Name: Key

13

Collaborative #7 – Spring 2011
Math 1a

Instructions: *In groups of 2 or 3 complete each of the following problems in curve sketching based upon critical points and inflections points and asymptotes.*

1. For $f(x) = x^3 - 3x^2$ on the interval $-1 \le x \le 3$ answer all the following questions. *(#18 p. 186 Business Calculus, Hughes-Hallet, Ed. 4)

+1 a) Find the first derivative.

$f'(x) = 3x^2 - 6x$

+1 b) Find the second derivative.

$f''(x) = 6x - 6$

+1 c) Find the critical points.

$f'(x) = 0 \quad 3x(x-2) = 0 \quad x = 0$ or $x = 2$

$f''(0) = 6(0) - 6 = -6$ concave down $\therefore$ max

$f''(2) = 6(2) - 6 = 12 - 6 = 6$ $\therefore$ concave up min

+1 d) Find the inflection points.

$f''(x) = 0 \quad 6x - 6 = 0 \quad x = 1$

+2 e) Evaluate the function at the critical points and endpoints

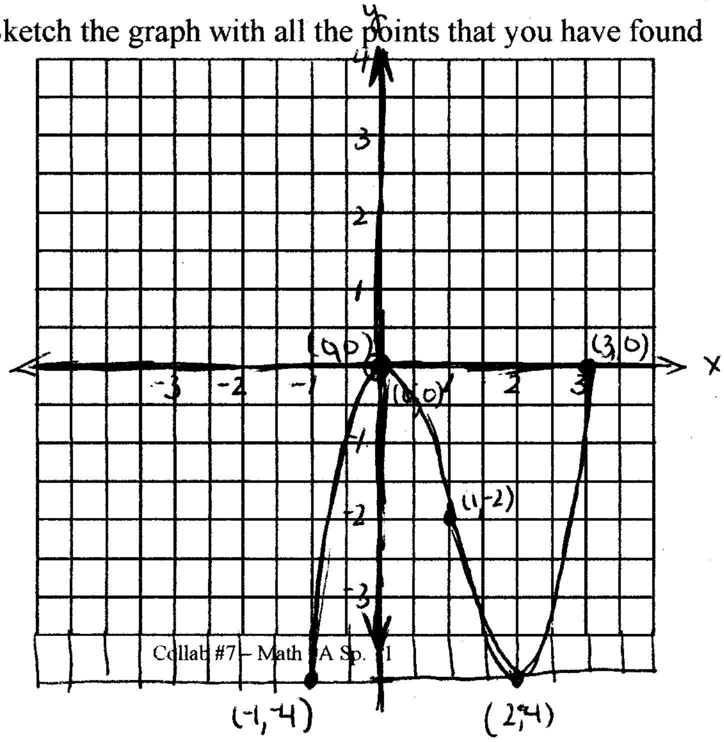
$f(-1) = (-1)^3 - 3(-1)^2 = -1 - 3(1) = -4$ $\quad f(0) = (0)^3 - 3(0)^2 = 0$ $\quad f(1) = 1^3 - 3(1) = -2$

$f(2) = 2^3 - 3(2)^2 = 8 - 12 = -4$

$f(3) = 3^3 - 3(3)^2 = 27 - 27 = 0$

+1 f) ID local and global max & min

+1 g) Sketch the graph with all the points that you have found



2. I or $f(x) = \dfrac{e^x}{1 + e^x}$ find the following

a) i) $\lim_{x \to -\infty} f(x)$ ii) $\lim_{x \to \infty} f(x)$

+2

As $x \to -\infty$ $e^x \to 0$

$\therefore$ $\lim_{x \to -\infty} \dfrac{e^x}{1+e^x} = \dfrac{0}{1+0} = 0$

Horizontal Asymptotes

@ $x=0$ & $x=1$

$\lim_{x \to \infty} \dfrac{e^x/e^x}{1/e^x + e^x/e^x}$

$= \lim_{x \to \infty} \dfrac{1}{e^{-x}+1} = \dfrac{1}{0+1} = 1$

As $x \to \infty$ $e^{-x} \to 0$

b) Find the inflection point for the function.

+2

$f'(x) = \dfrac{e^x(1+e^x) - e^x \cdot e^x}{(1+e^x)^2} = \dfrac{e^x}{(1+e^x)^2}$ ($e^x(1+e^x) = e^x + e^x \cdot e^x$)

Note: The $f'(x)$ can't be zero since $e^x \neq 0$

So there are no max/min values

$f''(x) = \dfrac{e^x(1+e^x)^2 - 2e^xe^x(1+e^x)}{(1+e^x)^4}$

$= \dfrac{\cancel{(1+e^x)}(e^x + e^{2x} - 2e^{2x})}{(1+e^x)^{\cancel{4}\,3}}$

$= \dfrac{e^x - e^{2x}}{(1+e^x)^3} = \dfrac{e^x(1-e^x)}{(1+e^x)^3}$

$f''(x) = 0$ when

$e^x = 1$ since $1-1=0$

$\therefore x = 0$

$f(0) = \dfrac{e^0}{1+e^0} = \dfrac{1}{1+1} = \dfrac{1}{2}$

c) Use horizontal asymptotes found in a) & the inflection point found in b) to graph the function.

+1