Problem 1:

① Solved for F already

② $F' = 0.25 + -3.4F^{-3}$

③ $F' = 0$ $\quad 0.25 - 3.4F^{-3} = 0$

④ Solve for F $\quad \frac{3.4}{F^3} = 0.25 \Rightarrow 0.25F^3 = 3.4$

$\Rightarrow F^3 = \frac{3.4}{0.25} = 13.6$

$\Rightarrow F = \sqrt[3]{13.6}$

$\Rightarrow F \approx 2.4$ hours

⑤ Make sure it's a min.

$F'(2) = 0.25 - \frac{3.4}{8} = -0.175$ $\searrow \nearrow$ $\therefore$ minimum

$F'(3) = 0.25 - \frac{3.4}{27} \doteq 0.124$

Problem 2:

① Solved for I

② $I' = \frac{192}{S/762} \cdot \frac{1}{762} - 1 = \frac{192}{S} - 1$

③ $I' = 0$ & Solve $\quad \frac{192}{S} - 1 = 0 \Rightarrow \frac{192}{S} = 1 \Rightarrow S = 192$

④ Solve for $I(S) = 192 \ln\left(\frac{192}{762}\right) - 192 + 763$

$= 306.337$

Note, even if it was 306.533 it would still be 306 since there weren't actually 307 infected

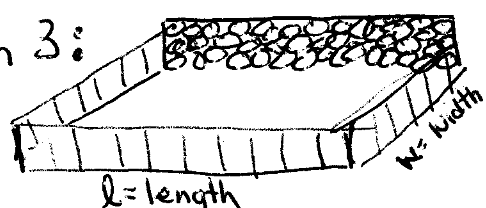
≈ 306 infected

⑤ Make sure it's a max

$I''(S) = \frac{-192}{S^2}$

$I''(306) = \frac{-192}{(306)^2} < 0 \therefore$ concave down & it's a max

Problem 3:



$P = 2w + l = 100\text{ft} \Rightarrow l = 100 - 2w$

$A = l \cdot w$

$= (100 - 2w)w = 100w - 2w^2$

$A' = 100 - 4w = 0 \Rightarrow 4w = 100$

$w = 25$

$l = 2(25) + l = 100 \Rightarrow l = 100 - 50 = 50$

$A = 25 \cdot 50 = \boxed{1250\text{ ft}^2}$

Check to make sure it's a max

$A'' = -4 < 0$ always concave down $\therefore$ max

Collab #8 cond p.2

Problem 4: x = height & $n(x)$ = ave # of drops

∴ Total height = #drops · height

$$H(x) = x(1+27x^{-2}) = x + 27x^{-1}$$

$$H'(x) = 1 + -27x^{-2} = 0 \Rightarrow \frac{27}{x^2} = 1 \Rightarrow x^2 = 27$$

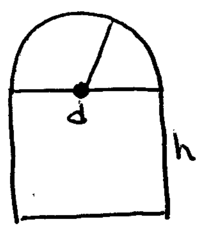
$x \approx 5.2$ feet or $3\sqrt{3}$ ft. $x = \sqrt{27}$

only + makes sense!!

check if min

$$H''(x) = 54x^{-3} = \frac{54}{(3\sqrt{3})^3} > 0 \text{ concave up } \therefore \text{ min}$$

Problem 5:



$$P = 2h + d + \tfrac{1}{2}\pi d = 30\text{ft}$$

$$A = dh + \tfrac{1}{2}\pi\left(\tfrac{d}{2}\right)^2$$

Solve P for h

$$2h = 30 - d - \frac{\pi d}{2}$$

$$h = 15 - \frac{d}{2} - \frac{\pi d}{4} = 15 - d\left(\frac{2+\pi}{4}\right)$$

Check max

$A'(8.3) = 0.1812$ ↗ ↘ thus max

$A'(8.5) = -0.1759$

$$A = d\left[15 - d\left(\frac{2+\pi}{4}\right)\right] + \frac{d^2\pi}{8}$$

$$A' = 15 - 2d\left(\frac{2+\pi}{4}\right) + \frac{d\pi}{4} = 0$$

$$\Rightarrow \frac{-4d + -2\pi d + d\pi}{4} = -15 \Rightarrow d\left(\frac{4+\pi}{4}\right) = 15$$

$$\Rightarrow d = \frac{15}{1} \cdot \frac{4}{4+\pi} = \boxed{\frac{60}{4+\pi} \approx 8.4\text{ft}}$$

Lastly, $h = -\frac{60}{4+\pi}\left(\frac{2+\pi}{4}\right) + 15 \doteq \boxed{4.2\text{ft}}$ or $h = 15 - \frac{1}{2} \cdot \frac{60}{4+\pi} - \frac{\pi}{4} \cdot \frac{60}{4+\pi}$

$$= 15 - \frac{30}{4+\pi} - \frac{15\pi}{4+\pi}$$

$$= \frac{15\pi + 60 - 30 - 15\pi}{4+\pi}$$

$$= \boxed{\frac{30}{4+\pi}\text{ ft.}}$$

Note: The height is exactly $\frac{1}{2}d$.

Collab#8 con'd p.3

Problem 6:

p = price = \$4 when demand is 4000 units

if $\Delta p = \$0.25$ then demand is up 200 units

This info tells us the slope & a pt.

$$m = \frac{\Delta d}{\Delta p} = \frac{200}{-0.25} = -800 \quad \text{when } (4, 4000)$$

Demand equation: $q = mp + \text{baseline}$

$$q - 4000 = -800(p - 4)$$

$$\Rightarrow q - 4000 = -800p + 3200 \Rightarrow q = -800p + 7200$$

Revenue $= p \cdot q = p(-800p + 7200)$

$$R(p) = -800p^2 + 7200p$$

$$R'(p) = -1600p + 7200 = 0$$

$$1600p = 7200 \Rightarrow p = \frac{7200}{1600} = 4.5$$

$$\boxed{\text{price} = \$4.50}$$

Check max

$R''(p) = -1600 < 0$ ∴ concave down always & is a max

$$q = -800(4.5) + 7200 = \boxed{3600 \text{ units}}$$

-3600