? If so, do it. If not, explain what is different about this function that makes this task difficult.

92. The Identity Function The function $I(x) = x$ is called the **identity function**. Show that for any function f we have $f \circ I = f$, $I \circ f = f$, and $f \circ f^{-1} = f^{-1} \circ f = I$. (This means that the identity function I behaves for functions and composition just the way the number 1 behaves for real numbers and multiplication.)

93. Solving an Equation for an Unknown Function In Exercise 69 of Section 2.6 you were asked to solve equations in which the unknowns were functions. Now that we know about inverses and the identity function (see Exercise 92), we can use algebra to solve such equations. For instance, to solve $f \circ g = h$ for the unknown function f, we perform the following steps:

$f \circ g = h$	Problem: Solve for f
$f \circ g \circ g^{-1} = h \circ g^{-1}$	Compose with g^{-1} on the right
$f \circ I = h \circ g^{-1}$	Because $g \circ g^{-1} = I$
$f = h \circ g^{-1}$	Because $f \circ I = f$

So the solution is $f = h \circ g^{-1}$. Use this technique to solve the equation $f \circ g = h$ for the indicated unknown function.

(a) Solve for f, where $g(x) = 2x + 1$ and $h(x) = 4x^2 + 4x + 7$.

(b) Solve for g, where $f(x) = 3x + 5$ and $h(x) = 3x^2 + 3x + 2$.

CHAPTER 2 | REVIEW

■ CONCEPT CHECK

1. Define each concept in your own words. (Check by referring to the definition in the text.)

(a) Function

(b) Domain and range of a function

(c) Graph of a function

(d) Independent and dependent variables

2. Sketch by hand, on the same axes, the graphs of the following functions.

(a) $f(x) = x$ **(b)** $g(x) = x^2$

(c) $h(x) = x^3$ **(d)** $j(x) = x^4$

3. (a) State the Vertical Line Test.

(b) State the Horizontal Line Test.

4. How is the average rate of change of the function f between two points defined?

5. What can you say about the average rate of change of a linear function?

6. Define each concept in your own words.

(a) Increasing function

(b) Decreasing function

(c) Constant function

7. Suppose the graph of f is given. Write an equation for each graph that is obtained from the graph of f as follows.

(a) Shift 3 units upward.

(b) Shift 3 units downward.

(c) Shift 3 units to the right.

(d) Shift 3 units to the left.

(e) Reflect in the x-axis.

(f) Reflect in the y-axis.

(g) Stretch vertically by a factor of 3.

(h) Shrink vertically by a factor of $\frac{1}{3}$.

(i) Stretch horizontally by a factor of 2.

(j) Shrink horizontally by a factor of $\frac{1}{2}$.