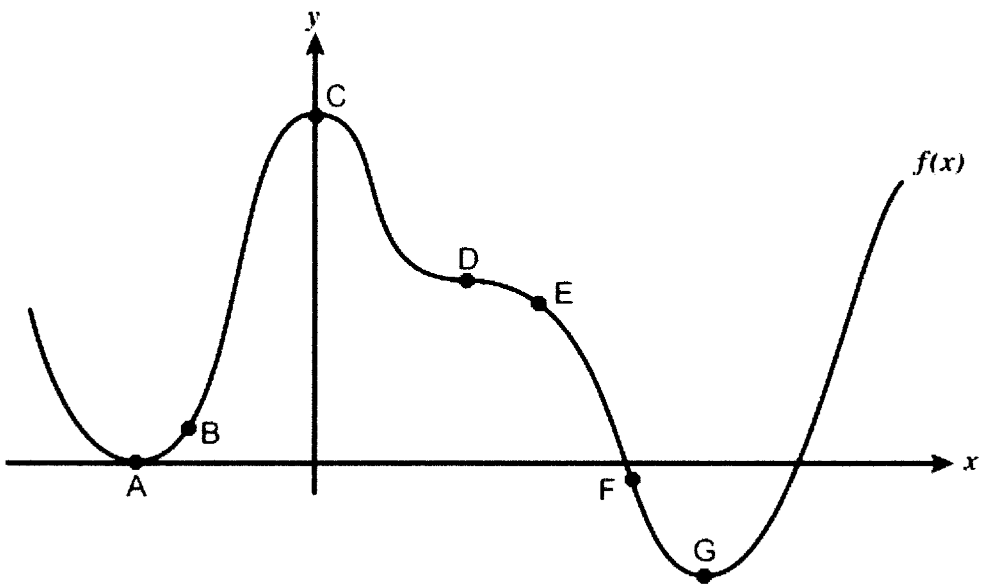
§2.1 Collab #2

Class Activity

Carefully examine the graph shown and answer the questions that follow.



(A) At which of the labeled points is the slope positive?

$\boxed{\text{B only}}$

(B) At which of the labeled points is the slope negative?

$\boxed{\text{E \& F}}$ It could be argued that D is also included here. It is a little difficult to tell.

(C) At which of the labeled points is the slope *approximately* zero?

$\boxed{\text{A, C \& G definitely, D is approx}}$

*(D) At which of the labeled points is the slope the greatest?

Referring to greatest as largest + integer → B

Referring to largest rate of change, absolute value of slope → F

*(E) At which of the labeled points is the slope the smallest?

Referring to smallest as − integer → F

Referring to smallest rate of change, absolute value of the slope → A,C,G

* Having not written these questions myself, I'm not sure what the reference is, but noting on E there are multiple possible answers based on abs. val., I'm assuming face value, smallest/largest integer, was the reference.

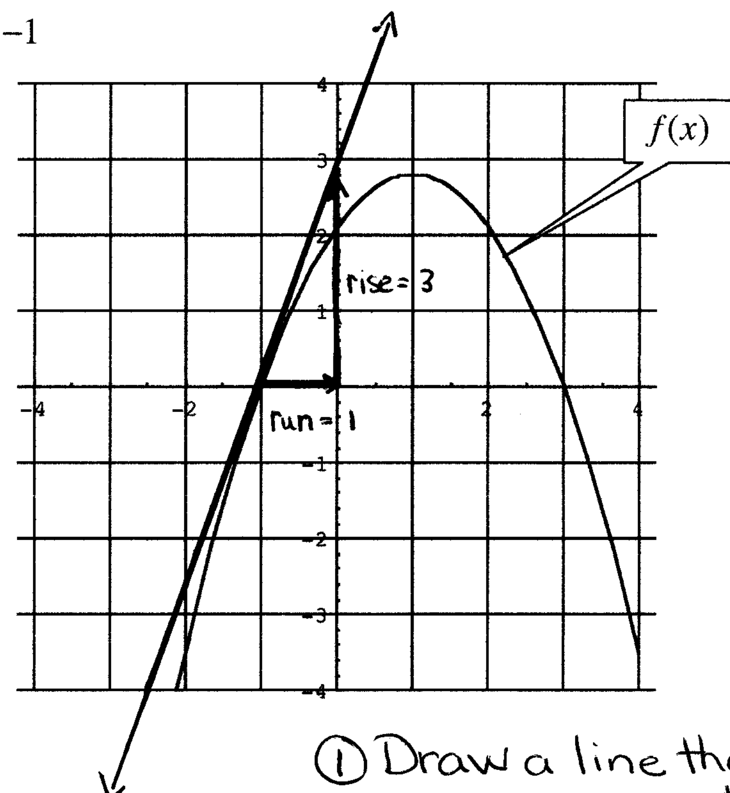
§2.1 Collab#2

Estimating the Slope of a Tangent Line From a Graph

Class Activity

The graph of a function is shown. Use a geometric construction on the graph to determine the slope of the tangent lines at the given points [Hint: use a straight–edge to draw a tangent line at the given point and build a right triangle whose side lengths you can use to estimate the slope.].

(A) At $x=-1$



① Draw a line that touches the curve @ x = -1 & follows the line of the curve as closely as possible for as long as possible.

② Use visual/geometric approach to find the line's slope.

$$\boxed{m_{\text{Tangent}\,x=-1} = \frac{\text{rise}}{\text{run}} = \frac{3}{1} = 3}$$

§2.1 Collab#2

Remark

Suppose that we are interested in the velocity of the tomato at the *instant* $t = 1$ second, i.e., the instantaneous velocity at $t = 1$. The expression for average velocity can only be used over an interval—not at a single point. To find the velocity at a single point in time we will need to use the idea of a limit developed earlier. We explore the concept of *instantaneous velocity* in a simpler setting next.

Class Activity

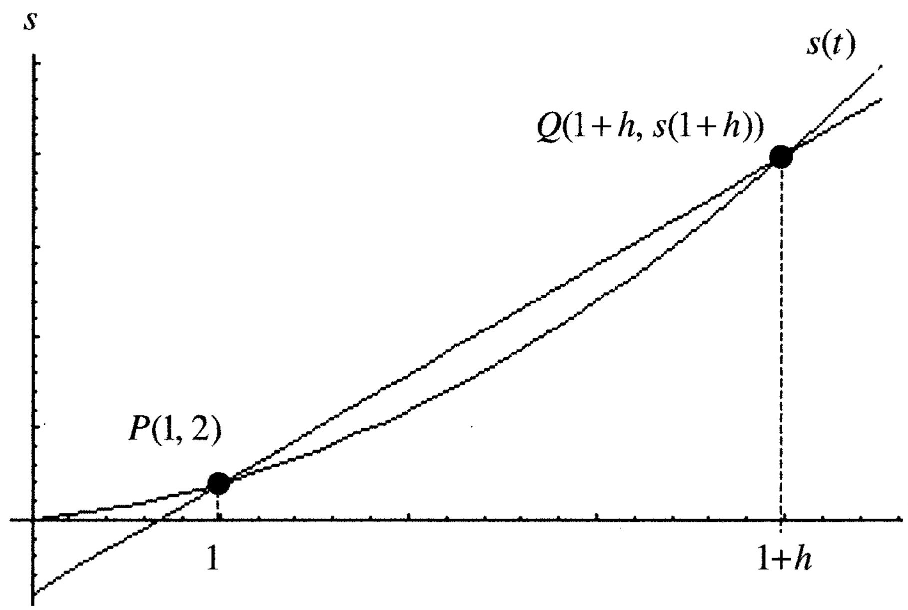
Suppose an object moves according to the position function $s(t)=t^2+t$ for $t \geq 0$ with t in seconds and s in meters. In this problem we are interested in determining how fast the object is traveling at exactly $t = 1$ second—i.e., the *instantaneous velocity* at $t = 1$.

(A) Find the average velocity of this object for each of the time intervals given in the table

Interval	Average velocity over this interval
[1,1.5]	$S(1) = 1^2 + 1 = 2 \Rightarrow (1,2)$; $S(1.5) = \underbrace{(1.5)^2}_{2.25} + 1.5 = \Rightarrow (1.5, 3.75)$; Ave $v(t) = m_{PQ}$ $\frac{3.75-2}{1.5-1} = \frac{1.75}{0.5} = 3.5$ m/s
[1,1.1]	$S(1) \Rightarrow (1,2)$; $S(1.1) = \underbrace{(1.1)^2}_{1.21} + 1.1 = 2.31 \Rightarrow (1.1, 2.31)$; Ave $v(t) = m_{PQ}$ $= \frac{2.31-2}{1.1-1} = \frac{0.31}{0.1} = 3.1$ m/s
[1,1.01]	$S(1) \Rightarrow (1,2)$; $S(1.01) \Rightarrow 1.01^2 + 1.01 = 2.0301$ $(1.01, 2.0301)$; Ave $v(t) = m_{PQ}$ $= \frac{2.0301-2}{1.01-1} = \frac{0.0301}{0.01} = 3.01$ m/s

§2.1 Collab #2

(B) Note that in part (A) we found average velocities over intervals of smaller and smaller width. We now investigate what happens to these *average velocities* as the width of the interval shrinks to zero. To focus our attention on the *width* of the interval we give it the variable name h. Once again with $s(t)=t^2+t$, find an expression for the average velocity of this object over an interval of the form $[1,1+h]$.



$3h$

$1+2h+h^2+1+h$

$$\text{Ave } v(t) = m_{PQ} = \frac{s(1+h) - s(1)}{(1+h)-1} = \frac{(1+h)^2 + (1+h) - 2}{h}$$

$$= \frac{h^2+3h}{h} = \frac{\cancel{h}(h+3)}{\cancel{h}} = \boxed{h+3}$$

(C) Substitute smaller and smaller values of h into the expression from part (B) and record the *average velocity*. Sample values of h to try are given in the table below.

h	Interval $[1, 1+h]$	Average Velocity $v_{[1,1+h]}$
0.1	[1, 1.1]	$0.1 + 3 = 3.1$ m/s
0.01	[1, 1.01]	$0.01 + 3 = 3.01$ m/s
0.001	[1, 1.001]	$0.001 + 3 = 3.001$ m/s

There really isn't any need for work here as you have a gen'l expression from B) for $v(t)$ on the interval $[1, 1+h]$.

(D) What appears to be happening to the average velocities as $h \rightarrow 0$?

$\lim_{h \to 0}$ appears to be approaching 3.

§2.2 Collab#2

Remark

Your reaction to the previous activity may have been something like: "What's all the fuss about? Can't I just plug the number 2 into the function to get the value of the limit?" The next activity should help clarify why the computation of limits is not always so "obvious."

Class Activity

Investigate the values of $y = f(x) = \frac{x^3 - 1}{x - 1}$ for values of x near 1. We have a partially completed table of values for $f(x) = \frac{x^3-1}{x-1}$. Fill in the remaining values of the table. Then answer parts (A) and (B).

x	$f(x) = \frac{x^3-1}{x-1}$ *
0	1
0.5	1.75
0.9	2.71
0.99	2.9701
0.999	2.997

x	$f(x) = \frac{x^3-1}{x-1}$
2.0	1
1.5	4.75
1.1	3.31
1.01	3.0301
1.001	3.003

(A) What value does y get close to as x gets close to the number 1?

$\lim_{x \to 1} f(x)$ appears to be 3 based on the tables above.

* Note: The tables were created using Y= & the Table features of the TI-84 calculator. They can be created manually by substituting & simplifying.

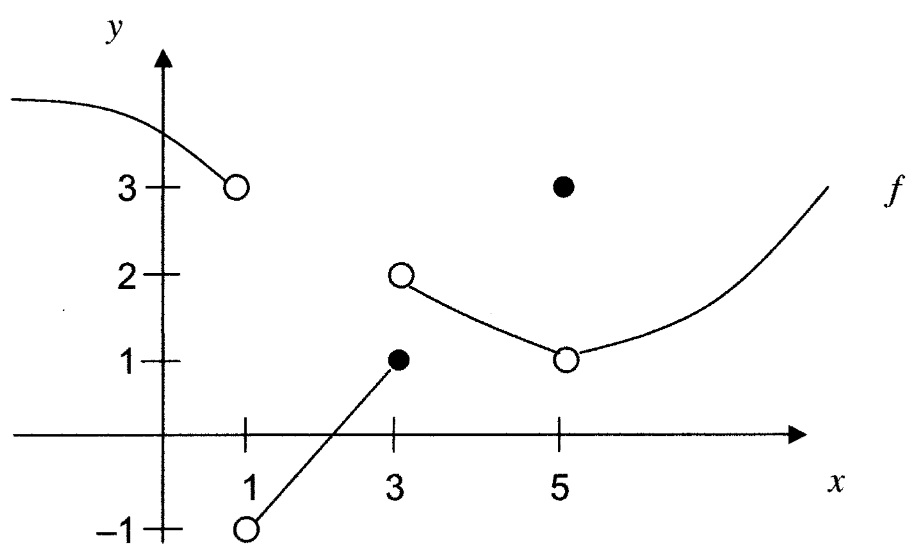
§2.2 Collab#2

Class Activity

Use the same graph (repeated below) to determine the following:

$\lim_{x \to 3^-} f(x) = \boxed{1}$ $\lim_{x \to 5^-} f(x) = \boxed{1}$

$\lim_{x \to 3^+} f(x) = \boxed{2}$ $\lim_{x \to 5^+} f(x) = \boxed{1}$



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Remark

A function does not even have to be defined at $x = c$ to have a limit at $x = c$. Notice that in finding limits we will often focus our attention on the x–values at which the function does something "peculiar," such as a *jump* from one value to another as in the cases above. We conclude this with a formal definition of one-sided limits.

+1

§2.3 Collab #2

Class Activity

Use the graphs on the next page and the properties of limits above to compute the limits.

A. $\lim_{x\to 3}(f(x)\cdot g(x)) = \left[\lim_{x\to 3} f(x)\right]\cdot\left[\lim_{x\to 3} g(x)\right]$
$= 2\cdot 0 = \boxed{0}$

B. $\lim_{x\to 5^-}\left(\frac{f(x)}{g(x)}\right) = \left[\lim_{x\to 5^-} f(x)\right]\div\left[\lim_{x\to 5^-} g(x)\right]$
$= 1 \div 2 = \boxed{\frac{1}{2} \text{ or } 0.5}$

C. $\lim_{x\to 5^+}\left(\frac{f(x)}{g(x)}\right) = \left[\lim_{x\to 5^+} f(x)\right]\div\left[\lim_{x\to 5^+} g(x)\right] = 1\div 3 = \boxed{\frac{1}{3} \text{ or } 0.\overline{33}}$

D. $\lim_{x\to 3}(f(x)+g(x)+\pi) = \lim_{x\to 3} f(x) + \lim_{x\to 3} g(x) + \pi$
$= 2 + 0 + \pi = \boxed{\pi + 2 \text{ or } \approx 5.14}$

E. $\lim_{x\to 5^-}(2f(x)-3g(x)) = 2\lim_{x\to 5^-} f(x) - 3\lim_{x\to 5^-} g(x)$

$= 2(1) - 3(2) = 2 - 6 = \boxed{-4}$

p37 will not appear in the key although it was essential in completing the problem.