§2.1 p. 94

①

x	5	10	15	20	25	30
V(gal)	694	444	250	111	28	0

V = Volume remaining in 1000 gal. tank after x minutes.

ⓐ If P is point $(15, 250)$ find the slopes of the secant lines PQ when Q is the point $t = 5, 10, 20, 25, 30$

$t=5$ $\quad m_{PQ} = \dfrac{250-694}{15-5} = \dfrac{-444}{10} = -44.4$

$t=10$ $\quad m_{PQ} = \dfrac{260-444}{15-10} = \dfrac{-194}{5} = -38.8$

$t=20$ $\quad m_{PQ} = \dfrac{250-111}{15-20} = \dfrac{139}{-5} = -27.8$

$t=25$ $\quad m_{PQ} = \dfrac{250-28}{15-25} = \dfrac{222}{-10} = -22.2$

$t=30$ $\quad m_{PQ} = \dfrac{250-0}{15-30} = \dfrac{250}{-15} = -16.\overline{66}$

ⓑ $m_P \approx \dfrac{m_{PQ_{10}} + m_{PQ_{20}}}{2} = \dfrac{-38.8 + -27.8}{2} = \dfrac{-66.6}{2} = -33.3$

ⓒ See attached graph paper

③ $P(1, \tfrac{1}{2})$ lies on $y = \dfrac{x}{1+x}$ ⓐ Use a calculator to find the slope of the secant lines (use 6 decimal app.)

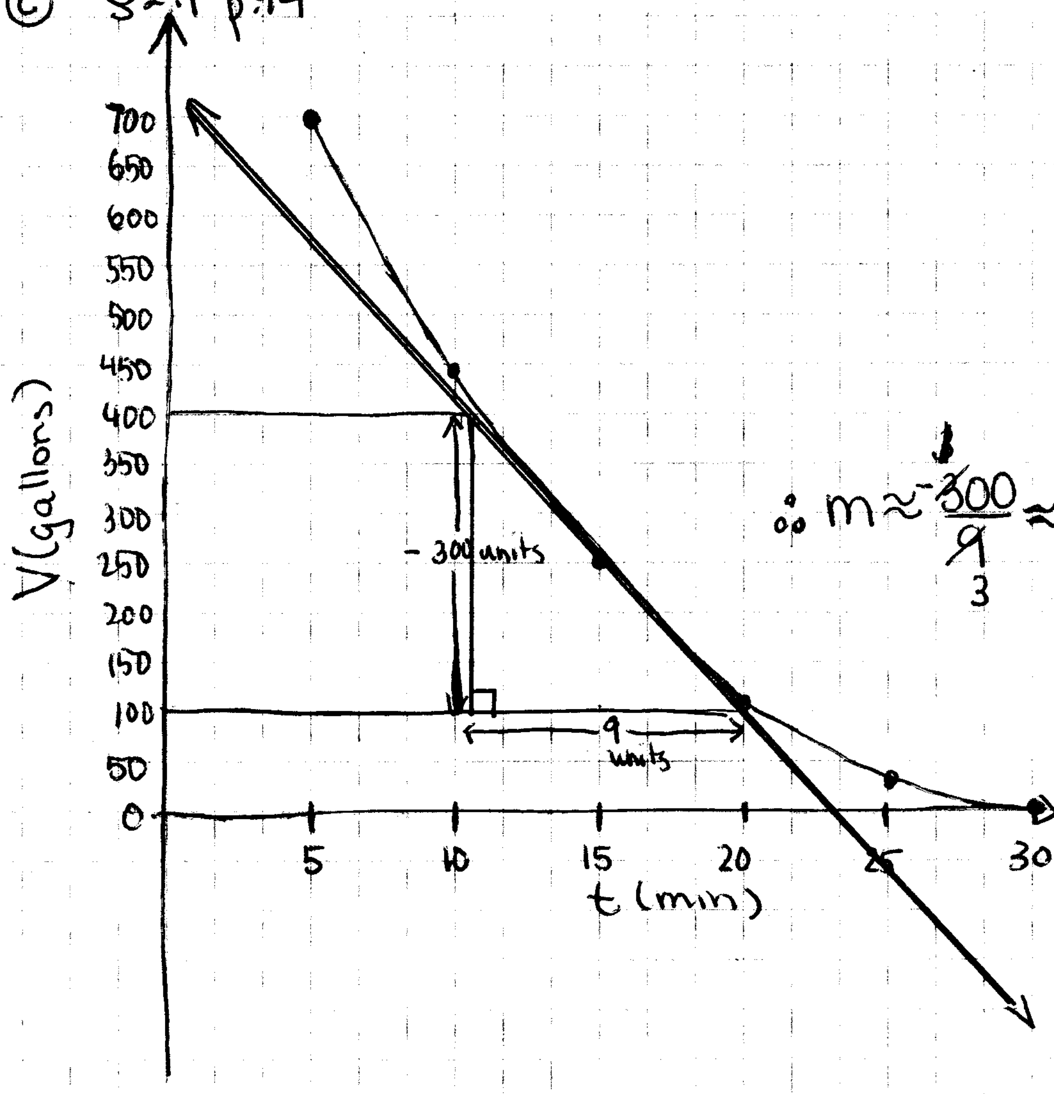
x	0.5	0.9	0.99	0.999	1.5	1.1	1.01	1.001
$f(x)$	0.333333	0.473684	0.497487	0.499750	0.6	0.523810	0.502488	0.500250
m_{PQ}	$\frac{\frac{1}{2}-}{1-\frac{1}{2}}$ $= 0.333333$	$\frac{\frac{1}{2}-0.473684}{1-0.9}$ $= 0.263158$	$\frac{\frac{1}{2}-0.497487}{1-0.99}$ $= 0.251256$	$\frac{\frac{1}{2}-0.499750}{1-0.999}$ 0.250125	$\frac{\frac{1}{2}-0.6}{1-1.5}$ $= 0.2$	$\frac{\frac{1}{2}-0.523810}{1-1.1}$ $= 0.238095$	$\frac{\frac{1}{2}-0.502488}{1-1.01}$ $= 0.248756$	$\frac{\frac{1}{2}-0.500250}{1-1.001}$ $= 0.249875$

ⓑ Since the slope of $m_{PQ_{0.999}} \approx m_{PQ_{1.001}} \approx 1/4$ I'd say that the slope of the tangent line at $x = 1$ is approx. $1/4$

ⓒ $y - \tfrac{1}{2} = \tfrac{1}{4}(x-1) \Rightarrow y - \tfrac{1}{2} = \tfrac{1}{4}x - \tfrac{1}{4} \Rightarrow \boxed{y = \tfrac{1}{4}x + \tfrac{1}{4}}$ is the eq. of tangent line at $(1, \tfrac{1}{2})$

$\quad +\tfrac{1}{2} \qquad +\tfrac{2}{4}$

① ⓒ §2.1 p.94



$\therefore\ m \approx \frac{-\cancel{300}^{100}}{\cancel{9}_{3}} \approx -33.\overline{33}$

§2.1 p.94

⑤ Position of a ball thrown in the air at an initial velocity of 40 m/s is given by $s(t) = 40t - 16t^2$

ⓐ Find the average velocity of the ball starting at $t=2$ and lasting

t	2	2.5	2.1	2.05	2.01
$s(t)$	16	0	13.44	14.76	15.7584
m_{pa}	X	$\frac{16-0}{2-2.5} = -32$ ft/sec	$\frac{16-13.44}{2-2.1} = -25.6$ ft/sec	$\frac{16-14.76}{2-2.05}$ -24.8 ft/sec	$\frac{16-15.7584}{2-2.01} = -24.16$ ft/sec

The average velocity is the slope of the secant line from P at (2,16)

but another way to look at it is average velocity between 2 and 2+h ∴

$$m_{ave} = \frac{f(2+h) - f(2)}{2+h-2} = \frac{40(2+h) - 16(\overset{4+4h+h^2}{2+h)^2} - 16}{h}$$

$$= \frac{80+40h+-64+-64h+-16h^2-16}{h}$$

$$= \frac{-24h-16h^2}{h} = \frac{-h(24+16h)}{h} = -(24+16h)$$

so $h=0.5 \Rightarrow -(24+\overset{8}{16(0.5)}) = -32$ ft/sec; $h=0.1 \Rightarrow -(24+\overset{1.6}{16(0.1)}) = -25.6$ ft/sec

$h=0.05 \Rightarrow -(24+\overset{0.8}{16(0.05)}) = -24.8$ ft/sec; $h=0.01 \Rightarrow -(24+\overset{0.16}{16(0.01)}) = -24.16$ ft/sec

ⓑ The instantaneous velocity at $t=2$ looks to be ≈ -24 ft/sec.

⑥ Rock thrown on Mars w/ velocity 10 m/s $s(t) = 10t - 1.86t^2$

Find ave velocities over time intervals

ⓐ Let's use the easy method of ave. between 1 & 1+h, $h \neq 0$

$$\text{ave velocity} = m_{ave} = \frac{f(1+h) - f(1)}{1+h-1} = \frac{10(1+h) - 1.86(\overset{1+2h+h^2}{1+h)^2} - 8.14}{h}$$

$$= \frac{10+10h-1.86-3.72h+-1.86h^2-8.14}{h} = \frac{6.28h-1.86h^2}{h} = 6.28-1.86h$$

i) $[1,2]$ $h=1 \Rightarrow m_{ave} = 6.28-1.86(1) = 4.42$ m/s ii) $[1,1.5]$ $h=0.5 \Rightarrow m_{ave} = 6.28-1.86(0.5) = 5.35$ m/s

iii) $[1,1.1]$ $h=0.1 \Rightarrow m_{ave} = 6.28-1.86(0.1) = 6.094$ m/s iv) $[1,1.01]$ $h=0.01 \Rightarrow 6.28-1.86(0.01) = 6.2614$ m/s

v) $[1,1.001]$ $h=0.001 \Rightarrow m_{ave} = 6.28-1.86(0.001) = 6.27814$ m/s

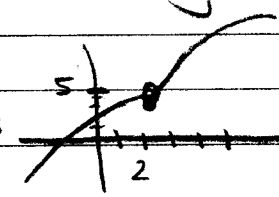
ⓑ Instantaneous velocity ∴ ≈ 6.28 m/s

§2.2 p.102

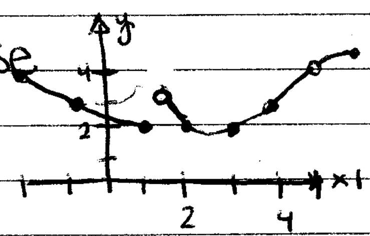
① What is meant by $\lim_{x\to 2} f(x) = 5$? Means that as x approaches 2 either from the right or left the limiting value is 5.

Is it possible for $f(2)=3$? Yes.

Example:



③ Use



ⓐ $\lim_{x\to 1^-} f(x) = 2$

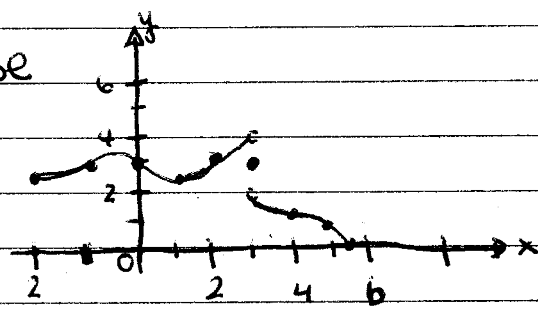
ⓑ $\lim_{x\to 1^+} f(x) = 3$

ⓒ $\lim_{x\to 1} f(x)$ d.n.e.

ⓓ $\lim_{x\to 5} f(x) = 4$

ⓔ $f(5) =$ d.n.e.

④ Use



ⓐ $\lim_{x\to 0} f(x) = 3$

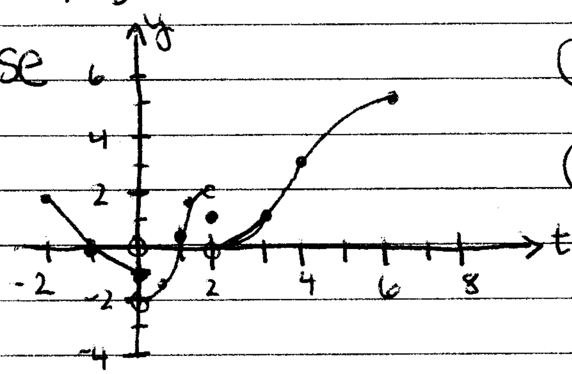
ⓑ $\lim_{x\to 3^-} f(x) = 4$

ⓒ $\lim_{x\to 3^+} f(x) = 2$

ⓓ $\lim_{x\to 3} f(x) =$ d.n.e.

ⓔ $f(3) = 3$

⑤ Use



ⓐ $\lim_{t\to 0^-} g(t) = -1$

ⓑ $\lim_{t\to 0^+} g(t) = -2$

ⓒ $\lim_{t\to 0} g(t) =$ d.n.e.

ⓓ $\lim_{t\to 2^-} g(t) = 2$

ⓔ $\lim_{t\to 2^+} g(t) = 0$

ⓕ $\lim_{t\to 2} g(t) =$ d.n.e.

ⓖ $g(2) = 1$

ⓗ $\lim_{t\to 4} g(t) = 3$

§2.2 cond p2

⑦ $f(x) = \begin{cases} 1+x & \text{if } x < -1 \\ x^2 & \text{if } -1 \le x < 1 \\ 2-x & \text{if } x \ge 1 \end{cases}$

$1+x$

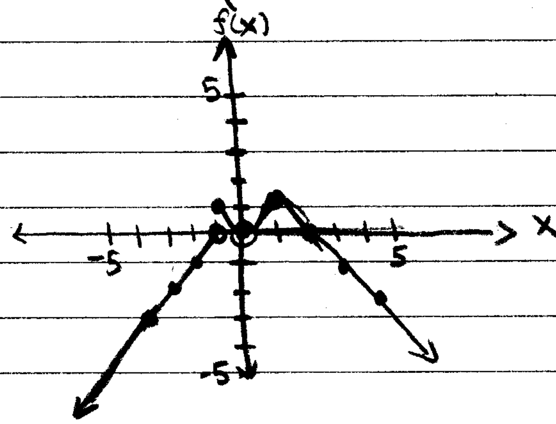
x	$f(x)$
-1	dne
-2	-1
-3	-2
-4	-3

x^2

x	$f(x)$
-1	1
0	0
1	dne

$2-x$

x	$f(x)$
1	1
2	0
3	-1



$\lim_{x \to a} f(x)$ exists for what a?

$\boxed{\text{For all } a \neq -1}$

b/c left & right limits are different there

⑧ $f(x) = \begin{cases} 1+\sin x & \text{if } x < 0 \\ \cos x & \text{if } 0 \le x \le \pi \\ \sin x & \text{if } x > \pi \end{cases}$

$1+\sin x$

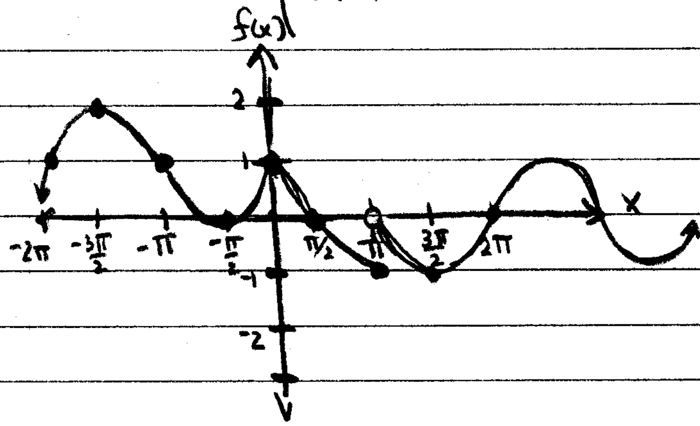
x	$f(x)$
0	dne(1)
$-\frac{\pi}{2}$	0
$-\pi$	1
$-\frac{3\pi}{2}$	2
-2π	1

$\cos x$

x	$f(x)$
0	1
$\frac{\pi}{2}$	0
π	-1

$\sin x$

x	$f(x)$
π	dne (0)
$\frac{3\pi}{2}$	-1
2π	0



$\lim_{x \to a} f(x)$ exists for what a?

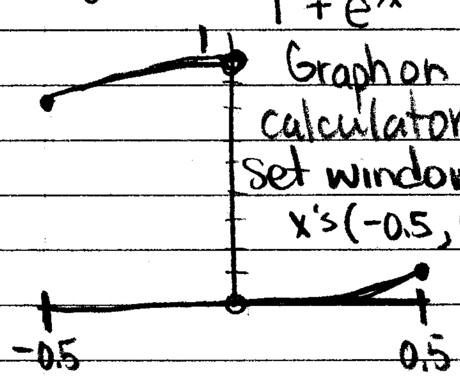
$\boxed{\text{For all } a \neq \pi}$

⑨ $f(x) = \dfrac{1}{1+e^{1/x}}$

Ⓐ $\lim_{x \to 0^-} f(x) = \boxed{1}$

Ⓑ $\lim_{x \to 0^+} f(x) = \boxed{0}$

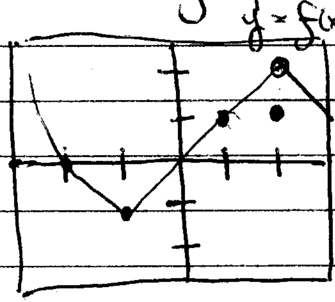
Ⓒ $\lim_{x \to 0} f(x) = \boxed{\text{d.ne}}$

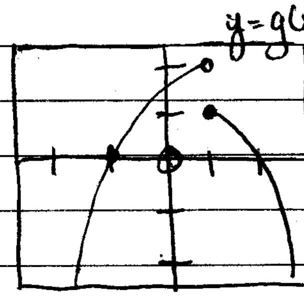


Graph on calculator
set window
x's (-0.5, 0.5) by 0.1
y's (-0.25, 1.2) by 0.1

§2.3 p.111

(2) Use the graphs to find the limits





(a) $\lim_{x\to 2} [f(x) + g(x)]$

$= \lim_{x\to 2} f(x) + \lim_{x\to 2} g(x)$

$= 2 + 0 = \boxed{2}$

(b) $\lim_{x\to 1} [f(x)+g(x)] = \lim_{x\to 1} f(x) + \lim_{x\to 1} g(x) = 1 + \underset{\lim_{x\to 1^-} \neq \lim_{x\to 1^+}}{\text{dne}} = \boxed{\text{dne}}$

(c) $\lim_{x\to 0} [f(x)g(x)] = \lim_{x\to 0} f(x) \lim_{x\to 0} g(x) = 0 \cdot (1.25) = \boxed{0}$

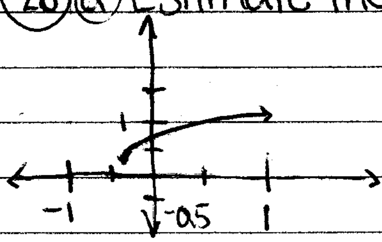
(d) $\lim_{x\to -1} f(x)/g(x) = \lim_{x\to -1} f(x) \div \lim_{x\to -1} g(x) = -1 \div 0 = \boxed{\text{dne}}$

(e) $\lim_{x\to 2} x^3 f(x) = \lim_{x\to 2} x^3 \lim_{x\to 2} f(x) = (2)^3 \cdot 2 = 2\cdot 2\cdot 2\cdot 2 = \boxed{16}$

(f) $\lim_{x\to 1} \sqrt{3+f(x)} = \sqrt{\lim_{x\to 1} 3 + \lim_{x\to 1} f(x)} = \sqrt{3+1} = \sqrt{4} = \boxed{2}$

(19) $\lim_{x\to -4} \dfrac{\frac{1}{4}+\frac{1}{x}}{4+x} = \lim_{x\to -4} \dfrac{\cancel{(4+x)}}{4x\cancel{(4+x)}} = \lim_{x\to -4} \dfrac{1}{4x} = \dfrac{1}{4(-4)} = \boxed{-\dfrac{1}{16}}$

(25) (a) Estimate the value of $\lim_{x\to 0} \dfrac{x}{\sqrt{1+3x}-1}$ by graphing it.



On the graph by zooming in closer & closer to zero and using the trace fcn

$\lim_{x\to 0} f(x) \approx 0.667$ or $2/3$

(b)

x	-0.001	-0.0001	-0.00001	-0.000001
f(x)	0.6661663	0.6666167	0.6666617	0.6666662

x	0.001	0.0001	0.00001	0.000001
f(x)	0.6671663	0.6667167	0.6666717	0.6666672

by plugging into $\dfrac{x}{\sqrt{1+3x}-1}$ & simplifying. $\lim_{x\to 0} f(x) \approx 0.666666 \approx 2/3$

c) $\lim_{x\to 0} \dfrac{x}{\sqrt{1+3x}-1} \cdot \dfrac{\sqrt{1+3x}+1}{\sqrt{1+3x}+1} = \lim_{x\to 0} \dfrac{x(\sqrt{1+3x}+1)}{\cancel{1}+3x\cancel{-1}} = \lim_{x\to 0} \dfrac{\sqrt{1+3x}+1}{3} = \dfrac{\sqrt{1+3(0)}+1}{3} = \dfrac{2}{3}$

Suggested HW
§2.3

§2.3 p. 3

(9) $\lim\limits_{x\to 5} \dfrac{x^2-6x+5}{x-5} = \lim\limits_{x\to 5} \dfrac{\cancel{(x-5)}(x-1)}{\cancel{(x-5)}} = \lim\limits_{x\to 5} (x-1)$

$= \lim\limits_{x\to 5} x - 1 = 5-1 = \boxed{4}$

(11) $\lim\limits_{x\to 5} \dfrac{x^2-5x+6}{x-5} = \lim\limits_{x\to 5} \dfrac{(x-3)(x-2)}{x-5} \therefore \boxed{\text{dne}}$

since $x-5\to 0$ & $(x-3)(x-2)\to 6$ as $x\to 5$

(12) $\lim\limits_{x\to -1} \dfrac{2x^2+3x+1}{x^2-2x-3} = \lim\limits_{x\to -1} \dfrac{(2x+1)\cancel{(x+1)}}{(x-3)\cancel{(x+1)}} = \dfrac{2\lim\limits_{x\to -1} x + 1}{\lim\limits_{x\to -1} x - 3}$

$= \dfrac{\overset{-2}{2(-1)} + 1}{-1-3} = \dfrac{-1}{-4} = \boxed{\dfrac{1}{4} \text{ or } 0.25}$

(13) $\lim\limits_{x\to -3} \dfrac{t^2-9}{2t^2+7t+3} \quad \lim\limits_{x\to -3} \dfrac{\cancel{(t+3)}(t-3)}{(2t+1)\cancel{(t+3)}} = \dfrac{\left(\lim\limits_{t\to -3} t\right) - 3}{\left(\lim\limits_{t\to -3} 2t\right) + 1}$

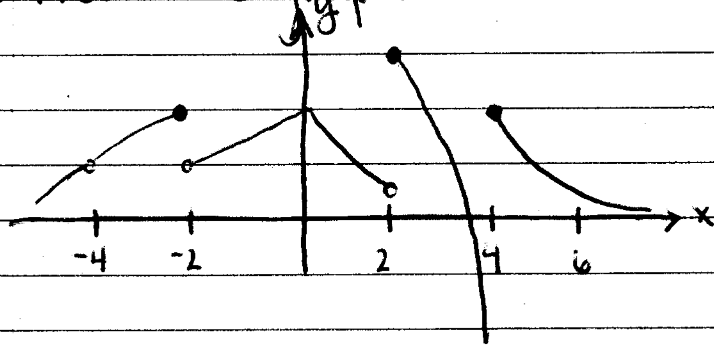
$= \dfrac{-3-3}{2(-3)+1} = \dfrac{-6}{-5} = \boxed{\dfrac{6}{5}}$

Suggest HW for §2.4 p.121 #1, 3, 4, 5, 7, 9, 15, 17, 37, 34, 35, 41

(1) Write an equation that expresses the fact that f is continuous at 4.

By definition: $\lim_{x\to 4} f(x) = f(4)$

(2) From the graph below



(a) State the numbers at which f is discontinuous & explain why.

@ -4 b/c $f(-4)$ dne

@ -2 b/c $f(-2)$ from the left $\lim_{x\to -2^-} f(x) = f(-2)$, $f(-2)$ from the right aren't equal

@ 2 for same reason as -2

@ 4 $f(4)$ from left dne, from right does

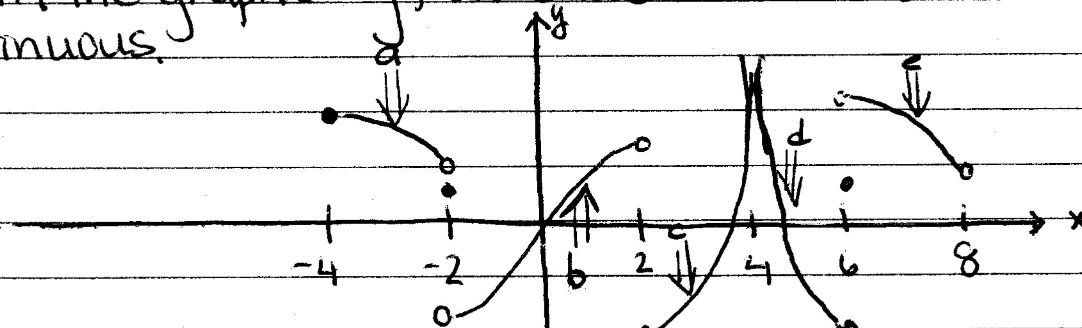
(b) -4 is not continuous either from the left or from the right

-2 is continuous from the left but not from the right $\lim_{x\to -2^-} f(x) = f(-2)$, $\lim_{x\to -2^+}$ dne

2 is continuous from the right but not from the left $\lim_{x\to 2^+} f(x) = f(2)$

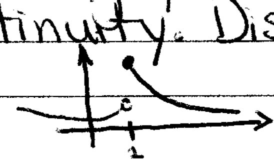
4 is continuous from the right but not the left $\lim_{x\to 4^+} f(x) = f(4)$

(4) From the graph of g, state the intervals on which g is continuous.



Intervals: (a) $[-4,-2)$ (b) $(-2,2)$ (c) $[2,4)$ (d) $(4,6)$ (e) $(6,8)$

(5) Sketch a graph that is continuous except for the stated discontinuity. Discontinuous, but cont. from right at 2



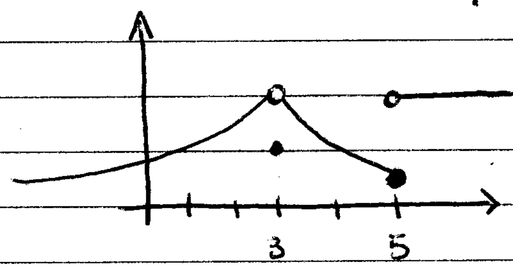
Note: $\lim_{x\to 2^+} f(x) = f(2)$ but $\lim_{x\to 2} f(x) \neq f(2)$

$\therefore$ discontinuous @ 2

§2.4 Suggested HW p.2

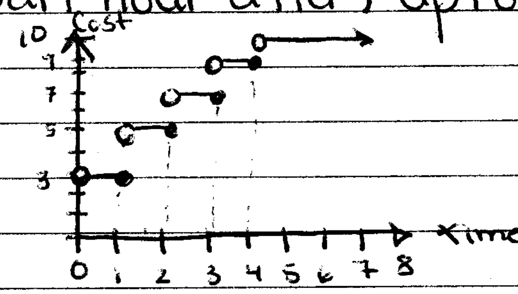
See instructions for #5

(7) Removable discontinuity at 3 & jump discontinuity @ 5



(9) A parking lot charges \$3 for the first hour (or part of an hour) and \$2 for each hour (or part hour after) up to a daily maximum of \$10.

(a) Sketch a graph of the cost of parking as a f(n) of time parked

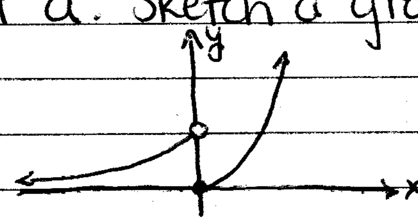


(b) Discuss discontinuity & significance to someone parking in the lot

The graph is discontinuous at integers [1,4]. This means if you are approaching the integer from the left (time below the integer hour) your cost will be continuous but discontinuous as you approach from right (above an hour.) There's a jump at the beginning of each hr.!!

(15) Why is the f(n) discontinuous at "a". Sketch a graph.

$$f(x) = \begin{cases} e^x & x < 0 \\ x^2 & x \geq 0 \end{cases} \qquad a=0$$



$e^0 = 1$ which is what value the f(n) approaches from the left

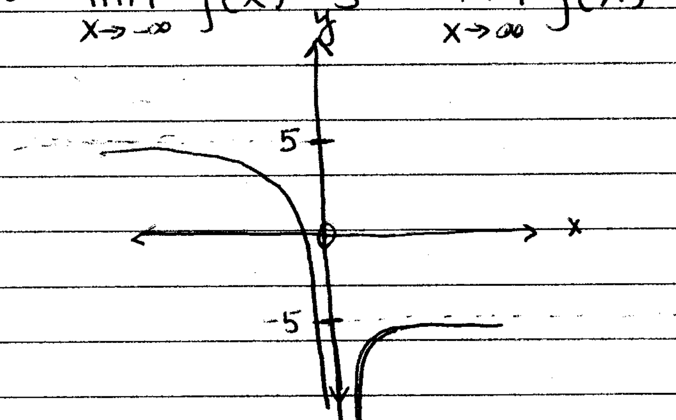
$(0)^2 = 0$ which is the value of the f(n) at zero

since $\lim_{x\to 0^-} f(x) \neq \lim_{x\to 0^+} f(x)$ the function isn't continuous

§2.5 Suggested HW cont'd p.2

In 5-10: Sketch the graph of an example f(n) satisfying the conditions

(5) $\lim_{x\to 0} f(x) = -\infty$ $\quad \lim_{x\to -\infty} f(x) = 5$ $\quad \lim_{x\to \infty} f(x) = -5$

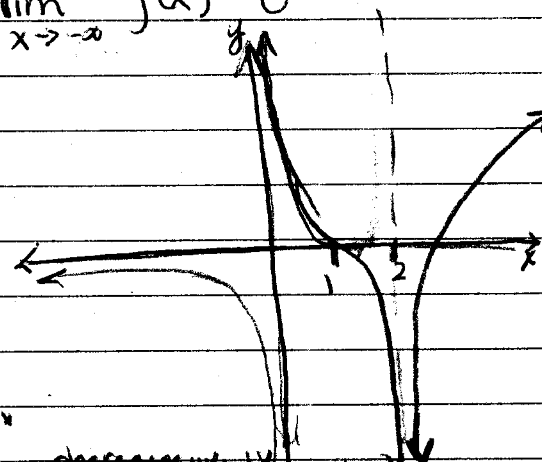


vertical asymptote @ 2 $\quad$ increases w/out bound $x\to\infty$ $\quad$ horizontal asymptote $y=0$ $x\to-\infty$

(7) $\lim_{x\to 2} f(x) = -\infty$ both left & right $\quad \lim_{x\to \infty} f(x) = \infty$ $\quad \lim_{x\to -\infty} f(x) = 0$

vertical asymptote at 0 up on right & down on left

$\lim_{x\to 0^+} f(x) = \infty$ $\quad \lim_{x\to 0^-} f(x) = -\infty$

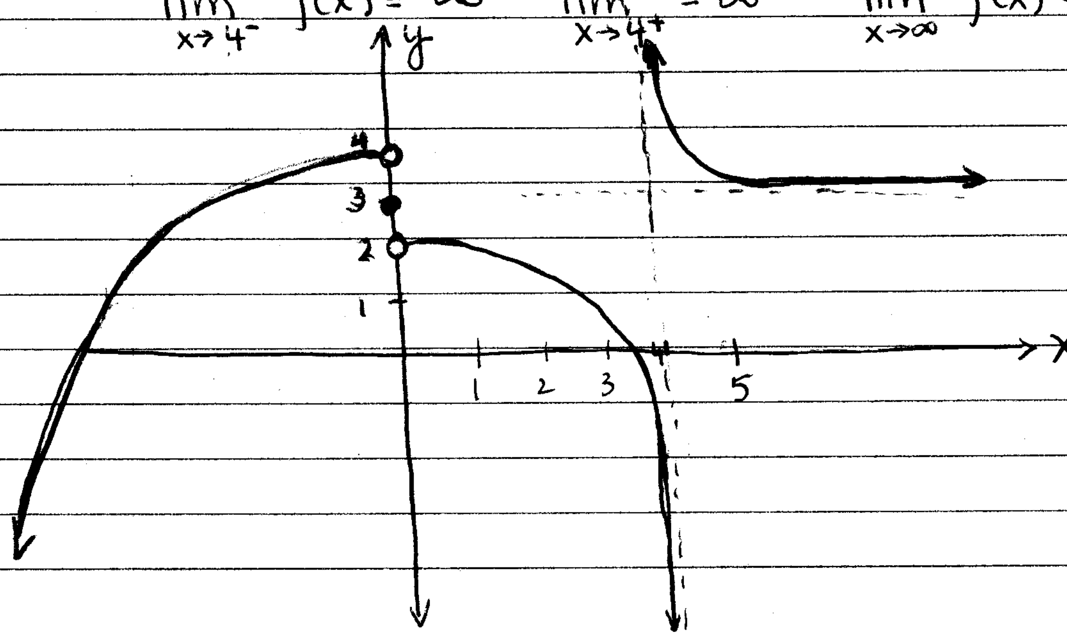


discontinuous @ 0 "Removable Discontinuity" $\quad$ decreases w/out bound as $x\to-\infty$

(9) $f(0) = 3$ $\quad \lim_{x\to 0^-} f(x) = 4$ $\quad \lim_{x\to 0^+} f(x) = 2$ $\quad \lim_{x\to -\infty} f(x) = -\infty$

vertical asymptote @ 4 $\quad$ horizontal asymptote as $x\to\infty$

$\lim_{x\to 4^-} f(x) = -\infty$ $\quad \lim_{x\to 4^+} = \infty$ $\quad \lim_{x\to \infty} f(x) = 3$



§2.5 p. 133

(12) $f(x) = \dfrac{1}{x^3-1}$

a)

x	$f(x)$	x	$f(x)$
0.5	-1.14	1.5	0.42
0.9	-3.69	1.1	3.02
0.99	-33.7	1.01	33.0
0.999	-333.7	1.001	333.0
0.9999	-3333.7	1.0001	3333.0
0.99999	-33,333.7	1.00001	33,333.3

Use table feature of calculator or plug in & simplify

e.g. $f(1.00001) = \dfrac{1}{(1.00001)^3-1}$

looks to be

$\lim_{x\to 1^-} f(x) = -\infty$

$\lim_{x\to 1^+} f(x) = \infty$

(b) x^3-1 When x is smaller than 1 but very close to one it will yield a small negative # & the reciprocal of a small neg. is a large negative number hence $\lim_{x\to 1^-} f(x) = -\infty$

When x is larger than 1 but very close to one it will yield a small positive # & the recip. is a large positive # hence $\lim_{x\to 1^+} f(x) = \infty$

(c) Using the TI to graph $y = (x^3-1)^{-1}$

Set window xmin 0, Xmax 2, scale 0.5; ymin −10, ymax 10, scale 1