Practice Midterm Solutions

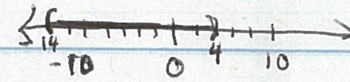
① $|x+5| < 9$

The value of $(x+5)$ must be fewer than 9 units from zero.

$$\begin{array}{rcccl} -9 & < & x+5 & < & 9 \\ -5 & & -5 & & -5 \\ \hline \end{array}$$

$$\boxed{-14 < x < 4}$$

$(-14, 4)$



② $|2x-9| \geq 5$

$(2x-9)$ must be more than 5 units away from zero.

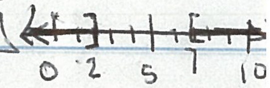
$2x - 9 \leq -5$ or $2x - 9 \geq 5$

$+9 \quad +9 \qquad\qquad +9 \quad +9$

$\frac{2x}{2} \leq \frac{4}{2} \qquad\qquad \frac{2x}{2} \geq \frac{14}{2}$

$$\boxed{x \leq 2 \quad \text{or} \quad x \geq 7}$$

$(-\infty, 2] \cup [7, \infty)$



③ $-2 \leq x \leq 2$ says that the value of x is within 2 units left or right of zero, so

$$|x| \leq 2$$

④ $g(m) = 11 - \frac{m}{50}$

Since the number of miles drive is given and is the independent value this problem requests finding

$$g(225) = 11 - \frac{225}{50} = 11 - 4.5 = \boxed{6.5 \text{ gallo}}$$

remain in the tank

⑤ $N(t) = -10t^2 + 100t$

Since this problem says that 210 micrograms remain, the request is to find t for which $N(t) = 210$, so

$$210 = -10t^2 + 100t \Rightarrow 10t^2 - 100t + 210 = 0$$

$$\Rightarrow 10(t^2 - 10t + 21) = 0 \Rightarrow 10(t-7)(t-3) = 0$$

(5) cont'd Since $t-3=0$ or $t-7=0$

$$\begin{array}{rcl} t-3 &=& 0 \\ +3 && +3 \\ \hline t &=& 3 \end{array} \qquad \begin{array}{rcl} t-7 &=& 0 \\ +7 && +7 \\ \hline t &=& 7 \end{array}$$

The problem asks for the longest time which would be $\boxed{7 \text{ hours.}}$

(6) $\dfrac{x}{14} - \dfrac{5x+2}{49} \geq \dfrac{3x-4}{7}$ LCD $= 2 \cdot 7^2 = 98$ used to clear not build higher terms!

$\overset{7}{\cancel{98}} \cdot \dfrac{x}{\underset{1}{\cancel{14}}} - \dfrac{\overset{2}{\cancel{98}}(5x+2)}{\cancel{49}} \geq \dfrac{(3x-4)}{\cancel{7}} \cdot \overset{14}{\cancel{98}} \Rightarrow 7x - 10x - 4 \geq 42x - 56$

$\begin{array}{r} 14 \\ \times 3 \\ \hline 42 \end{array} \quad \begin{array}{r} 14 \\ \times 4 \\ \hline 56 \end{array}$

$$\begin{array}{rcl} \Rightarrow -3x - 4 &\geq& 42x - 56 \\ +3x && +3x \\ +56 && +56 \\ \hline \end{array}$$

$\dfrac{52}{45} \geq \dfrac{45x}{45} \Rightarrow \dfrac{52}{45} \geq x \Rightarrow x \leq \dfrac{52}{45}$

Interval $\boxed{\left(-\infty, \frac{52}{45}\right]}$

(7) $\dfrac{-\frac{9}{4}}{-15/18} = -\dfrac{9}{4} \div -\dfrac{15}{18} = -\dfrac{\overset{3}{\cancel{9}}}{\underset{2}{\cancel{4}}} \times -\dfrac{\overset{9}{\cancel{18}}}{\underset{5}{\cancel{15}}} = \boxed{\dfrac{-27}{10}}$

(8) $P(x) = 2x-3$ so $P(x) \cdot Q(x) = (2x+3)(2x-3)$

$Q(x) = 2x+3$ $= \boxed{4x^2 - 9}$

since this is a product of conjugates.

(9) $R(x) = 0.5x - 1$ so $[R(x)]^2 = (0.5x-1)^2$ $\begin{array}{r} 0.5 \\ \times 2 \\ \hline 1.0 \end{array}$

$= \boxed{0.25x^2 - x + 1}$

This is a binomial squared $(a-b)^2$

$= a^2 - 2ab + b^2$ where $a = 0.5$

$b = 1$

(10) length = 2width + 5 = 2x + 5

A = length × width = 900 ft²   width = x

$$x(2x+5) = 900$$

$$2x^2 + 5x - 900 = 0$$

$2 \cdot 900 = 1800$

$1 \cdot 1800$
$2 \cdot 900$
$3 \cdot 600$
$4 \cdot 450$
$45 \cdot 20$

$$(2x+45)(x-20) = 0$$

$$2x + 45 = 0 \quad \text{or} \quad x - 20 = 0$$

$$2x = {}^{-}45 \qquad\qquad x = 20$$

extraneous root

so length $= 2(20) + 5 = 45$

The width is $\boxed{20 \text{ feet}}$ and the length is $\boxed{45 \text{ feet.}}$

(11) $\dfrac{1 + \frac{2}{x}}{1 - \frac{4}{x^2}}$ The LCD $= x^2$

$$\frac{x^2 \cdot 1 + \frac{2}{\cancel{x^2}} \cdot \cancel{x}}{x^2 \cdot 1 - \frac{4}{\cancel{x^2}} \cdot \cancel{x^2}} = \frac{x^2 + 2x}{x^2 - 4}$$

$$= \frac{x\cancel{(x+2)}}{\cancel{(x+2)}(x-2)}$$

$$= \boxed{\frac{x}{x-2}}$$

or

$$\frac{\frac{x}{x} + \frac{2}{x}}{\frac{x^2}{x^2} - \frac{4}{x^2}} = \frac{\frac{(x+2)}{x}}{\frac{(x^2-4)}{x^2}} = \frac{(x+2)}{x} \div \frac{(x^2-4)}{x^2} = \frac{\cancel{x+2}}{\cancel{x}} \cdot \frac{x^{\cancel{2}}}{\cancel{(x+2)}(x-2)}$$

$$= \boxed{\frac{x}{x-2}}$$

(12) $$\frac{|5| - |-2+17|}{2} = \frac{5 - |15|}{2} = \frac{5 - 15}{2} = \frac{-10}{2} = \boxed{-5}$$

2 errors are commonly made. (1) Absolute value isn't the opposite $|5| \neq -5$. It is the distance, so it is always positive.

(2) Absolute value of sum isn't absolute of each term $|{}^{-}2 + 17| \neq |{}^{-}2| + |17|$

Errors in 12 con'd

3) $-|-2+17| \neq |2-17|$ or any crazy scenario such as this because the negative doesn't distribute over the definition of absolute value!

13)
$$\begin{array}{r|l} & \boxed{x + 3 + \frac{2}{2x-1}} \\ 2x-1 & 2x^2 + 5x - 1 \\ & \underline{-2x^2 + x} \\ & 6x - 1 \\ & \underline{-6x + 3} \\ & 2 \end{array}$$

$\frac{2x^2}{2x} = x$

$\ominus\ x(2x-1) = -2x^2 + x$

$\frac{6x}{2x} = 3$

$\ominus 3(2x-1) = -6x+3$

Check: $(2x-1)(x+3) + 2$

$= 2x^2 + 6x - x - 3 + 2 = 2x^2 + 5x - 1$ which is the dividend so this is a check

14)
$$\begin{array}{r|l} & \boxed{x^2 - 6x - 19 - \frac{16}{x-1}} \\ x-1 & x^3 - 7x^2 - 13x + 3 \\ & \underline{-x^3 + x^2} \\ & -6x^2 - 13x \\ & \underline{6x^2 - 6x} \\ & -19x + 3 \\ & \underline{19x - 19} \\ & -16 \end{array}$$

$\frac{x^3}{x} = x^2 \quad \ominus x^2(x-1) = -x^3 + x^2$

$\frac{-6x^2}{x} = -6x \quad \ominus 6x(x-1) = 6x^2 - 6x$

$\frac{-19x}{x} = -19 \quad \ominus 19(x-1) = 19x - 19$

or $x - c \Rightarrow x - 1$ so $c = 1$ to use synthetic division

$$\begin{array}{r|rrrr} 1 & 1 & -7 & -13 & 3 \\ & & 1 & -6 & -19 \\ \hline & 1 & -6 & -19 & -16 \end{array}$$

$$\boxed{x^2 - 6x - 19 - \frac{16}{x-1}}$$

* Note a common error is failing to order the dividend.

15. $20(5xy-2)-(30xy-28) = 100xy-40-30xy+28$

$= \boxed{70xy-12}$

*A common error is failing to distribute the subtraction all the way through $(30xy-28)$

16. $\left|\frac{2x-1}{3}\right| = |-4|$ simplify 1st! $\Longrightarrow \left|\frac{2x-1}{3}\right| = 4 \Rightarrow \frac{2x-1}{3} = \frac{-4}{1}$ or $\frac{2x-1}{3} = \frac{4}{1}$

$2x-1=-12$ | $2x-1=12$
$\underline{\ \ +1 \quad +1}$ | $\underline{\ \ +1 \quad +1}$
$2x = -11$ | $2x = 13$
$\boxed{x = \frac{-11}{2}}$ or $\boxed{x = \frac{13}{2}}$

17. $5-2|3x-4| = -5 \Rightarrow \frac{-2|3x-4|}{-2} = \frac{0}{-2} \Rightarrow |3x-4| = 0$
$+5 \qquad\qquad +5$

Only one possible solution $\quad 3x-4=0 \Rightarrow \frac{3x}{3} = \frac{4}{3} \Rightarrow \boxed{x = \frac{4}{3}}$
$\qquad\qquad +4 \quad +4$

18. $3x-6y=5 \Rightarrow \frac{-6y}{-6} = \frac{-3x}{-6} + \frac{5}{-6} \Rightarrow y = \frac{1}{2}x - \frac{5}{6}$ so $m_\perp = -2$

neg. recip.

Thru $(3,-2)$ with $m=-2$

$y-(-2) = -2(x-3) \Rightarrow y+2 = -2x+6$
$\qquad +2x \quad -2 \quad +2x \quad -2$

$\Rightarrow \boxed{2x+y=4}$ std. form

or

$y = -2x+b$ so if $x=3$, $y=-2$

$-2 = -2(3)+b$
$-2 = -6+b$
$\ \ 6 \qquad 6$
$4 = b$

so $y = -2x+4$
and finish to std. form
$\boxed{2x+y=4}$

19) $[5x-(3y+6)][5x+(3y+6)]$ represent conjugates so the result is

$= (5x)^2 - (3y+6)^2 = 25x^2 - (9y^2+36y+36)$

$= \boxed{25x^2 - 9y^2 - 36y - 36}$

20) $f(t)$ means substitute the variable t in for the variable x, so $\boxed{f(t) = 6t - 9}$

21) $f(-2)$ means substitute -2 in for x and then you will simplify to find the value of the function.

$$f(-2) = -3(-2)^2 - 5(-2) + 3$$
$$= -3(4) + 10 + 3$$
$$= -12 + 10 + 3 = -2 + 3 = \boxed{1}$$

22) Finding $f(x) = 9$ means set $f(x)$ equal to 9, and then solve for x in $f(x) = 2x - 1$

$$\underset{+1}{9} = 2x \underset{+1}{-1} \Rightarrow \frac{10}{2} = \frac{2x}{2} \Rightarrow \boxed{x = 5}$$

23)

	D =	R ·	T
down	d	p+6	4hr
up	d	p−6	10hr

p = Kahla's paddle rate

$d = 4(p+6)$

$d = 10(p-6)$

Since distance up and back are the same

$$4(p+6) = 10(p-6)$$
$$\underset{-4p\ +60}{4p + 24} = \underset{-4p\ +60}{10p - 60}$$
$$\frac{84}{6} = \frac{6p}{6} \Rightarrow p = 14$$

Kahla paddles at 14 km/hr.