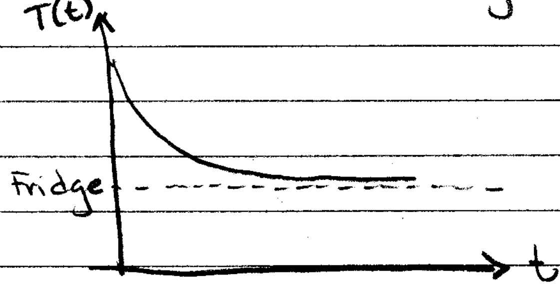
§2.6 Suggested HW Con'd

(41) Warm Soda → cold fridge



Intial rate of change will be greater

(45) $C(x) = 5000 + 10x + 0.05x^2$

(a) Average rate of change

i) $x = 100$ to 105 $\quad$ $\text{ave}\Delta = \dfrac{C(105) - C(100)}{105-100} = \dfrac{5000 + 10(105) + (0.05)(105)^2 -}{5}$

$= \dfrac{6601.25 - 6500}{5} = \dfrac{101.25}{5} = \$20.25/\text{unit}$

ii) $x = 100$ to 101 $\quad$ $\text{ave}\Delta = \dfrac{C(101) - C(100)}{101-100} = \dfrac{6520.1 - 6500}{1} = 20.1 \ \$/\text{unit}$

(b) Instantaneous Rate of Change @ $x = 100$

$$\lim_{h\to 0} \frac{\left[\overbrace{5000 + 10(100+h)}^{5000+1000+10h+} + 0.05\overbrace{(100+h)^2}^{10000+200h+h^2}\right] - \overbrace{\left[5000 + 10(100) + 0.05(100)^2\right]}^{6500}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{6500} + 20h + 0.05h^2 - \cancel{6500}}{h} = \lim_{h\to 0} \frac{\cancel{h}(20 + 0.05h)}{\cancel{h}}$$

$= \$20/\text{unit}$

(47) (a) $f'(x)$ is the rate of change in producing from x^{th} ounce of gold to the $(x+1)^{th}$ ounce. Units are \$/oz.

(b) $f'(800) = 17$ means that it will cost an additional \$17/oz to produce the 801^{st} oz of gold

(c) Short term they will as the equipment is put into place & a process is achieved but in the long run they will as the gold gets harder to find.

§2.6 Suggested HW Con'd

(50) (a) $f'(8)$ is the rate of change of the amount of coffee sold wrt price per pound when the price is \$8.
The units are pounds/(dollars/pound)
(b) $f'(8)$ is negative since the quantity of coffee sold will decrease as the price/pound increases. This means that people as less likely to purchase a product as the price increases.

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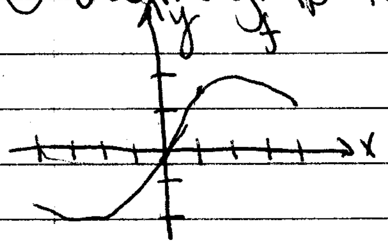
① Use the graph to estimate & the sketch $f'(x)$



$f'(-3) = \frac{rise}{run} = \frac{-1\frac{1}{3}}{+5} = -\frac{4}{15}$

$f'(1) = \frac{\frac{9}{2}}{5} = \frac{9}{10}$

$f'(-2) = 0$

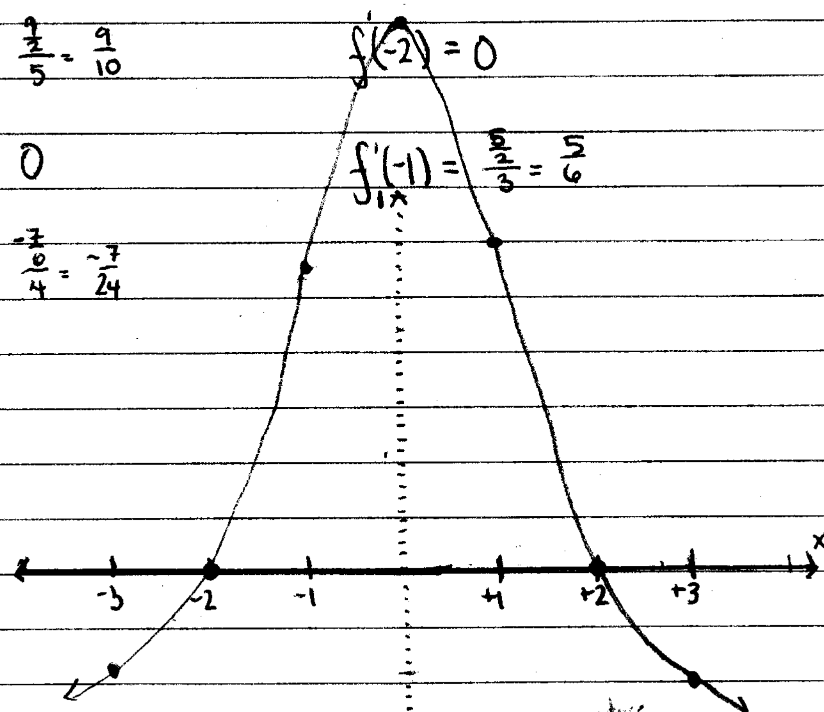
$f'(2) = 0$

$f'(-1) = \frac{\frac{5}{2}}{3} = \frac{5}{6}$

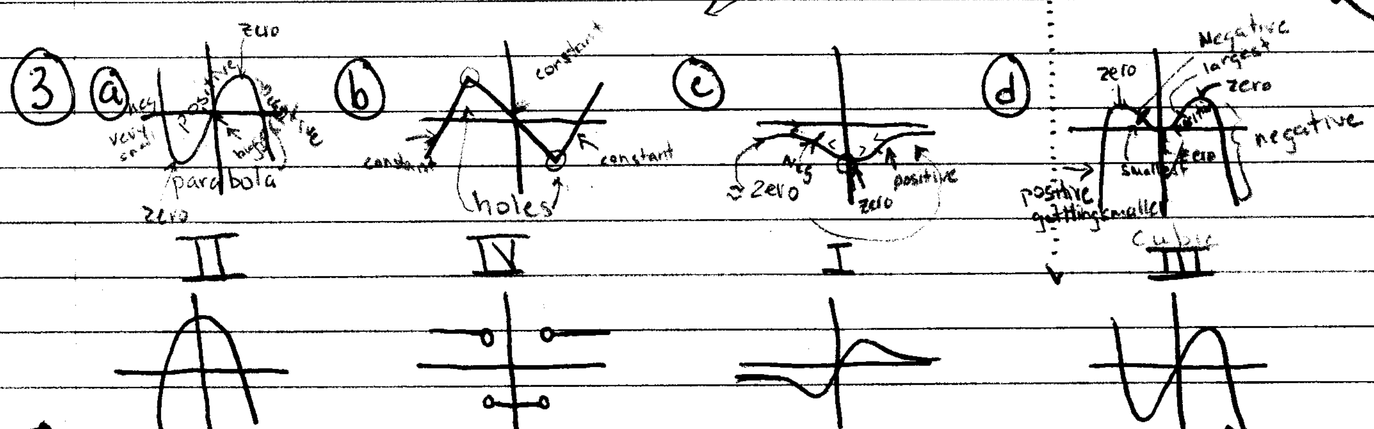
$f'(0) = \frac{3}{2}$

$f'(3) = \frac{\frac{-7}{6}}{4} = \frac{-7}{24}$

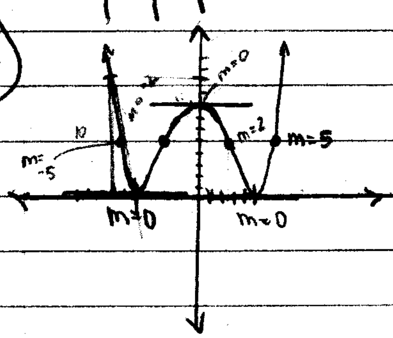
Note y-axis tick marks are 1/30th of 1



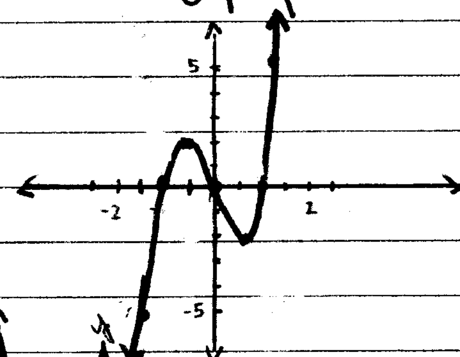
③



④

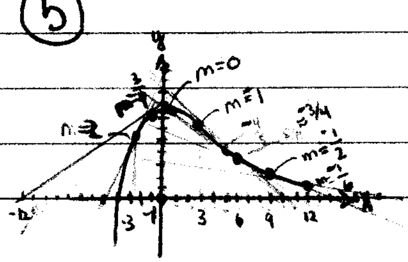


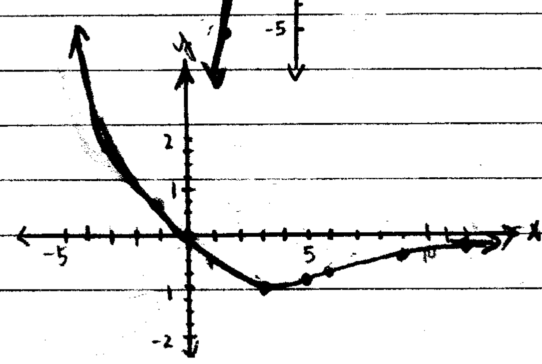
Sketch f' as in Example 1



⑤

Sketch f' as in Example 1





§2.7 Suggested HW con'd

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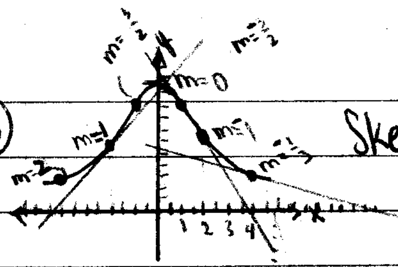
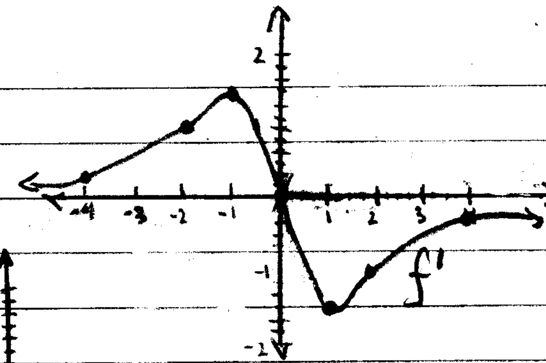


Sketch f' as in Example 1



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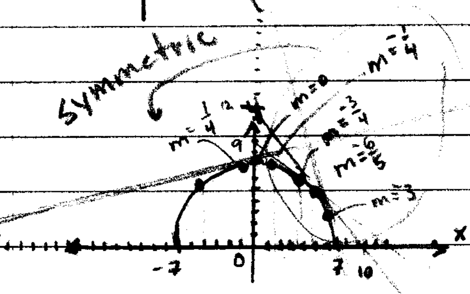

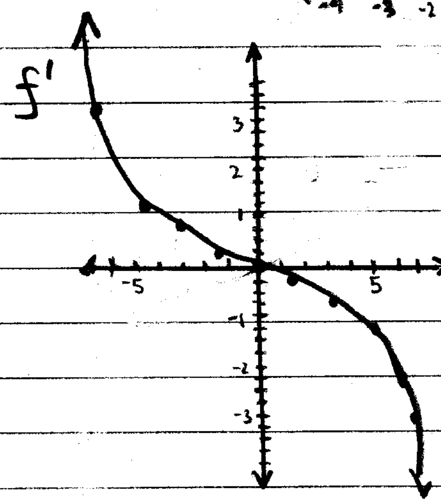

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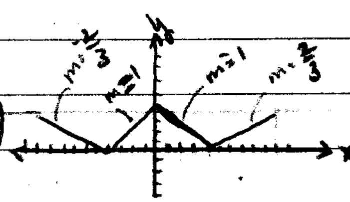
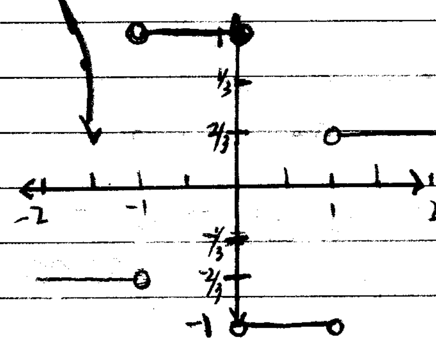




§2.7 p.156 Suggested HW Con'd

(19) $f(x) = \frac{1}{2}x - \frac{1}{3}$ $\quad \boxed{D_f : \mathbb{R}}$

$$f'(x) = \lim_{h\to 0} \frac{\left[\frac{1}{2}(x+h) - \frac{1}{3}\right] - \left[\frac{1}{2}x - \frac{1}{3}\right]}{h} = \lim_{h\to 0} \frac{\frac{1}{2}h}{h} = \boxed{\frac{1}{2}}$$

$\boxed{D_{f'} : \mathbb{R}}$

(21) $f(t) = 5t - 9t^2$ $\quad \boxed{D_f : \mathbb{R}}$

$$f'(t) = \lim_{h\to 0} \frac{\left[\overset{\cancel{5t}+5h}{5(t+h)} - \overset{\cancel{-9t^2}-18th-9h^2}{9(t+h)^2}\right] - \left[\overset{\cancel{-5t}+\cancel{9t^2}}{5t - 9t^2}\right]}{h} = \lim_{h\to 0} \frac{5h - 18th - 9h^2}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(5 - 18t - 9h)}{\cancel{h}} = \boxed{5 - 18t}$$

$\boxed{D_{f'} : \mathbb{R}}$

(23) $f(x) = x^2 - 2x^3$ $\quad \boxed{D_f : \mathbb{R}}$

$$f'(x) = \lim_{h\to 0} \frac{\left[\overset{\cancel{x^2}+2xh+h^2}{(x+h)^2} - 2(x+h)^3\right] - \left[\overset{\cancel{-x^2}+2x^3}{x^2 - 2x^3}\right]}{h} = \lim_{h\to 0} \frac{-6x^2h - 6xh^2 - 2h^3 + 2xh + h^2}{h}$$

$x^2 + 2xh + h^2$
$x + h$
$x^2h + 2xh^2 + h^3$
$x^3 + 2x^2h + xh^2$
$x^3 + 3x^2h + 3xh^2 + h^3$
$-2x^3 - 6x^2h - 6xh^2 - 2h^3$
$+2x^3 \quad +2xh + h^2$

$$= \lim_{h\to 0} \frac{\cancel{h}(-6x^2 - 6xh - 2h^2 + 2x + h)}{\cancel{h}} = \boxed{-6x^2 + 2x}$$

$\boxed{D_{f'} : \mathbb{R}}$

(25) $g(x) = \sqrt{1 + 2x}$ $\quad \boxed{D_f : \{x \mid x \in \mathbb{R},\ x \geq -\tfrac{1}{2}\} \text{ or } [-\tfrac{1}{2}, \infty)}$

$$g'(x) = \lim_{h\to 0} \frac{\sqrt{1 + 2(x+h)} - \sqrt{1+2x}}{h} \cdot \frac{\sqrt{1+2x+2h} + \sqrt{1+2x}}{\sqrt{1+2x+2h} + \sqrt{1+2x}}$$

$$= \lim_{h\to 0} \frac{\cancel{1} + \cancel{2x} + 2h - \cancel{1} - \cancel{2x}}{h(\sqrt{1+2x+2h} + \sqrt{1+2x})} = \lim_{h\to 0} \frac{2\cancel{h}}{\cancel{h}(\sqrt{1+2x+2h} + \sqrt{1+2x})}$$

$$= \frac{2}{2\sqrt{1+2x}} = \boxed{\frac{1}{\sqrt{1+2x}}}$$

$\boxed{D_{f'} : (-\tfrac{1}{2}, \infty)}$

§2.7 cont'd

$f(x) = \frac{1}{2}x - \frac{1}{3}$

(19) $f'(x) = \lim_{h\to 0} \frac{\frac{1}{2}(x+h) - \cancel{\frac{1}{3}} - \cancel{\frac{1}{2}x} + \cancel{\frac{1}{3}}}{h} = \lim_{h\to 0} \frac{\cancel{h}}{2\cancel{h}} = \boxed{\frac{1}{2}}$

$D_f : \{x \mid x \in \mathbb{R}\}$

$D_{f'} : \{x \mid x \in \mathbb{R}\}$

$f(t) = 5t - 9t^2$

(21) $f'(t) = \lim_{h\to 0} \frac{5(t+h) - 9(t+h)^2 - \cancel{5t} + \cancel{9t^2}}{h} = \lim_{h\to 0} \frac{5h - 18ht - 9h^2}{\cancel{h}}$

$\lim_{h\to 0} 5 - 18t - 9h = \boxed{5 - 18t}$ $\quad D_f : \{t \mid t \in \mathbb{R}\}$ $\quad D_{f'} : \{t \mid t \in \mathbb{R}\}$

$f(x) = x^2 - 2x^3$

(23) $f'(x) = \frac{(x+h)^2 - 2(x+h)^3 - x^2 + 2x^3}{h} = \lim_{h\to 0} \frac{2xh + h^2 - 6x^2h - 6xh^2 - 2h^3}{\cancel{h}}$

$= \boxed{2x - 6x^2}$ $\quad \boxed{D_f : \mathbb{R} \quad D_{f'} : \mathbb{R}}$

(25) $g(x) = \sqrt{1+2x}$ $\quad D : \{x \mid x \geq -\frac{1}{2}\}$ since $1+2x \geq 0$

$g'(x) = \lim_{h\to 0} \frac{\sqrt{1+2(x+h)} - \sqrt{1+2x}}{h} \cdot \frac{\sqrt{1+2x+2h} + \sqrt{1+2x}}{\sqrt{1+2x+2h} + \sqrt{1+2x}}$

$= \lim_{h\to 0} \frac{\cancel{1} + \cancel{2x} + 2h - \cancel{1} - \cancel{2x}}{h(\sqrt{1+2x+2h} + \sqrt{1+2x})} = \lim_{h\to 0} \frac{2\cancel{h}}{\cancel{h}(\sqrt{1+2x+2h} + \sqrt{1+2x})}$

$= \frac{2}{\sqrt{1+2x} + \sqrt{1+2x}} = \frac{\cancel{2}}{\cancel{2}\sqrt{1+2x}} = \boxed{\frac{1}{\sqrt{1+2x}}}$ $\quad D_{f'} : \{x \mid x > -\frac{1}{2}\}$

§2.7 Suggested HW Con'd

(27) $G(t) = 4t/t+1$ $\boxed{D_G : \{t \mid t \in \mathbb{R},\ t \neq -1\}}$

$$4t^2+\cancel{4t}+\cancel{4ht}+4h-\cancel{4t^2}-\cancel{4ht}-\cancel{4t}$$

$$G'(t) = \lim_{h\to 0} \frac{\left[\frac{4(t+h)}{t+h+1}\right] - \left[\frac{4t}{t+1}\right]}{h} = \lim_{h\to 0} \frac{4(t+h)(t+1) - 4t(t+h+1)}{h(t+h+1)(t+1)}$$

$$= \lim_{h\to 0} \frac{4\cancel{h}}{\cancel{h}(t+h+1)(t+1)} = \boxed{\frac{4}{(t+1)^2}} \quad \boxed{D_{G'} : \{t \mid t \neq -1\}}$$

(30) $f(x) = x + 1/x$

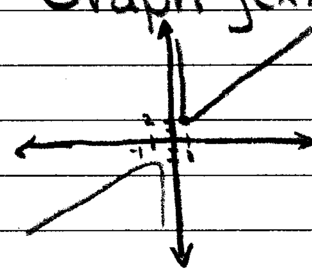
$$x^3+2x^2h+xh^2+\cancel{x}-x^3-x^2h-\cancel{x}-h$$

$$f'(x) = \lim_{h\to 0} \frac{\left[(x+h) + 1/(x+h)\right] - \left[x + 1/x\right]}{h} = \frac{x(x+h)^2 + x - x^2(x+h) - (x+h)}{xh(x+h)}$$

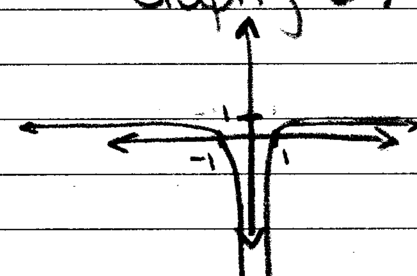
$$= \lim_{h\to 0} \frac{x^2h + xh^2 - h}{xh(x+h)} = \lim_{h\to 0} \frac{\cancel{h}(x^2+xh-1)}{\cancel{h}(x)(x+h)}$$

$$= \boxed{\frac{x^2-1}{x^2}}$$

Graph $f(x)$



Graph $f'(x)$



Yes, they match. @ x±1 the slope of the line tangent to f(x) would be zero & that is what f'(±1) shows. Furthermore the slope of lines tangent to the curve after x=±1 (to left of -1 & right of +1) approach a slope of 1 and again this is shown in f'(x). Finally, f(x)≠0 and tangent lines' slopes become increasingly smaller as x comes in toward zero & this too is shown in f'(x).

$x^2 + 2xh + h^2$
$x^2 + 2xh + h^2$
$x^2h^2 + 2xh^3 + h^4$
$2x^3h + 4x^2h^2 + 2xh^3$
$x^4 + 2x^3h + x^2h^2$
$\cancel{x^4} + 4x^3h + 6x^2h^2 + 4xh^3 + h^4$
$\cancel{2x} + 2h$ $\quad$ $\cancel{-x^4} - \cancel{2x}$

§2.7 Suggested HW con'd

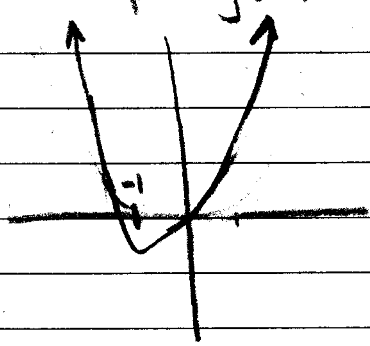
(31) $f(x) = x^4 + 2x$

$$f'(x) = \lim_{h\to 0} \frac{[(x+h)^4 + 2(x+h)] - [x^4 + 2x]}{h}$$

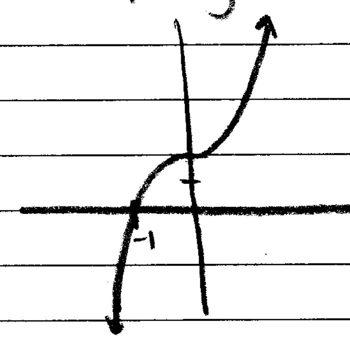
$$= \lim_{h\to 0} \frac{4x^3h + 6x^2h^2 + 4xh^3 + h^4 + 2h}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(4x^3 + 6x^2h + 4xh^2 + h^3 + 2)}{\cancel{h}} = \boxed{4x^3 + 2}$$

Graph $f(x)$



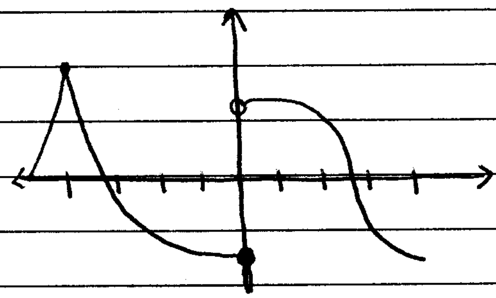
Graph $f'(x)$



Yes it appears accurate since to the left of -1 the slopes of the tangent lines are very small neg values but are increasing in value. Furthermore @ $x \approx -1$ the slope of the line tangent to $f(x)$ appears to be zero, after which the slopes become positive. Finally at $x = 0$ the slope of the tangent line does appear to be app. 2 and the slopes from that point become increasingly larger.

§2.7 cont'd

(35)



The function isn't differentiable @

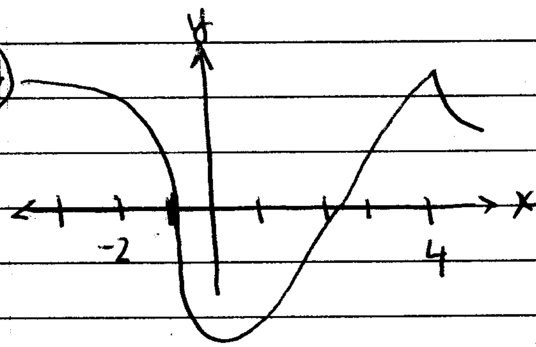
$x = -4$ b/c there is a corner

$\lim_{x\to 4^-} f(x) \neq \lim_{x\to 4^+} f(x)$

$x = 0$ b/c there is a jump discontinuity

$\lim_{x\to 0^-} f(x) \neq \lim_{x\to 0^+} f(x)$

(34)



The function is not differentiable @

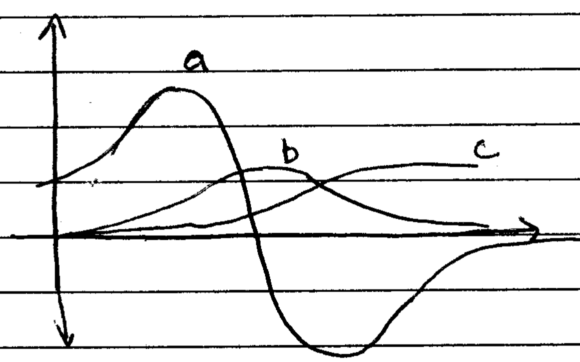
$x = -1$ b/c there is a vertical tangent

$\lim_{x\to -1} f(x) =$

$x = 4$ b/c there is a corner

$\lim_{x\to 4^-} f(x) \neq \lim_{x\to 4^+}$

(43)



c is position

It has all positive slope w/ one inflection pt.

b is velocity

It reflects all positive slopes in the position f(n) & the inflection point as a max

a is acceleration

It reflect positive & negative slopes of velocity and the maximum of the velocity is where it crosses the x-axis

Furthermore, a must be the acceleration since it has a horizontal tangent and neither c nor b is equal to zero. Also a=0 at the point b has a horizontal tangent so b must be the graph of velocity. So we know b' = a and therefore c is position