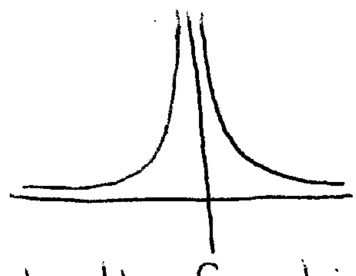
§4.4 p.288 #9

① Graph the function on your calculator & you'll see



in the standard view for

$f(x) = 1 + \frac{1}{x} + \frac{8}{x^2} + \frac{1}{x^3}$

② Let's investigate the function with our current powers.

ⓐ Where are there critical pts?

$f'(x) = -1/x^2 + -16/x^3 + -3/x^4 = -1/x^4(x^2+16x+3)$

and ∴ by use of the quadratic formula

$x = \frac{-16 \pm \sqrt{16^2-4(1)(3)}}{2(1)} = \frac{-16 \pm \sqrt{244}}{2} \doteq \frac{-16 \pm 15.62}{2}$ so $\boxed{\begin{matrix} x \approx -0.19 \\ x \approx -15.81 \end{matrix}}$

$x = 0$ is also a critical pt.

ⓑ What's going on at $x = 0$?

$\lim_{x \to 0^-} 1 + \frac{1}{x} + \frac{8}{x^2} + \frac{1}{x^3}$ after investigation of small neg values $x = -0.001$ & $x = -0.0001$ it is found that $\lim_{x \to 0^-} f(x) = -\infty$

$\lim_{x \to 0^+} f(x)$ after investigation using small positive values $x = 0.001$ & $x = 0.0001$ it is found that $\lim_{x \to 0^+} = \infty$

ⓒ Are $x \approx -0.19$ & $x \approx -15.81$ max or min values?

$f'(-0.5) = -4 + 16(8) + -3(16) > 0$ & $f'(-0.1) = -100 + -16(1000) + 3(10000) < 0$

∴ $\boxed{x \approx -0.19 \text{ is a maximum}}$

$f'(-16) = -\frac{1}{256} + \frac{1}{256} - \frac{3}{65536} < 0$ ∴ $\boxed{x \approx -15.81 \text{ is a minimum}}$

ⓓ What are the values of the function at these critical points?

$f(-15.81) \approx 0.9685$
$f(-0.19) \approx 71.55$
} Use your calculator's Table to get values

ⓔ What's happening with the function to the right & left as the indep. goes to $\pm\infty$?

$\lim_{x \to \infty} 1 + \cancel{1/x}^{\to 0} + \cancel{8/x^2}^{\to 0} + \cancel{1/x^3}^{\to 0} = 1$

$\lim_{x \to -\infty} 1 + 1/x^{\to 0} + 8/x^2^{\to 0} + 1/x^3^{\to 0} = 1$

(f) Where is the function increasing & decreasing?

Found to be decreasing to the left of -15.81 $(-\infty, -15.81)$

Found increasing between -15.81 & -0.19 $(-15.81, -0.19)$

Found decreasing to right of -0.19 $(-0.19, 0)$

But what's happening on the right side?

$f'(1) = -1 + -16 + -3 < 0$ & $f'(16) = -\frac{1}{256} + \frac{-16}{4096} + \frac{-3}{65536} < 0$

so on the right side it's a decreasing fun) $(0, \infty)$

(g) What about inflection points?

$f''(x) =$

$f''(x) = 0$ reveals $x = -24$ & $x = -0.25$

$x \approx$ or $x \approx$

$f''(\ ) =$ > 0

$f''(\ )$ > 0 $f''(\ ) =$ > 0

I ran out of time To finish checks of concavity f(-24) & f(-0.25)

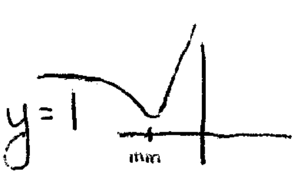
(h) What about x-intercepts? We know there are no y-int. This is one place our calculator could help. We can zoom in between the max at ≈ -0.19 and zero and use the calculator to find the value $\boxed{\text{Calc}} \rightarrow \boxed{\text{zero}} \rightarrow$ left bound -0.19

right bound trace to below x axis

$\rightarrow x = -0.1267538$ $y = 0$

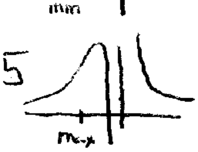
(i) View on your calculator from

$x = -100$ to $x = -1$ with $y = 0.95$ to $y = 1$



&

$x = -1$ to $x = 1$ with $y = -10$ to $y = 75$



§4.5 p. 296 #9 & 11 (Due Mon, 6/6)

(9) $\lim_{x\to 0} \frac{e^t - 1}{t^3}$ $\quad \frac{1-1}{0^3} = \frac{0}{0}$ indeterminate form ∴ use l'Hospital's Rule

$\lim_{x\to 0} \frac{e^t}{3t^2} \cdot \frac{1}{0} = \boxed{\infty}$

Note: You can't use l'Hospital's Rule again b/c it is not an indeterminate form.

(11) $\lim_{x\to\infty} \frac{\ln x}{\sqrt{x}}$ $\quad \frac{\infty}{\infty}$ indeterminate form ∴ use l'Hospital's Rule

$\lim_{x\to\infty} \frac{1/x}{1/2\sqrt{x}} = \lim_{x\to\infty} \frac{2\sqrt{x}}{x} = 2 \lim_{x\to\infty} \frac{1}{x^{1/2}} = 2 \cdot 0 = \boxed{0}$

§4.5 p. 297 #43 (Due Wed June 8)

43) $\lim_{x\to\infty} x^{1/x}$ $\quad \infty^0$ indeterminate form

$$y = x^{1/x} \Rightarrow \ln y = \frac{1}{x}\ln x \Rightarrow \ln y = \frac{\ln x}{x}$$

$\frac{\infty}{\infty}$ indeterminate form ∴ l'Hospital's

$$\lim_{x\to\infty} \frac{\ln x}{x} = \lim_{x\to\infty} \frac{1/x}{1} = \lim_{x\to\infty} \frac{1}{x} = 0$$

but we didn't have $\ln y = \frac{\ln x}{x}$ originally, so to get back to our original function's limit $e^{\ln y} = e^{\frac{\ln x}{x}}$ and we've just shown that as $x \to \infty$ $\frac{\ln x}{x} \to 0$, so make a substitution call $t = \frac{\ln x}{x}$ ∴ $\lim_{t\to 0} e^t = e^0 = \boxed{1}$ is the limit of our original function.

§4.5 p. 296 # 17, 18 & 41 (Due Mon, 6/13)

(17) $\lim\limits_{t\to 0} \dfrac{5^t - 3^t}{t}$ $\quad \dfrac{5^0 - 3^0}{0} = \dfrac{0}{0}$ indeterminate so use l'Hospital's

$\lim\limits_{t\to 0} \dfrac{5^t \ln 5 - 3^t \ln 3}{1} = 5^0 \ln 5 - 3^0 \ln 3 = \ln 5 - \ln 3 = \boxed{\ln \frac{5}{3}}$ or $\approx \boxed{0.5108}$

(18) $\lim\limits_{u\to\infty} \dfrac{e^{u/10}}{u^3}$ $\quad \dfrac{e^{\infty/10}}{\infty^3} = \dfrac{e^{\infty}}{\infty} = \dfrac{\infty}{\infty}$ indeterminate so use l'Hospital's

$\lim\limits_{u\to\infty} \dfrac{\frac{1}{10}e^{u/10}}{3u^2} = \lim\limits_{u\to\infty} \dfrac{e^{u/10}}{30u^2}$ which is still $\frac{\infty}{\infty}$ so l'Hospital's

$\lim\limits_{u\to\infty} \dfrac{\frac{1}{10}e^{u/10}}{60u} = \lim\limits_{u\to\infty} \dfrac{e^{u/10}}{600u}$ which is still $\frac{\infty}{\infty}$ so l'Hospital's

$\lim\limits_{u\to\infty} \dfrac{\frac{1}{10}e^{u/10}}{600} = \lim\limits_{u\to\infty} \dfrac{e^{u/10}}{6000} = \boxed{\infty}$

(41) $\lim\limits_{x\to 0} (1-2x)^{1/x}$ $\quad [1-2(0)]^{1/0} = 1^{\infty}$ indeterminate power so L'Hospital's

$y = (1-2x)^{1/x} \Rightarrow \ln y = \ln (1-2x)^{1/x} \Rightarrow \ln y = \frac{1}{x} \ln(1-2x)$

$\lim\limits_{x\to 0} \dfrac{\ln(1-2x)}{x} = \lim\limits_{x\to 0} \dfrac{-2/(1-2x)}{1} = \lim\limits_{x\to 0} \dfrac{-2}{1-2x} = \dfrac{-2}{1-2(0)} = -2$

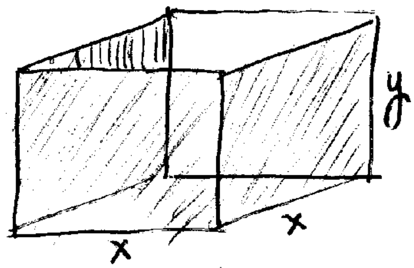
but we aren't finished $y = (1-2x)^{1/x}$ not $\ln y$ so

$e^{\ln y} = e^{\frac{1}{x}\ln(1-2x)}$ so we found as $x \to 0$ what happened to the exponent of e ∴

$\lim\limits_{y\to -2} e^y = \boxed{e^{-2}}$

§4.6 p. 306 #11 (Due Mon 6/13)

(11)



$A_s = \underset{\uparrow \text{ bottom}}{x^2} + \underset{\uparrow \text{ 4sides}}{4xy} = 1200\text{ cm}^2$

$V = x^2 y$

& $y = \frac{1200 - x^2}{4x}$ from solving A_s for y

$\therefore V = x^2\left(\frac{1200 - x^2}{4x}\right) = \frac{1200x - x^3}{4} = 300x - \frac{1}{4}x^3$

$V' = 300 - \frac{3}{4}x^2$ to maximize $V' = 0 \Rightarrow 300 = \frac{3}{4}x^2$

$\Rightarrow \frac{4}{\cancel{3}} \cdot \cancel{300}^{100} = x^2 \Rightarrow x^2 = 400$

$\Rightarrow x = \pm\sqrt{400} = \pm 20$

but -20 makes no sense so disregard

Show that $x = 20$ is a max $V'' = -\frac{3}{2}x$ & $V''(20) = -30 < 0$

$\therefore$ by 2nd derivative test it is a maximum since concave down

or $V'(19) = 300 - \frac{3}{4}(19)^2 = \text{"}+\text{"}$ & $V'(21) = 300 - \frac{3}{4}(21)^2 = \text{"}-\text{"}$

$\therefore$ by 1st derivative test $\nearrow \bullet \searrow$ it's a max

Now, find $y = \frac{1200 - 400}{4(20)} = \frac{800}{80} = 10$ $\therefore V = (20)^2(10)$

$= 400(10)$

$= \boxed{4{,}000\text{ cm}^3}$

§4.7 p. 315 #5 & 15 (Due Mon 6/13)

(5) $x^3+2x-4=0 \qquad x_1=1 \qquad f'=3x^2+2$

$f(1)=1^3+2(1)-4=1+2-4=-1 \qquad f'(1)=3(1)^2+2=3+2=5$

$$x_2=x_1-\frac{f(x_1)}{f'(x_1)}=1+\frac{1}{5}=1.2$$

$f(1.2)=(1.2)^3+2(1.2)-4=1.728+2.4-4=0.128 \quad f'(1.2)=3(1.2)^2+2=4.32+2=6.32$

$$x_3=x_2-\frac{f(x_2)}{f'(x_2)}=1.2-\frac{0.128}{6.32}\doteq 1.179746835\approx\boxed{1.1797}$$

(15) Find the roots of $(x-2)^2=\ln x \Rightarrow \underbrace{\overset{x^2-4x+4}{(x-2)^2}-\ln x=0}_{\text{std form to find roots}}$

Use your calculator to get an idea of where to start

left-hand root (on pix)

$x_1=1.5$

$f(1.5)=(1.5-2)^2-\ln(1.5)=0.25-0.4054651081$
$=-0.1554651081$

$f'=2(x-2)-1/x$

$f'(1.5)=2(1.5-2)-1/1.5=-1\tfrac{2}{3}=-5/3$

$x_2=1.5-\frac{-0.1554651081}{-5/3}=1.5-0.932790649$
$=1.406720935$

$f(1.406720935)=0.0107186308$
$f'(1.406720935)=-1.897431179$

$x_3=1.406720935+0.0056490222$
$=1.412369957$

$f(1.412369957)=0.00003995327424$
$f'(1.412369957)=-1.883289873$

right-hand root (on pix)

$x=3$

$f(3)=-0.098612289$
$f'(3)=1.666666667=1\tfrac{2}{3}=5/3$

$x_2=3-\frac{-0.098612289}{5/3}$
$=3.059167373$

$f(3.059167)=0.003692746$
$f'(3.059167)=1.791448415$

$x_3=3.059167-\frac{0.003692746}{1.791448415}$
$=3.057106$

$f(3.057106)=0.00004378$
$f'(3.057106)=1.787105253$

§4.7 con'd

(15) con'd

$x_4 = 1.412369957 + (2.12146175 \times 10^{-5})$
$= 1.412391172$

$f(1.412391172) = 4.9449 \times 10^{-10}$
$f'(1.412391172) = -1.883236808$

$x_5 = 1.412391172 + (2.6257452 \times 10^{-10})$
$= 1.412391172$ is same as x_4

$\boxed{\text{1 root is} \approx 1.412391}$

$x_4 = 3.057106 - \frac{0.000004378}{1.787105253}$
$= 3.057104$

$f(3.057104)$
$f'(3.057104)$

$x_5 = 3.057104 - \frac{}{}$
$= 3.057104 \approx x_4$

$\boxed{\text{2nd root is} \approx 3.057104}$

I ran out of time to complete everything on this right side but the left is thourough & hopefully you get the idea