Name: ______________________________

Hand in on Monday, 6/20

When you walk in!! Before quiz!

Individual Only (No Groups)

Collaborative #10 & #11

Instructions: This is a practice final. It is much longer than the actual final will be, but I wanted to give you many questions to practice in addition to others that you have already worked throughout the quarter. You will turn this in as credit for collaborative exercise #11 on Monday, as an individual exercise. On Wednesday, during class, if we have the time, you will work problems in groups of 2 or 3 and hand in your work on binder paper at the end of the period as Collab #10. On Monday, after our quiz we will go over as many problems as we can and I will post a key to all problems on my website.

1. What is the value of this limit? Show all work in achieving the answer.

$$\lim_{x \to 0} \frac{e^x - x - 1}{x^2}$$

2. Assume y = f(x) is defined implicitly by the equation $y^2 + 2xy = -x^3$ Use implicit differentiation to find equation of the tangent line at the point (1, -1)

3. Find $f'(x)$ for $f(x) = \sin(x + \cos 2x)$. Show all work. Simplify as needed.

4. An 8 foot ladder is leaning against a vertical wall. The foot of the ladder is pulled away from the base of the wall with a constant velocity of 3 feet per second. When the foot of the ladder is 3 feet from the base of the wall, at what rate, in feet per second (rounded to the nearest hundredth), is the top of the ladder moving vertically down the wall?

5. Suppose that f(2) = 3, $f'(2) = -2$, and $f''(2) = 1$. Based on this information alone, which statement below should be the <u>best</u> description of the value of f(2.2)?

i)	About 3	ii)	< 3	iii)	About 2.8
iv)	Slightly > 2.6	v)	Slightly < 3.4	vi)	> 3.2

6. Consider the local linearization, $L(x) = f(x) + f'(x)\Delta x$, of the function $f(x) = -e^{-x^2}$ near $x = -1$. What is the value of $L(-0.8)$?

7. In problem number 6, is the value $L(-0.8)$ an overestimate or an underestimate or neither of $f(-0.8)$? Justify your answer with derivative information.

8. Suppose that $y = f(x)$, where the units are y(dollars) and x(meters). If $g(x) = [f(x)]^2$, what are the units of $g'(x)$?

9. If $y = f'(x)$ changes sign from negative to positive at $x = 3$, then f must have a local ____________________ at $x = 3$.

10. Draw a right triangle to find the value of $\tan(\sin^{-1} x)$

11. If $f'(3) = 0$, $f''(2) = 2$ and $f''(4) = 2$, then f must have a local ______________ a $x = 3$.

16. Suppose function f has first derivative $f'(x) = 2(x - 2)^3$. How many inflection points does function f (not its derivative, f') have?

17. All of the functions below have a local minimum at $x = 0$. The Second Derivative Test can be used to prove this fact for some of the functions but not others. Circle all the functions for which the Second Derivative Test can be used.

i) $y = \sqrt{x}$ ii) $y = \sqrt[3]{x^2}$ iii) $y = |x|$ iv) $y = -e^{-x^2}$

v) $y = -\cos x$

18. If the second derivative of a function f exists at $x = a$ then is it TRUE or FALSE that f is continuous at $x = a$.

19. Circle **ALL** true statements.
   i) There are some functions with no critical points.
   ii) There are some functions with an infinite number of critical points.
   iii) Every cubic polynomial has at least one critical point
   iv) Every function defined on a closed interval [a, b] has at least 2 critical pts.
   v) All local extrema of function f are critical points of f.

20. Investigate the function $f(x) = -x^2 + bx + 3$. Which one description below <u>best</u> describes the role of parameter b?

i) b determines the value of f at $x = 0$

ii) b determines the slope of f at $x = 0$

iii) b determines the concavity of f at $x = 0$

iv) b determines the concavity of f as $x \to \pm\infty$

v) b determines the sign of f as $x \to \pm\infty$

21. *Minimize* the **distance** between the point (2, 3) and the curve $f(x) = {}^1/_x$. Hint: The use of your calculator will be necessary to complete this problem.

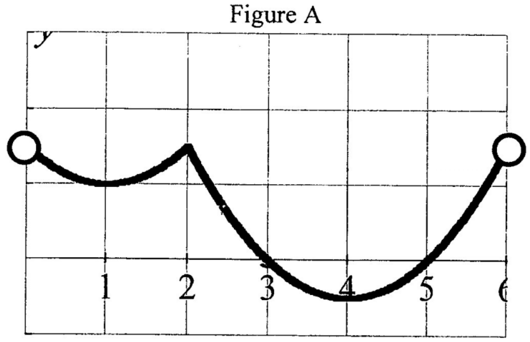
22. The Mean Value Theorem states that if $f(x) = 1/x$ is continuous on [1, 5] and differentiable on (1, 5) then there is a c with $1 < c < 5$ so that $f'(c) = \dfrac{f(5) - f(1)}{5 - 1}$. Find the value of c for this situation.

23. Define $f(x) = \begin{cases} |x - 2|, & x < 1 \\ x^2 + C, & x \geq 1 \end{cases}$. What value of C makes f continuous at $x = 1$?

24. Does a value exist, and if so, what is that value, that makes f differentiable at $x = 1$ in problem 23.

For problems 25-30 you will be using Figure A below to answer the questions.

Figure A



25. Suppose Figure A shows the first derivative, $f'(x)$, of a function $f(x)$ (not function $f(x)$ itself). How many **local minimums** does $f(x)$ have on the interval $0 < x < 6$?

26. Suppose again that Figure A shows the first derivative, $f'(x)$, of a function $f(x)$ (not function $f(x)$ itself). How many **inflection points** does $f(x)$ have on the interval $0 < x < 6$?

27. Suppose now that Figure A shows $g(x)$. Give the best estimate of the average rate of change of $g(x)$ over the interval $1 \le x \le 4$. Show your work.

28. Suppose now that Figure A shows function $h(x)$. Define $F(x) = x^2 \bullet h(x)$. What's the value of $F'(1)$?

29. Suppose now that Figure A shows function $j(x)$. The Second Derivative Test can be used to prove that $y = j(x)$ has a local maximum at $x = 2$. TRUE or FALSE.

30. Suppose now that Figure A shows function $k(x)$. Give your best estimate of $k'(3)$. Show work.

31. Fact: The function $f(x) = x^x$, $x > 0$ is neither exponential nor a power function, so the differentiation rules for those functions don't apply. Give the value of $f''(2)$ using the technique of logs and implicit differentiation.

32. Functions $f(x) = \ln x$ and $g(x) = x - 1$ are continuous on $[1, 5]$ and differentiable on $(1, 5)$. The Racetrack Principle can be used to show:
    i) $f(x) \le g(x)$ on $[1, 5]$
    ii) $f(x) \ge g(x)$ on $[1, 5]$
    iii) Nothing; the conditions are not met for the Racetrack Principle Theorem

*We didn't cover this problem and I wouldn't ask it on a final, but you might want to investigate it for future reference. If $f(a) = f(a)$ and $f'(x) \le g'(x)$ for $x \ge a$, then $f(x) \le g(x)$ for $x \ge a$. Although I had not heard this referred to as the Racetrack Principle it is a commonly applied principle.
http://www.matheverywhere.com/mei/courseware/calculus/growth/racesG/

33. An inflection point of a function **can't** also be a critical point. Show that this is false by using a picture of the derivative of a function.

36. Suppose that an oil spill spreads out in the shape of a circle on the sea's surface. If the radius of the oil slick is increasing at the rate of $^1/_{10}$ kilometer per hour when the radius of the circle is 5 kilometers, at what rate, in square kilometers per hour is the **area** of the circle increasing at this time?

37. Consider line L_1, tangent to $f(x)= x^2$ at $x =2$, and line L_2, tangent to $f(x) = x^2$ at $x = -3$. Where in the xy=plane do L_1 and L_2 intersect?

For problems 38-40 refer to TABLE A.

X	F(x)	F ′(x)	G(x)	G ′(x)
1	3	-4	2	-3
2	1	5	-2	8
3	-1	2	1	-2

38. Refer to TABLE A. If $h(x) = \dfrac{f(x)}{g(2x)}$, find the value of $h'(1)$.

39. Refer to TABLE A. If $j(x) = f(g(x))$, find the value of $j'(3)$.

40. Refer to TABLE A. If $k(x) = [f(x)]^3$, find the value of $k'(3)$.

41. Select the function that has the <u>greatest</u> first derivative at $x = 0$.

i) $f(x) = 2^x$ ii) $f(x) = \tan x$ iii) $f(x) = x^2$

iv) $f(x) = \sin\left({}^{x}/_{2}\right)$ v) $f(x) = {}^{1}/_{(x+2)}$

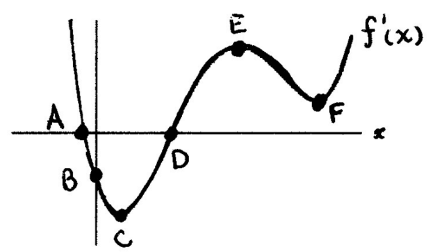
42. The first derivative of $f(x) = a^{x+b}$, $a > 0$, is:

i) Always "+" ii) Always "–" iii) Can be "+/–/0"

43. Consider the function $f(x) = x^3 - x$. Over which interval is $f''(x) < 0$?

44. The perimeter, P, of a window in the shape of a semicircle placed atop a rectangle is fixed at P = 1 meter. What is the formula that gives the area of the window, A (in m^2), as a function of the radius, r (in m), of the circle? Find the maximum possible area of the window in m^2 to the nearest thousandth.

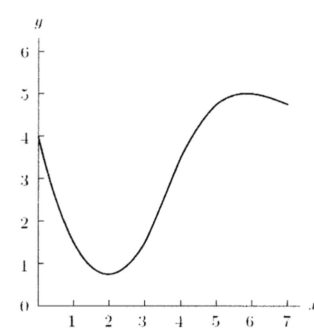
45. The following figure is a graph of the derivative function $f'(x)$. Use the graph to answer the following questions.



a) Which point(s) represent inflection points?
b) Which point(s) represent critical points?
c) Which point(s) represent minimums?
d) Which point(s) represent maximums?

46. For the following diagram answer the questions that follow:

a) Which is larger $f'(5)$ or $f'(4)$? Support your answer.



b) Which is larger $f''(1)$ or $f''(4)$? Support your answer.

47. Find the derivatives of each of the following:

a) $y = \sqrt{x^2 + 3}$ b) $y = x(x + a)^7$ c) $t = x \ln x$

d) $w = \dfrac{x - 2}{x^2 + 8}$ e) $y = e^{(e^x + 4)}$

f) $f(x) = 4x^3 - 5x^2 + 4x + 10$ g) $f(x) = \dfrac{3}{x} + \sqrt{9x + 1}$

48. A water park finds that at an admission price of $17, attendance is 450 per day. For every $1 decrease in price, 30 more people visit the park per day. What is the park attendance when admission prices are set to maximize revenue (to the nearest person)? Hint: Use derivatives not guess and check.

49. For the function $f(x) = -2x^3 + 3x^2 + 12x$ find all the following and graph the function labeling all points found with appropriate ordered pairs.

a) Find all critical points using Calculus. Give the ordered pair.

b) Indicate which critical points are maximum/minima based on the second derivative test.

c) Find all potential points of inflection and give as ordered pairs.

d) Use the second derivative to show that the points in c) are inflection points.

e) Find the y-intercept and give it as an ordered pair.

f) Use the quadratic formula to approximate the remaining 2 x-intercepts (not y-intercept).

g) Graph the function.

50. Find the antiderivative of each of the following.

a) $f(x) = x^2 + 5$

b) $f(x) = \dfrac{5e^x}{5 + e^x}$

c) $f(x) = x^2(2x - 1)$

51. Find an antiderivative F(x) for $f(x) = x^2 - \dfrac{4}{x} + \dfrac{8}{x^3}$

52. Find the antiderivative of G(z) with G′ (z) = g(z) and G(0)=4, given that $g(z) = z - \sqrt{z}$

53. Find the horizontal asymptote(s) of $f(x) = \dfrac{x^3 + 3x}{1 - x^4}$

54. Find the vertical asymptote(s) of $f(x) = \dfrac{x^2 - 4}{x^2 - 3x - 10}$