Keys Midterm 1a & 1b

#1a & #10b Finish the table w/ ave accelerations

t	0	1	2	3	4	5	6
ave accel	$\frac{18-0}{1-0}=18$	$\frac{47-18}{2-1}=29$	$\frac{77-47}{3-2}=30$	$\frac{104-77}{4-3}=27$	$\frac{132-104}{5-4}=28$	$\frac{163-132}{6-5}=31$	

6 to 7 $\frac{184-163}{7-6}=21$ 7 to 8 $\frac{205-184}{8-7}=24$

ⓐ Ave Rate of Accel. is $\boxed{\text{greatest on } [5,6]}$ Ave $= \frac{v(6)-v(5)}{6-5} = \frac{163-132}{6-5} = \boxed{31\ \text{ft/s}^2}$

ⓑ Instantaneous is best found by ave. above & below ave. so $[3,4]$ & $[4,5]$ for $t=4$

$[3,4] \Rightarrow \frac{104-77}{4-3} = 27$ & $[4,5] \Rightarrow \frac{132-104}{5-4} = 28$ $\therefore$ Instant. Accel @ $t=4 \approx \frac{27+28}{2} = \boxed{27.5\ \frac{ft}{s^2}}$

#2a & #13b Use an appropriate table of values to determine the indicated limits

ⓐ $\lim_{x\to -3^+} f(x)$

x	f(x)
-2.9	-79
-2.99	-799
-2.999	-7999
-2.9999	-79,999

ⓑ $\lim_{x\to -3^-}$

x	f(x)
-3.1	81
-3.01	801
-3.001	8001
-3.0001	80,001

ⓒ $\lim_{x\to -3} f(x)$ does not exist b/c $\lim_{x\to -3^-} \neq \lim_{x\to -3^+}$

#3a & #9b ⓐ Values from left $x = 2, 1.5, 1.1, 1.01, 1.001$ values from right $x = 0, 0.5, 0.9, 0.99, 0.999$

$m_{PQ} = \frac{f(1.001)-f(1)}{1.001-1} = \frac{1.7314731-\sqrt{3}}{0.001} \doteq -0.577708$ w/ round off could get -0.578

or

$m_{PQ} = \frac{f(1)-f(0.999)}{1-0.999} = \frac{\sqrt{3}-1.7326278}{0.001} \doteq -0.576992$ w/ round off could get -0.577

ⓑ This could be $m_{PQ_{1.001}}$ or $m_{PQ_{0.999}}$ or better still an ave.

$m_{ave} = \frac{m_{PQ_{1.001}} + m_{PQ_{0.999}}}{2} = \frac{-0.577708 + -0.576992}{2} \doteq -0.577350 \approx \boxed{-0.58}$

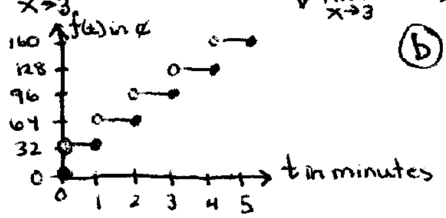
ⓒ $y-\sqrt{3} = -0.58(x-1) \Rightarrow y-\sqrt{3} = -0.58x + 0.58$
$\quad +\sqrt{3} = \quad + 1.73$
$\boxed{y = -0.58x + 2.31}$

#4a & #12b ⓐ $\lim_{x\to 3} x^2 f(x) = (\lim_{x\to 3} x^2)(\lim_{x\to 3} f(x)) = (3^2)(5) = 9\cdot 5 = \boxed{45}$

ⓑ $\lim_{x\to 3} \frac{2h(x)}{f(x)-h(x)} = \frac{2\lim_{x\to 3} h(x)}{\lim_{x\to 3} f(x) - \lim_{x\to 3} h(x)} = \frac{2(-8)}{5-(-8)} = \boxed{\frac{-16}{13}}$

ⓒ $\lim_{x\to 3} \sqrt[3]{h(x)} = \sqrt[3]{\lim_{x\to 3} h(x)} = \sqrt[3]{-8} = \boxed{-2}$

#5a & #11b ⓐ



ⓑ $\lim_{x\to 2^-} f(t) = 64¢$ $\lim_{t\to 2^+} f(t) = 96¢$

ⓒ A limit dne for any integer. It means that the phone user must watch their min. usage b/c there is a jump in cost for every fraction of usage over an integer.

#6a & #14b Continuous on $[-5,1)$, $[1,3)$ & $(3,5]$

Note which endpts are included & which are not & not that corners don't effect continuity (but they do effect limits).

Key Midterm 1a & 1b cond

#7a & #1b) $P \Rightarrow s(0) = 0^3 - 0 \Rightarrow (0,0)$ & $Q \Rightarrow s(1) = 1^3 = 1 \Rightarrow (1,1)$ m_{PQ} = ave velocity = $\frac{1-0}{1-0} = \frac{1}{1} = 1$ m/s

$\boxed{\text{Answer on a is iv) 1 \& answer on b is viii) 1}}$

#8a & #4b) a & d respectively — From left & right it goes to -1 $\therefore \lim_{x\to -2} f(x) = -1$ — on test a) iii) -1; on test b) vi) -1

b & a respectively — From left is goes to 2 $\therefore \lim_{x\to 3^-} f(x) = 2$ — on test a) vi) 2; on test b) iii) 2

c & b respectively — From right it goes to -2 $\therefore \lim_{x\to 3^+} f(x) = -2$ — on test a) ii) -2; on test b) vi) -2

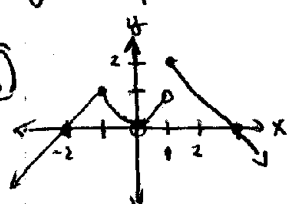
d & c respectively — Since left & right $\neq$ dne $\therefore \lim_{x\to 3} f(x) =$ dne — on test a) viii) dne; on test b) viii) dne

e & e respectively — The dot is on $(-2,-3)$ $\therefore f(-2) = -3$ — on test a) i) -3; on test b) v) -3

f & g respectively — Since $\lim_{x\to -2} f(x) \neq f(-2)$ it's not continuous — on test a) ii) No; on test b) ii) No

g & f respectively — There is jump on the graph — on test a) ii) Jump; on test b) ii) Jump

#9a & #5b)



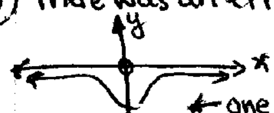
See the graph to the left to answer question of discontinuity for this piecewise defined $f(x)$ — on test a) iii) 1; on test b) v) 1

#10a & #6b) $\lim_{x\to 1} \frac{(2x+3)\cancel{(x-1)}}{x\cancel{(x-1)}} = \frac{2(1)+3}{1} = 5$ by direct substitution — on test a) i) 5; on test b) v) 5

#11a & #2b) Simplify $\frac{1}{x^2} - 1$ to be $\frac{1}{x^2} - \frac{x^2}{x^2} = \frac{1-x^2}{x^2}$ & divide $\frac{1-x^2}{x^2} \div (x-1)$ — $\lim_{x\to 1} \frac{-(x+1)\cancel{(x-1)}}{x^2\cancel{(x-1)}} = \frac{-(1+1)}{1^2} = -2$

getting $\frac{1-x^2}{x^2} \cdot \frac{1}{x-1} = \frac{1-x^2}{x^2(x-1)}$ or rationalize & get same answer after mult top & bottom by x^2 — on test a) ii) -2; on test b) vi) -2

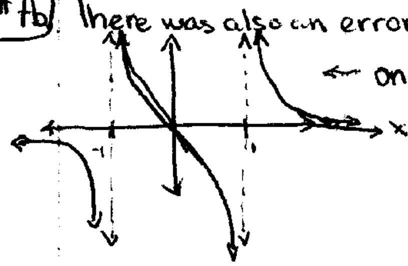
#12a & #8b) There was an error on test b) i) Horiz Asym at $y=0$, ii) Vert Asym at $x=0$ } Should have read



test a) ii) Horizontal asymptote @ $y=0$

test b) Technically it should have been iv) none of the above

#13a & #7b) There was also an error on test b) $x \Rightarrow$ should be y's & vice versa for all parts eg. Horiz Asym $y=1$ etc.

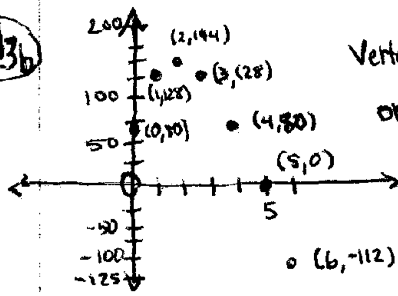


test a) iv) vertical asymptote at $x = \pm 1$

test b) Technically it should have been iv) none of the above

Key for Midterm #1a & #1b con'd

#14a & #3b



Vertex $x = -b/2a = \frac{-64}{2(-16)} = \frac{-64}{-32} = 2$ so $y = -16(4)+64(2)+80 = 144$

or as we know now $f'(x) = 0$ will give the max $f'(x) = -32t+64 = 0 \Rightarrow t = 2$ & $f(2) = 144$

a) The ball can't fall below the ground ∴ (6, -112) is physically impossible $\boxed{\text{on test a) vii) 6 \& test b) vii) 6}}$

b) Vertex of parabola [we also know it to be max] $\boxed{\begin{array}{l}\text{on test a) iii) 2} \\ \text{on test b) iii) 2}\end{array}}$

c) $h(t) = 0$ for both $t = 5$ & $t = -1$ and it's not possible to have a time < 0

∴ $\boxed{\begin{array}{l}\text{on test a) i) -1} \\ \text{on test b) i) -1}\end{array}}$