Practice Final

(1) $\lim\limits_{x\to 0} \dfrac{e^x - x - 1}{x^2}$

$\lim\limits_{x\to 0} e^x - x - 1 = 1 - 0 - 1 = 0$

$\lim\limits_{x\to 0} x^2 = 0$

$\frac{0}{0}$

Apply L'Hospital's Rule

$\lim\limits_{x\to 0} \dfrac{e^x - 1}{2x}$

$\lim\limits_{x\to 0} e^x - 1$

$\lim\limits_{x\to 0} 2x$

$\frac{0}{0}$

Apply L'Hospital's Rule

$$\lim_{x\to 0} \frac{e^x}{2} = \boxed{\frac{1}{2}}$$

(2) $y = f(x)$ is defined implicitly by $y^2 + 2xy = -x^3$. Find $\frac{dy}{dx}$ at the point $(1,-1)$

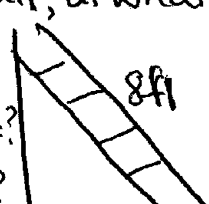
$$2y\frac{dy}{dx} + 2y + 2x\frac{dy}{dx} = -3x^2 \Rightarrow (2x+2y)\frac{dy}{dx} = -2y - 3x^2$$

$$\Rightarrow \frac{dy}{dx} = \frac{-2y-3x^2}{2x+2y} \Rightarrow \left.\frac{dy}{dx}\right|_{\substack{x=1\\y=-1}} = \frac{-2(-1)-3(1)^2}{2(1)+2(-1)} = \boxed{\text{undefined}}$$

(3) Find $f'(x)$ for $f(x) = \sin(x + \cos 2x)$

$$f'(x) = [+\cos(x + \cos 2x)](1 + -2\sin 2x) = \boxed{(1-2\sin 2x)(\cos(x+\cos 2x))}$$

(4) An 8ft ladder is leaning against a vertical wall. The foot is pulled away from the wall with constant velocity 3ft/sec. When the foot is 3ft from the wall, at what rate is the top of the ladder moving vertically down?



$\frac{dy}{dt} = ?$

$y = ?$

$x = 3\text{ft}$ $\frac{dx}{dt} = 3\frac{\text{ft}}{\text{sec}}$

$\boxed{\text{Moving down at rate of } 1.21 \text{ ft/sec}}$

$$x^2 + y^2 = 8^2 \qquad 2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

since $x^2 + y^2 = 64 \Rightarrow y = \sqrt{64-9} = \sqrt{55}$ ft

$$\frac{dy}{dt} = \frac{-2x\frac{dx}{dt}}{2y} = \frac{-x\frac{dx}{dt}}{y}$$

thus $\frac{dy}{dt} = \frac{-(3)(3)}{\sqrt{55}} = \frac{-9}{\sqrt{55}} \approx -1.21$ ft/sec

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(5) If $f(2)=3$, $f'(2)=-2$ and $f''(2)=1$ meaning that the function is decreasing at 2 at a rate of 2 units of y per 1 unit of x.

$$f(2.2) = f(2) + f'(2)\Delta x$$
$$= 3 + -2(0.2) = 3 - 0.4 = 2.6$$

but the second derivative tells us that the function is concave up at 2 and the function is decreasing but the rate of change for the slope is ~~leveling off~~ increasing meaning that it won't be quite as negative as -2 in the next unit

$$f(2.2) = f(2) + f'(3)\Delta x = 3 + -1(0.2) = 3 - 0.2 = 2.8$$

so it's somewhere between 2.6 and 2.8, but closer to 2.6 since 2.2 is closer to 2 than to 3.

$\boxed{\text{Slightly greater than 2.6}}$

(6) Local linearization, $L(x)$, of $f(x) = -e^{-x^2}$ near $x=-1$.

What is the value of $L(-0.8)$ $\qquad f'(x) = 2xe^{-x^2}$

$$L(-0.8) \approx f(-1) + f'(x)\Delta x$$
$$\doteq -e^{-1} + 2(-1)e^{-1}(0.2)$$
$$\doteq -1.4e^{-1} \doteq \boxed{-0.515}$$

-0.15

(7) Is $L(-0.8)$ an over or underestimate of $f(-0.8)$?

$f''(x) = -4x^2e^{-x^2} + 2e^{-x^2} = 2e^{-x^2}(1-2x^2) < 0$ when $x = -0.8$

concave down $\qquad f'(x) = 2xe^{-x^2} < 0$ when $x = -0.8$ decreasing $f(n)$

$\frac{1}{2} = x^2$

$x =$

so the slopes are getting more & more negative thus $f'(x)$ may be -0.75 instead of -0.74 thus giving an estimate like -0.517 which is less than our estimate, so our estimate is an $\boxed{\text{overestimate}}$

Practice Final

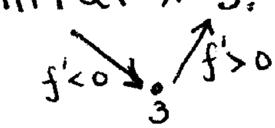
(8) Suppose that $y = f(x)$, where y's units are in dollars and x's units are in meters. If $g(x) = [f(x)]^2$, then what are the units of $g'(x)$?

$[f(x)]^2$ is in squared dollars ~~and~~ is y's units for $g(x)$. ~~are in~~ The x units are still in meters

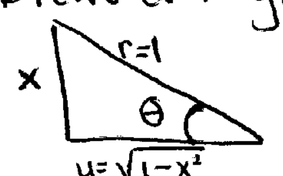
$\therefore$ $\boxed{g'(x) = \dfrac{\Delta y}{\Delta x} = \dfrac{\text{square dollars}}{\text{meter}}}$

(9) If $f'(x)$ changes sign at $x=3$ and goes from neg. to pos. then f must have a local min at $x=3$.

This is a true statement



(10) Draw a right Δ to find $\tan(\arcsin x)$

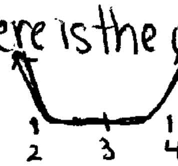


Since $x^2 + y^2 = r^2$
$y = \sqrt{1-x^2}$

The arcsin x is the angle that yields $\frac{x}{1}$, shown as θ in the diagram at the left. The $\tan\theta$ is opp/adj which is therefore $\boxed{\dfrac{x}{\sqrt{1-x^2}}}$

(11) Given $f'(3) = 0$, $f''(2) = 2$ and $f''(4) = 2$ then f must have a local min at $x=3$

This is a false statement Here is the counterexample



(16) f has $f'(x) = 2(x-3)^3$. How many inflections points does f have?

$f''(x) = 6(x-3)^2$ $\qquad$ $f''(x) = 0$ when $x = 3$

but concavity doesn't change so $\boxed{\text{zero}}$

$f''(2) > 0$ & $f''(4) > 0$