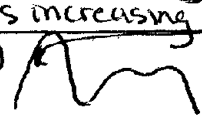
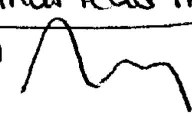
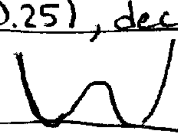
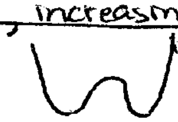
Remainder of Problems from Midterm #2a & #2b

Recall that I put all problems w/ derivatives in HW solutions before your 3rd midterm.

#1a & #16b
$f'(x) < 0$ means decreasing fcn
$f'(x) > 0$ means increasing fcn ← can't be any that lead in as decreasing!

on test a) iii)  on test b) i) 

#2a & #15b
increasing $(-2,-1.5)$ since $f' > 0$, zero @ -1.5, decreasing $(-1.5,-0.25)$ $f' < 0$
zero @ $(-0.25, 0.25)$, decreasing $(0.25, 1.5)$ $f' < 0$, increasing $(1.5, 2)$ since $f' > 0$

on test a) iii)  on test b) iv) 

#3a
#12b
$f' > 0$ means an increasing fcn ∴ $\boxed{\text{i) increasing}}$
$f' < 0$ means a decreasing fcn ∴ $\boxed{\text{iii) decreasing}}$

#4a
#13b
$f'' < 0$ means concave down (slopes are decreasing) $\boxed{\text{iv) Concave Down}}$
$f'' > 0$ mean concave up (slopes are increasing $\boxed{\text{i) Concave Up}}$

#5a & #14b
$f'(x) = 0$ says the slope is zero [we also now know that it is a max or min when it crosses the x axis]

on test a) ii) zero slope
on test b) ii) zero slope

#6a & #10b
$C'(x) = 16x + 14$ is marginal cost $C'(7) = 126$ but it is in thousands of dollars

on test a) iv) \$126,000
on test b) iv) \$126,000

#7a & #11b
$P'(3) = 110 \frac{\text{people}}{\text{day}}$ ← P(x) units, ← x units
$P'(x)$ is the rate of change & $x=3$ is Feb 3

on test a) ii) $P'(3) = 110$
on test b) iv) $P'(3) = 110$

#8a & #17b
a) $f(21) = 30$ $f(x)$ = # of computers (in 1000s) x = price per computer (in \$100)

on test a) iii) 30,000 computers sold when price is \$2100
on test b) i) " " " "

b) $f'(21) = -8$ $f'(x)$ = change in price per computer (still in 1000's) when x = price per computer (\$100)

on test a) i) when price is \$2100 for every (unit of change in x) \$100 price increase, the sales will decrease 8,000 computers

on test b) ii)

Key Midterm #2a & 2b con'd

#9a) $y = \frac{\cancel{2x+1}}{(\cancel{2x+1})(x+3)}$ ← $x=-3$ is a potential asymptote. We'd have to find $\lim_{x\to -3^+}$ & $\lim_{x\to -3^-} = \pm\infty$ to show for sure

$\boxed{iii)\ -3}$

#20b) $y = \frac{(x+4)\cancel{(x-4)}}{(x-1)\cancel{(x-4)}}$ $x=1$ is a potential We'd have to show $\lim_{x\to 1^+}$ & $\lim_{x\to 1^-} = \pm\infty$ to show asymptote

$\boxed{vi)\ 1}$

#10a) $f(4) = \sqrt{4} = 2$ $f'(a) = \frac{f(x)-f(a)}{x-a}$ $\boxed{iii)\ \lim_{x\to 4} \frac{\sqrt{x}-2}{x-4}}$

#18b) $f(9) = \sqrt{9} = 3$ $f'(a) = \frac{f(x)-f(a)}{x-a}$ $\boxed{i)\ \lim_{x\to 9} \frac{\sqrt{x}-3}{x-9}}$

#11a & #9b) $f(1) = 1^2 - 2(1) = -1$ $f'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$ $\boxed{v)\text{ on test a) \& b) } \lim_{h\to 0} \frac{(1+h)^2-2(1+h)+1}{h}}$

Part a) is on key in HW Keys from Test #3

#16a & #3b) b) (Test a)
$2x^2+4xh+2h^2 \div x^2+2xh+h^2$

$$\lim_{h\to 0} \frac{\frac{-2}{3(x+h)^2} - \frac{-2}{3x^2}}{h} \xrightarrow{\text{added}} \lim_{h\to 0} \frac{-2x^2+2(x+h)^2}{3x^2h(x+h)^2} = \lim_{h\to 0} \frac{4xh+2h^2}{3hx^2(x+h)^2} = \lim_{h\to 0} \frac{\cancel{h}(4x+2h)}{3\cancel{h}x^2(x+h)^2}$$

$$= \lim_{h\to 0} \frac{4x+2h}{3x^2(x+h)^2} = \frac{4x}{3x^4} = \frac{4}{3x^3}$$

$3x^2+6xh+3h^2+x^2+2xh+h^2$ $h(6x+3h)$

b) (Test b)

$$\lim_{h\to 0} \frac{\frac{-3}{2(x+h)^2} - \frac{-3}{2x^2}}{h} \xrightarrow{\text{added}} \lim_{h\to 0} \frac{-3x^2+3(x+h)^2}{2x^2h(x+h)^2} = \lim_{h\to 0} \frac{6xh+3h^2}{2x^2h(x+h)^2} = \lim_{h\to 0} \frac{6x+3h}{2x^2(x+h)^2} = \frac{6x}{2x^4} = \boxed{\frac{3}{x^3}}$$

#17a & #1b) $x=-3$ There's a corner, $x=-1$ There's a jump, $x=1$ There's infinite discont., $x=2$ $\lim_{x\to 2} f(x) \neq f(2)$

$x=4$ There's a corner.

#18a) & #4b) Draw the lines tangent to the curve a multiple places & use $m = \frac{\text{rise}}{\text{run}}$ to find their approx. slopes which are the derivatives of the function.

$m_{-4} \approx -6$, $m_{\approx -3} = 0$, $m_{-2} \approx 3$, $m_{-1} \approx 2$, $m_0 = 0$, $m_1 \approx -1$, $m_{2.25} = 0$, $m_4 = 7$

Plot these pts on a scaled coordinate system & draw a smooth curve of $f'(x)$