Test #2 Derivatives

#12 2a, #9 2b $f(x) = 2\sqrt{x}$ $\boxed{f'(x) = 2\cdot\frac{1}{2}\cdot x^{-1/2} = 1/\sqrt{x}}$

#13 2a, #6 2b $y = (2x-7)/(3x-2)$ $\boxed{dy/dx = \frac{\overset{6x-4}{2(3x-2)} - \overset{-6x+21}{3(2x-7)}}{(3x-2)^2} = \frac{17}{(3x-2)^2}}$

#14 2a, #7 2b $f(x) = \sin^2 x$ $\boxed{df/dx = 2\sin x\cos x}$

#15 2a, #8 2b $y = \frac{2x-(x^2+1)^7}{3} = \frac{2}{3}x - \frac{1}{3}(x^2+1)^7$ $\boxed{\frac{d}{dx} = \frac{2}{3} - \frac{1}{3}\cdot 7(x^2+1)^6\cdot 2x = \frac{2}{3} - \frac{14x}{3}(x^2+1)^6}$ $[2-14x(x^2+1)^6]$ $= \frac{2[1-(7x)(x^2+1)^6]}{3}$

#16 2a $f(x) = \frac{-2}{3x^2} = \frac{-2}{3}x^{-2}$ $\boxed{f' = \frac{-2}{3}\cdot -2x^{-3} = \frac{4}{3x^3}}$

#3 2b $f(x) = \frac{-3}{2x^2} = \frac{-3}{2}x^{-2}$ $\boxed{f' = \frac{-3}{2}\cdot -2\cdot x^{-3} = \frac{3}{x^3}}$

#19 2a, #2 2b $y = \sqrt{2x^2+1}$ $y' = \frac{1}{2}(2x^2+1)^{-1/2}\cdot 4x = \frac{2x}{\sqrt{2x^2+1}}$

#20 2a, #5 2b (a) (b) $s(t) = \frac{1}{\sqrt[3]{t}} = t^{-1/3}$ $\boxed{v(t) = -\frac{1}{3}t^{-4/3} = \frac{-1}{3t^{4/3}} = \frac{-1}{3t\sqrt[3]{t}}}$

#20 2a, #5 2b (b) (d) $w = 7t^2 - 19\sqrt{t} + 23$ $dw/dt = 14t - \frac{19}{2}t^{-1/2}$ $\boxed{d^2w/dt^2 = 14 + \frac{19}{4}t^{-3/2} = 14 + \frac{19}{4t^{3/2}}}$ $= 14 + \frac{19}{4t\sqrt{t}}$

#20 2a, #5 2b (c) (f) $D_x F(x) = \frac{\tan x}{x^3}$ $\boxed{D_x F(x) = \frac{x^3\sec^2 x - 3x^2\tan x}{(x^3)^2} = \frac{x^2(x\sec^2 x - 3\tan x)}{x^6} = \frac{x\sec^2 x - 3\tan x}{x^4}}$

#20 2a, #5 2b (d) (g) $y = \frac{e^{(3-2x)}}{3} = \frac{1}{3}e^{(3-2x)}$ $\boxed{\frac{dy}{dx} = \frac{1}{3}e^{3-2x}\cdot -2 = \frac{-2e^{(3-2x)}}{3}}$

#20 2a, #5 2b (e) (h) $f(x) = 2\ln x^e$ $\boxed{d/dx\, f(x) = \frac{2}{x^e}\cdot e x^{e-1} = \frac{2e\cancel{x^e}x^{-1}}{\cancel{x^e}} = \frac{2e}{x}}$

#20 2a, #5 2b (f) (c) $g(x) = (x^3+1)(3x^2-1) = 3x^5 - x^3 + 3x^2 - 1$ $\boxed{g'(x) = 15x^4 - 3x^2 + 6x}$

#20 2a, #5 2b (g) (i) except it is int $f(x) = x^3\ln x$ $\boxed{\frac{d}{dx} = 3x^2\ln x + \frac{x^3}{x} = 3x^2\ln x + x^2 = x^2(3\ln x + 1)}$

#20 2a, #5 2b (h) $f(x) = 3^4 = 81$ $\boxed{f'(x) = 0}$

(a) $f(x) = 7^3 = 343$ $f'(x) = 0$

Quiz #7

(1a) Show $\frac{d}{dx}\cot x = -\csc^2 x$

Step1: Identity $\frac{d}{dx}\frac{\cos x}{\sin x}$ Step2: Quotient Rule $= \frac{-\sin x \sin x - \cos x \cos x}{\sin^2 x}$

Step3: Algebra $= \frac{-(\sin^2 x + \cos^2 x)}{\sin^2 x}$ Step4: Identity $\frac{-1}{\sin^2 x}$

Step5: Identity $= \boxed{-\csc^2 x}$

(1b) Show $\frac{d}{dx}\sec x = \sec x \tan x$

Step1: Identity $\frac{d}{dx}\frac{1}{\cos x} = (\cos x)^{-1}$ Step2: Power Rule & Chain Rule $= -1(\cos x)^{-2} \cdot -\sin x$

Step3: Algebra $= \frac{\sin x}{\cos^2 x} = \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x}$ Step4: Identities $= \boxed{\tan x \cdot \sec x}$

(2a) Show $\frac{d}{dx}\csc x = -\csc x \cot x$

Step1: Identity $\frac{d}{dx}\frac{1}{\sin x} = (\sin x)^{-1}$ Step2: Quotient Rule $= \frac{0(\sin x) - 1 \cdot \cos x}{\sin^2 x}$

Step3: Algebra $= -\frac{\cos x}{\sin x} \cdot \frac{1}{\sin x}$ Step4: Identity $= \boxed{-\cot x \csc x}$

(2b) Show $\frac{d}{dx}\cot x = -\csc^2 x$ See (1a)

(3a) a) $f(x) = \frac{(2x^2+5x-9)^{-2/3}}{3} = \frac{1}{3}(2x^2+5x-9)^{-2/3}$ $f'(x) = \frac{1}{3} \cdot \frac{-2}{3} \cdot (2x^2+5x-9)^{-5/3} \cdot (4x+5)$ $= \boxed{\frac{-2(4x+5)}{9(2x^2+5x-9)^{5/3}}}$

(3a) b) $y = \log_7 x$ $\boxed{y' = \frac{1}{x\ln 7}}$ (3a) c) $g(z) = \cos^{-1} z$ $\boxed{\frac{dg}{dz} = \frac{-1}{\sqrt{1-z^2}}}$

(3a) d) $F(x) = 3^x$ $\boxed{F'(x) = 3^x \ln 3}$ (3a) e) $h(t) = \ln(\sin t)$ $\boxed{h'(t) = \frac{1}{\sin t} \cdot \cos t = \cot t}$

(3b) a) $f(x) = \frac{(3x^2+2x-9)^{-4/5}}{5} = \frac{1}{5}(3x^2+2x-9)^{-4/5}$ $f'(x) = \frac{1}{5} \cdot \frac{-4}{5} \cdot (3x^2+2x-9)^{-9/5} \cdot (6x+2)$ $= \boxed{\frac{-4(6x+2)}{25(3x^2+2x-9)^{9/5}}} = \frac{-8(3x+1)}{25(3x^2+2x-9)^{9/5}}$

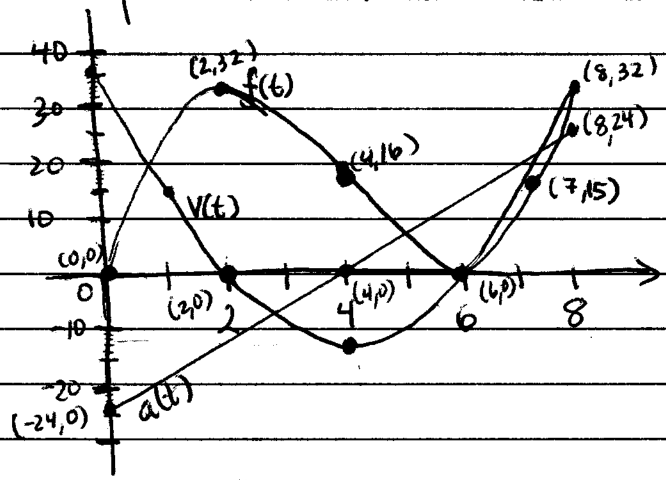
b) $F(x) = 7^x$ $\boxed{F'(x) = 7^x \ln 7}$

c) $g(z) = \sin^{-1} z$ $\boxed{\frac{dg}{dz} = \frac{1}{\sqrt{1-z^2}}}$ d) $y = \log_3 x$ $\boxed{y' = \frac{1}{x\ln 3}}$

e) $h(t) = \ln(\cos t)$ $h'(t) = \frac{1}{\cos t} \cdot -\sin t = \boxed{-\tan t}$

§3.8 p.237 Collab #6

h)



$f''(t) = 6t - 24 = 0 \Rightarrow t = 4$

i) Particle speeds up when v(t) is positive & increasing (a(t) is positive)
v(t) is negative & decreasing (a(t) is negative)

(2,4) v(t) negative & decreasing (a(t) neg.)

(6,8) v(t) positive & increasing (a(t) positive)

Particle slows down when signs of a(t) & v(t) are opposite

[0,2) v(t) positive & a(t) negative

(4,6) v(t) negative & a(t) positive

⑦ The position f(n) of a particle is given by

$s = t^3 - 4.5t^2 - 7t, \quad t \geq 0$

$s' = 3t^2 - 9t - 7$

a) When does particle reach velocity of 5 m/s

$3t^2 - 9t - 7 = 5 \Rightarrow 3t^2 - 9t - 12 = 0 \Rightarrow 3(t^2 - 3t - 4) = 0$
$\quad -5 \quad -5$

$\Rightarrow 3(t-4)(t+1) = 0$

$\boxed{t = 4 \text{ sec}}$

$t = 4$ or $t = -1$
extraneous

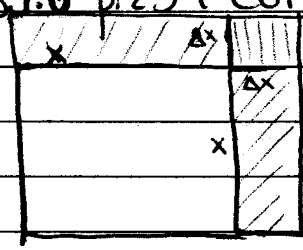
b) When is acceleration 0?

$s'' = 6t - 9 = 0 \Rightarrow 6t = 9 \Rightarrow \boxed{t = 3/2 \text{ sec}}$

The significance?

This is an inflection point of position. It is the point where it reaches its minimum (absolute) velocity and velocity increases

§3.8 p.239 con'd

(11) 

ⓐ $A(x) = x^2 \quad \therefore \quad A'(x) = 2x$

$A'(15) = 2(15) = 30\, mm^2/mm$ The area changes by $30\, mm^2$ wrt. to side length, when the side length is 15mm.

ⓑ The $P = 4x$ & $\frac{1}{2} \cdot 4x = 2x \quad \therefore \quad A'(x) = \frac{1}{2} \cdot P(x)$

The area of the figure above changes from $A(x)$ by the increase of the shaded regions shown as x changes by Δx.

Thus $\underbrace{x^2}_{\text{old}} + \overbrace{2(x \cdot \Delta x) + (\Delta x)^2}^{\text{increase} = \Delta A}$

another way of looking at it is $A(x) = x^2$ is old area

or $A_n(x) = (x + \Delta x)^2$ is new area

$\Delta A = \text{new} - \text{old}$

$= [x^2 + 2x\Delta x + (\Delta x)^2] - x^2 = 2x\Delta x + (\Delta x)^2$

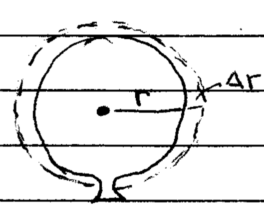
$\therefore$ either way

$\Delta A = 2x\Delta x + (\Delta x)^2$

and if Δx is very small $(\Delta x)^2 \approx 0$

so

$\Delta A \approx 2x\,\Delta x$ & solving $\frac{\Delta A}{\Delta x} \approx 2x$

(15) 

$A = 4\pi r^2 \qquad \frac{\Delta A}{\Delta r} = 8\pi r$

a) when $r = 1ft \qquad \frac{\Delta A}{\Delta r} = 8\pi(1) = 8\pi\ ft^2/ft$

b) when $r = 2ft \qquad \frac{\Delta A}{\Delta r} = 8\pi(2) = 16\pi\ ft^2/ft$

c) when $r = 3ft \qquad \frac{\Delta A}{\Delta r} = 8\pi(3) = 24\pi\ ft^2/ft$

Notice that the change in Area wrt radius is a linear function, therefore for every foot increase in radius the Area changes (increases) by $8\ ft^2$.

Collab #6 §3.8 p. 239 cont'd

(23) A bacteria population triples every hour and begins with 400 bacteria.

$$P(t) = n_0 r^t$$

$\therefore$ since $\frac{P(t)}{n_0} = 3$ The ratio of new population to the old is 3 when $t=1$

$3 = r^1 \Rightarrow r = 3$

so $P(t) = 400\, 3^t$

thus

$$P'(t) = 400\, 3^t \ln 3$$

and therefore the rate of change (growth) in the population after 2.5 hours is

$$P'(2.5) = 400\, 3^{2.5} \ln 3 \approx 6850.268286 \approx \boxed{6850 \frac{\text{bateria}}{\text{hr}}}$$

Like Collaborative #7

1) For $f(x) = x^3 - 6x^2 + 9x + 2$ on $[-1, 4]$

a) Find the 1st derivative $\quad f'(x) = 3x^2 - 12x + 9$

b) Find the 2nd derivative $\quad f''(x) = 6x - 12$

c) Find the critical pts. $\quad f'(x) = 3x^2 - 12x + 9 = 0$

$= 3(x^2 - 4x + 3) = 0 \Rightarrow 3(x-3)(x-1) = 0$

$\therefore \boxed{C.V. = 3, 1}$ i.e. since for $f'(x) \Rightarrow x \in \mathbb{R}$ no others

d) Find inflection pts. $\quad f''(x) = 6x - 12 = 0$

$\Rightarrow 6x = 12 \Rightarrow \boxed{x = 2}$

e) $f(-1) = (-1)^3 - 6(-1)^2 + 9(-1) + 2 = -1 - 6 - 9 + 2 = -14$ ← Global Min

$f(1) = (1)^3 - 6(1)^2 + 9(1) + 2 = 1 - 6 + 9 + 2 = 6$ ← local max/Global max

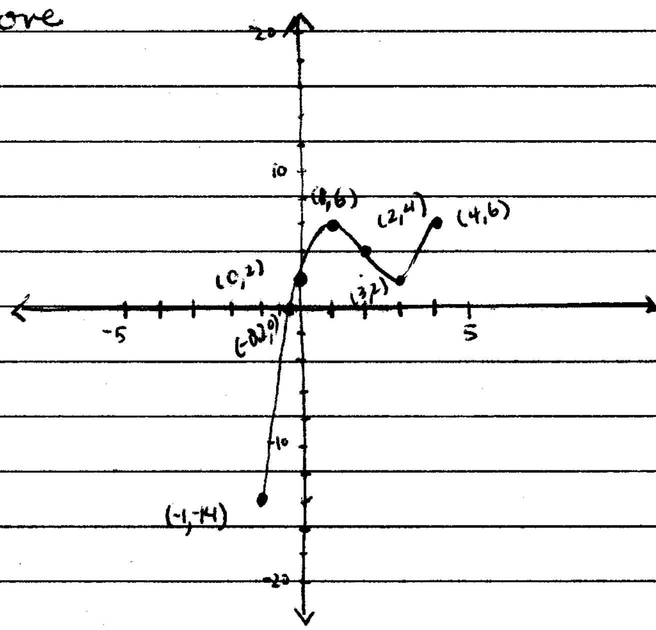
$f(2) = (2)^3 - 6(2)^2 + 9(2) + 2 = 8 - 24 + 18 + 2 = 4$

$f(3) = (3)^3 - 6(3)^2 + 9(3) + 2 = 27 - 54 + 27 + 2 = 2$ ← local/global min

$f(4) = (4)^3 - 6(4)^2 + 9(4) + 2 = 64 - 96 + 36 + 2 = 6$ ← Global Max

f) See above

g) Sketch



y-int of $f(x) = 2$

x int of $f(x) \doteq -0.1958$

Like Collab #7 cond

(2) $f(x) = e^{\tan^{-1} x}$

a) i) $\lim\limits_{x \to -\infty} f(x) = \lim\limits_{t \to -\pi/2} e^{t} = e^{-\pi/2} \approx 0.21$

as $x \to -\infty$ $\tan^{-1} x \to -\pi/2$

ii) $\lim\limits_{x \to \infty} f(x) = \lim\limits_{t \to \pi/2} e^{t} = e^{\pi/2} \approx 4.81$

as $x \to \infty$ $\tan^{-1} x \to \pi/2$

Recall that a horizontal asymptote is the [illegible] a function approaches as $x \to \pm\infty$ so we've just shown the horizontal asymptotes of $f(x)$

Also recall that $\lim\limits_{x \to a} f(x) = \pm\infty$ is a vertical asymptote and there are no such "a's" for $f(x)$

b) $f'(x) = e^{\tan^{-1} x} \cdot \frac{1}{1+x^2}$ $\quad x \in \mathbb{R}$

$f''(x) = e^{\tan^{-1} x} \cdot \frac{1}{(1+x^2)^2} + \frac{-1}{(1+x^2)^2} \cdot 2x\, e^{\tan^{-1} x} = \frac{e^{\tan^{-1} x}(-2x+1)}{(1+x^2)^2}$

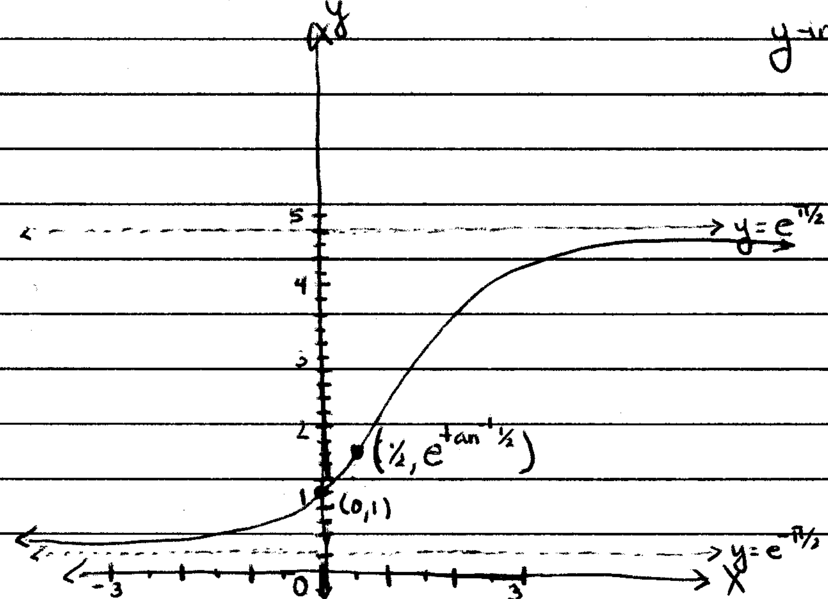
$f''(x) = 0$ only when $(-2x+1) = 0 \therefore x = \tfrac{1}{2}$

since $e^{t} \neq 0$ & $(1+x^2) \neq 0$ $\quad x, t \in \mathbb{R}$

$\boxed{\text{Inflection Pt } \left(\tfrac{1}{2}, e^{\tan^{-1} \frac{1}{2}}\right)}$ $\quad$ Note: $e^{\tan^{-1} \frac{1}{2}} \approx 1.6$

y-int $= e^{\tan^{-1} 0} = 1$

(c) Graph



§3.7 p. 226 #47

Show $\lim_{x\to 0} \frac{\ln(1+x)}{x} = 1$ using definition of derivative

Recall $f'(a) = \lim_{x\to a} \frac{f(x)-f(a)}{x-a}$ where $a=0$ & $f(x)=\ln(1+x)$

My Way

$\therefore$

$$f'(0) = \lim_{x\to 0} \frac{\ln(1+x) - \ln(1+0)}{1-0} = \lim_{x\to 0} \frac{\ln(1+x) - 0}{x}$$

$$= \lim_{x\to 0} \frac{\ln(1+x)}{x} = \lim_{x\to 0} \frac{1}{x}\cdot \ln(1+x)$$

$$= \lim_{x\to 0} \ln(1+x)^{1/x} = \lim_{t\to e} \ln t = \ln(e) = 1$$

and as $x\to 0$ $(1+x)^{1/x} \to e$

or if we say $f(x) = \ln(1+x)$ then it follows $f(0) = \ln(1+0) = 0$

$f'(x) = \frac{1}{1+x}$ & $f'(0) = \frac{1}{1+(0)} = 1$

Books Way

and by definition $f'(a) = \lim_{x\to a} \frac{f(x)-f(a)}{x-a}$ and we say $a=0$

then $f'(0) = \lim_{x\to 0} \frac{f(x) - \overset{\to 0}{f(0)}}{x-0} = \lim_{x\to 0} \frac{f(x)}{x} = \lim_{x\to 0} \frac{\ln(1+x)}{x} = 1$

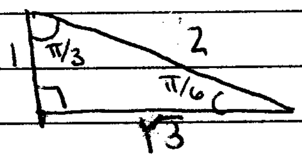
by substitution from follows above & simplifying

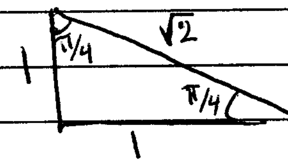
by subst.

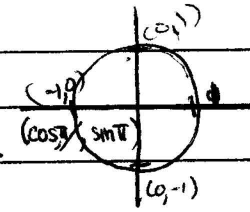
by subst of above

§3.6 p.220 Suggested HW

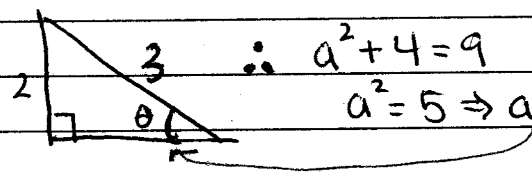
① a) $\sin^{-1}(\sqrt{3}/2) = \pi/3$ b) $\cos^{-1}(-1) = \pi$







⑤ $\tan\left(\sin^{-1}\left(\frac{2}{3}\right)\right) = 2/\sqrt{5} = 2\sqrt{5}/5$



$\therefore\ a^2 + 4 = 9$

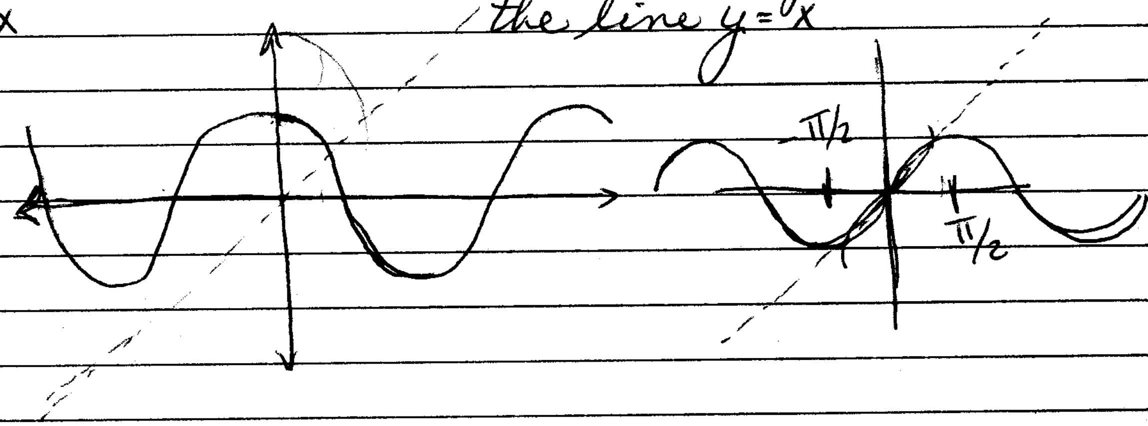
$a^2 = 5 \Rightarrow a = \sqrt{5}$

⑬ $y = \sin x \quad (-\pi/2 \le x \le \pi/2)$

$y = \sin^{-1} x$

$y = x$

These are inverse functions which are symmetric across the line $y = x$



⑮ Prove Formula 2 by the same method as Formula 1

Prove $\frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}} \qquad -1 < x < 1$

$y = \cos^{-1} x \Rightarrow \cos y = x \qquad -\sin y\, y' = 1 \Rightarrow y' = \frac{1}{-\sin y}$

By implicit differentiation

$y' = -\frac{1}{\sqrt{1-\cos^2 y}} = -\frac{1}{\sqrt{1-x^2}}$

by substitution of original

$\sin^2 y + \cos^2 y = 1$

$\sin^2 y = 1 - \cos^2 y$

$\sin y = \sqrt{1-\cos^2 y}$

§3.6 p.220 Suggested HW con'd

(17) $y = (\tan^{-1} x)^2$ $\qquad$ $y' = \dfrac{2\tan^{-1} x}{1+x^2}$

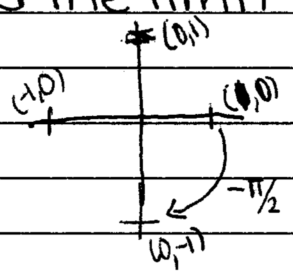
(28) $y = \arctan \sqrt{\dfrac{1-x}{1+x}}$

$$y' = \frac{1}{1+\left(\sqrt{\frac{1-x}{1+x}}\right)^2} \cdot \frac{1}{2}\left(\frac{1+x}{1-x}\right)^{1/2}\left(\frac{\overset{-2}{-1(1+x)-1(1-x)}}{(1+x)^2}\right)$$

$$= \frac{1}{\frac{1+x+1-x}{\cancel{1+x}}} \cdot \frac{1}{\cancel{2}}\left(\frac{1+x}{1-x}\right)^{1/2}\left(\frac{\cancel{-2}}{(1+x)^2}\right) = \frac{(\cancel{1+x})}{2} \cdot \frac{-1}{(1+x)^{\cancel{2}}} \cdot \frac{(1+x)^{1/2}}{(1-x)^{1/2}}$$

$$= \frac{-1}{2\sqrt{(1+x)(1-x)}} = \frac{-1}{2\sqrt{1-x^2}}$$

(37) Find the limit $\displaystyle\lim_{x \to -1^+} \sin^{-1} x = \sin^{-1}(-1) = \frac{3\pi}{2} \text{ or } -\frac{\pi}{2}$



not since $\sin^{-1} x$

$-\frac{\pi}{2} \le x \le \frac{\pi}{2}$

§3.7 Suggested HW p. 226

(2) $f(x) = x\ln x - x$ $\qquad f'(x) = \overset{=1}{\frac{x}{x}} + \ln x - 1 = \boxed{\ln x}$

(3) $f(x) = \sin(\ln x)$ $\qquad f'(x) = \boxed{\dfrac{\cos(\ln x)}{x}}$

Incorporate neg into denom

(5) $f(x) = \log_2(1-3x)$ $\qquad f'(x) = \boxed{\dfrac{-3}{(1-3x)\ln 2} = \dfrac{3}{(3x-1)\ln 2}}$

(11) $F(t) = \ln \dfrac{(2t+1)^3}{(3t-1)^4}$ $\qquad F'(t) = \dfrac{(3t-1)^4}{(2t+1)^3} \cdot \dfrac{3(2t+1)^2 \cdot 2 \cdot (3t-1)^4 - 4(3t-1)^3 \cdot 3(2t+1)^3}{(3t-1)^8}$

$$= \frac{6\cancel{(3t-1)^4}\cancel{(3t-1)^3}\cancel{(2t+1)^2}\left[(3t-1) - \overset{-4t-2}{2(2t+1)}\right]}{\cancel{(2t+1)^3}\,(3t-1)^{\cancel{8}}} \qquad 6t^2+t-1$$

$$= \boxed{\frac{-6(t+3)}{(2t+1)(3t-1)}}$$

incorporate into denom.

(15) $y = \ln[2 - x - 5x^2]$ $\qquad y' = \dfrac{1}{2-x-5x^2} \cdot \overset{-(10x+1)}{(-1-10x)}$

$$= \boxed{\frac{10x+1}{5x^2+x-2}}$$

(19) $y = 2x\log_{10}(\sqrt{x})$ $\qquad y' = 2\log_{10}(\sqrt{x}) + \dfrac{\cancel{2x}}{\sqrt{x}\ln 10} \cdot \dfrac{1}{\cancel{2\sqrt{x}}}$

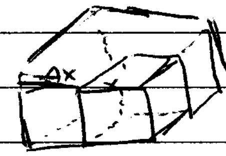
$= 2x\log_{10} x^{1/2}$ $\qquad = 2\log_{10}\sqrt{x} + \dfrac{1}{\ln 10} = \dfrac{1}{2} \cdot 2\log_{10} x + \dfrac{1}{\ln 10}$

$= 2 \cdot \frac{1}{2} x \log_{10} x$ $\qquad = \log_{10} x + \dfrac{1}{\ln 10}$

$= x\log_{10} x$ $\qquad y' = 1 \cdot \log_{10} x + \dfrac{x}{x\ln 10} = \boxed{\log_{10} x + \dfrac{1}{\ln 10}}$

§4.1 Related Rates p. 260

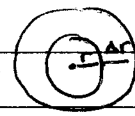
*① V of Cube $\Rightarrow$ $V = x^3$



$$\boxed{\frac{dV}{dt} = 3x^2\frac{dx}{dt}} \Rightarrow \frac{dx}{dt} = \frac{1}{3x^3}\frac{dV}{dt}$$

Example

② ⓐ $A = \pi r^2$

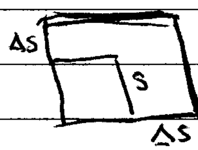


$$\boxed{\frac{dA}{dt} = 2r\pi\frac{dr}{dt}} \Rightarrow \frac{dr}{dt} = \frac{1}{2\pi r}\frac{dA}{dt}$$

ⓑ $\frac{dr}{dt} = 1\ m/s$ when $r = 30m$ $\quad \frac{dA}{dt} = 2(30m)\pi \cdot 1\ m/s$

$$\frac{dA}{dt} = \boxed{60\pi\ m^2/s}$$

*③ $A = s^2$



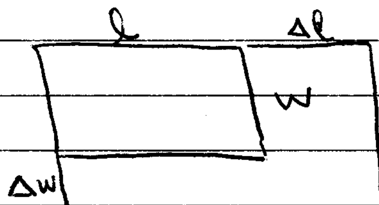
$$\frac{dA}{dt} = 2s\frac{ds}{dt} \Rightarrow \frac{dA}{dt} = 2(6\tfrac{cm}{s})(4cm)$$

$S = \sqrt{A} = \sqrt{16} = 4$ $\qquad = \boxed{48\ cm^2/s}$

They try

④ $A = l \cdot w$

$W =$

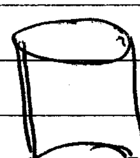


$$\frac{dA}{dt} = w\frac{dl}{dt} + l\frac{dw}{dt}$$

$\frac{dl}{dt} = 8\frac{cm}{s}$ $\quad \frac{dw}{dt} = 3\frac{cm}{s}$ $\quad l = 20cm$, $w = 10cm$ $\quad \frac{dA}{dt} = \overset{80}{10 \cdot 8} + \overset{60}{3 \cdot 20}$

$$= \boxed{140\ cm^2/s}$$

*⑤ $V = \pi r^2 h$



$r = 5m$

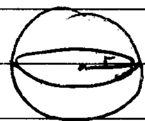
$\frac{dV}{dt} = 3\frac{m^3}{min}$

$$\frac{dV}{dt} = r^2\pi\frac{dh}{dt}$$

$$\Rightarrow 3\frac{m^3}{min} = (5m)^2\pi\frac{dh}{dt} \Rightarrow \frac{3}{25\pi} = \frac{dh}{dt}$$

$$\boxed{\frac{dh}{dt} \approx 0.038197186\ m/min = \frac{3}{25\pi}\frac{m}{min}}$$

§4.1 con'd

(6) 

$V = \frac{4}{3}\pi r^3$ $\qquad \frac{dV}{dt} = \frac{4}{3}\pi \cdot 3r^2 \frac{dr}{dt}$

$\frac{dr}{dt} = 4\frac{mm}{s}$

$d = 80mm \Rightarrow r = 40mm$

$\frac{dV}{dt} = 4\pi(40mm)^2(4\ mm/s)$

$= \boxed{25{,}600\pi \ \frac{mm^3}{s}}$

(7) $y = \sqrt{2x+1}$ $\quad x$ & y are fns of t

(a) If $dx/dt = 3$ find dy/dt when $x = 4$

$\frac{dy}{dt} = \frac{1 \cdot \cancel{2}}{\cancel{2}\sqrt{2x+1}} \frac{dx}{dt} \Rightarrow \frac{dy}{dt} = \frac{1}{\sqrt{2(4)+1}} \cdot 3 = \boxed{1 \text{ unit/time}}$

(b) If $dy/dt = 5$ find dx/dt when $x = 12$

$5 = \frac{1}{\sqrt{2(12)+1}} \cdot \frac{dx}{dt} \Rightarrow \frac{dx}{dt} = \boxed{25}$

(8) If $x^2 + y^2 = 25$ and $dy/dt = 6$ find dx/dt when $y = 4$

Solve for x

$2x\frac{dx}{2t} + 2y\frac{dy}{dt} = 0 \Rightarrow 2x\frac{dx}{dt} + 2(4)\cdot 6 = 0$

Use as example

$\Rightarrow 2x\frac{dx}{dt} = -48 \Rightarrow \frac{dx}{dt} = \frac{-48}{2\sqrt{25-y^2}} = \frac{-48}{\pm 2 \cdot 3} = \boxed{\pm 8}$

$y = 4 \Rightarrow 16$

$25 - 16 = 9$

*(9) $z^2 = x^2 + y^2$ $\quad dx/dt = 2$ $\quad dy/dt = 3$ $\quad dz/dt = ?$

$x = 5$ & $y = 12$

They try...

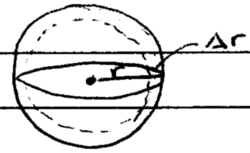
$2z\frac{dz}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt} \Rightarrow 2z\frac{dz}{dt} = \overset{20}{2(5)(2)} + \overset{72}{2(12)(3)}$

42

4.1

§4.1 TBA Cond

11



$\frac{\Delta A_s}{\Delta t} = -1 \frac{cm^2}{min}$ when $d = 10\,cm$

$A = 4\pi r^2 \Rightarrow \frac{dA}{dt} = 4\pi \cdot \cancel{2}\left(\frac{d}{\cancel{2}}\right)\cdot\frac{1}{2}\frac{dd}{dt}$

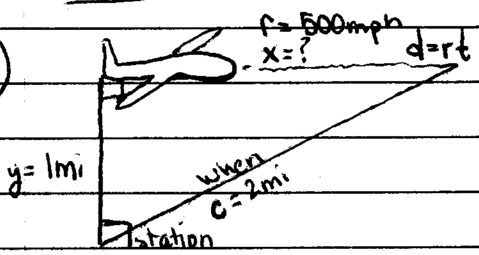
$r = \frac{d}{2}$

$-1 = 2\pi d \frac{dd}{dt} \Rightarrow \frac{1}{20\pi} = \frac{dd}{dt}$

$d = 10$

$\boxed{-\frac{1}{20\pi}\frac{cm}{min}}$ or $\boxed{\text{decreasing } \frac{1}{20\pi}\frac{cm}{min}}$

13



therefore $r = \frac{dx}{dt} = 500\,mi/hr$

when $y=1$ & $c=2$ $\quad x^2+y^2=c^2$

$X = \sqrt{4-1} = \sqrt{3}$

$c^2 = 1 + x^2$

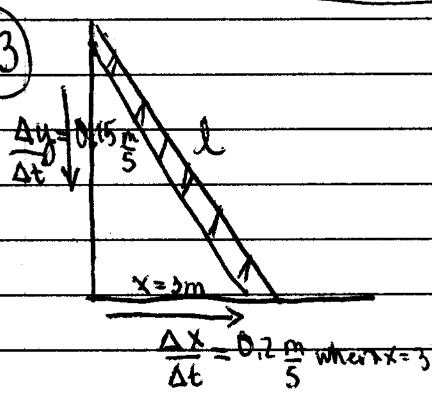
$2c\frac{dc}{dt} = 0 + 2x\frac{dx}{dt} \Rightarrow \frac{dc}{dt} = \frac{x}{c}\frac{dx}{dt}$

$\frac{dc}{dt} = \frac{x}{2}\cdot 500$

$= \frac{\sqrt{3}}{2}\cdot 500 \frac{mi}{hr}$

$\boxed{\frac{dc}{dt} = 250\sqrt{3}\ mi/hr} \approx 433\,mi/hr$

23



$l^2 = y^2 + x^2$ but l is constant

$0 = 2y\frac{dy}{dt} + 2x\frac{dx}{dt}$ $\quad 2y\frac{dy}{dt} = -2x\frac{dx}{dt}$

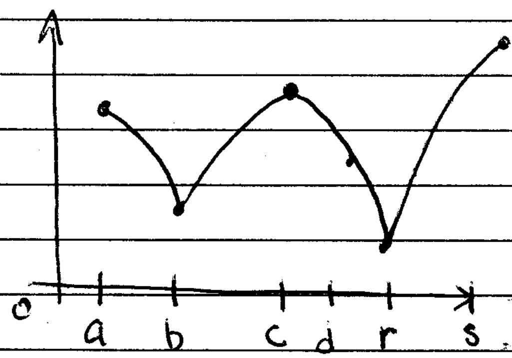
$2y(-0.15) = -2(0.2)(3)$

$y = \frac{-1.2}{-0.3} = 4$

$\therefore\ l = \sqrt{4^2+3^2} = \sqrt{25} = \boxed{5m}$

§4.2 p. 268

(3)



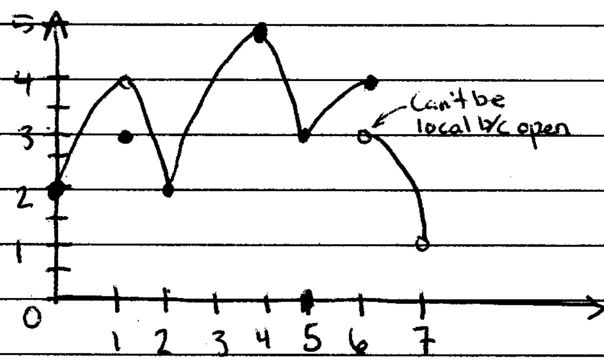
absolute max s

local max c

local min b

absolute min r

(5)



absolute max $f(4) = 5$

local max $f(4)=5$ & $f(6)=4$

local min $f(2)=2$, $f(1)=f(5)=3$

absolute min none

(25) $f(x) = x^3 + 3x^2 - 24x$

$f'(x) = 3x^2 + 6x - 24 = 3(x^2 + 2x - 8) = 3(x+4)(x-2)$

$f'(x) = 3(x+4)(x-2) = 0 \Rightarrow \therefore\ x = -4$ & 2

and since $f'(x)$ is defined on $\mathbb{R}$

$\boxed{CV\ \ x = -4 \ \&\ x = 2}$

(41) $f(x) = 12 + 4x - x^2$ on $[0, 5]$

$f'(x) = 4 - 2x$ and since $f'(x)$ is defined on $\mathbb{R}$ no asymptotes

$f'(x) = 4 - 2x = 0 \Rightarrow x = 2$ is a CV.

since $f'(1) = 4 - 2(1) = 2$

& $f'(3) = 4 - 2(3) = -2$

$\nearrow \cdot \searrow$ ← $x=2$ means $x=2$ is max

$f(0) = 12 + 4(0) - (0)^2 = 12$

$f(2) = 12 + 4(2) - (2)^2 = 12 + 8 - 4 = 16$

$f(5) = 12 + 4(5) - (5)^2 = 12 + 20 - 25 = 7$

thus Absolute Max $f(2) = 16$

Absolute Min $f(5) = 7$