§3.8 p.237 Collab #6 problems #1,7,11,15,23

① $f(t) = t^3 - 12t^2 + 36t$

$$f'(t) = 3t^2 - 24t + 36$$

$$f''(t) = 6t - 24$$

a) Velocity is the 1st derivative of position

$$\boxed{v(t) = f'(t) = 3t^2 - 24t + 36}$$

b) $v(3) = 3(3)^2 + -24(3) + 36 = 3(9) - 72 + 36 = \boxed{-9 \text{ ft/s}}$

c) $f'(t) = 0$ is when the particle is at rest

$$v(t) = f'(t) = 3(t^2 - 8t + 12) = 3(t-6)(t-2) = 0$$

when $\boxed{t = 6 \text{ or } 2 \text{ sec}}$

d) moving in a positive direction when velocity is positive

$v(t) = f'(t) = 3(t-6)(t-2) > 0$ $\quad t>6$ and $t>2 \Rightarrow \boxed{t>6}$

and

$t<6$ and $t<2 \Rightarrow t<2 \Rightarrow \boxed{0 \le t < 2}$

e) total distance during first 8 sec.

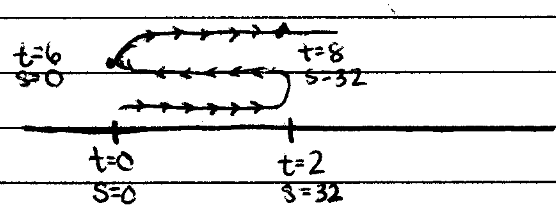
$$|f(2) - f(0)| + |f(6) - f(2)| + |f(8) - f(6)|$$

$$[2^3 - 12(2^2) + 36(2) - 0] + [6^3 - 12(6^2) + 36(6) - [32]] + \text{"}$$

$$= |32| + |-32| + [8^3 - 12(8^2) + 36(8)) - 0]$$

$$= |32| + |-32| + |32| = 3(32) = \boxed{96 \text{ ft}}$$

f)



g) acceleration at time t is second derivative of position (derivative of velocity)

$$\boxed{a(t) = v'(t) = f''(t) = 6t - 24}$$

acceleration at 3 sec

$$a(3) = 6(3) - 24 = 18 - 24 = \boxed{-6 \text{ ft/s}^2}$$

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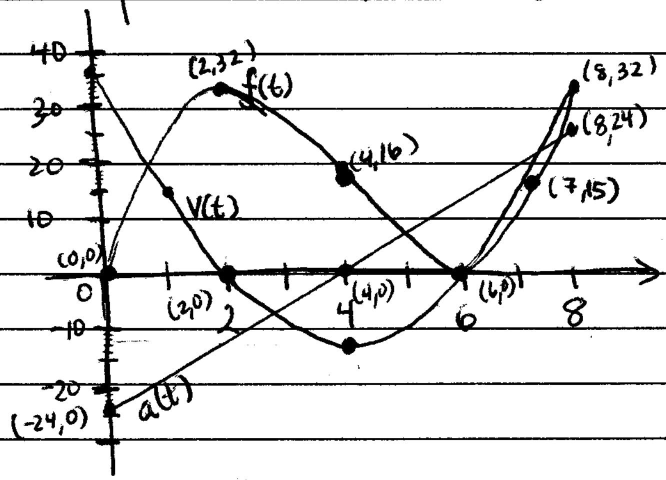
h) $f''(t) = 6t - 24 = 0 \Rightarrow t = 4$



i) Particle speeds up when v(t) is positive & increasing (a(t) is positive)
v(t) is negative & decreasing (a(t) is negative)

(2,4) v(t) negative & decreasing (a(t) neg.)

(6,8) v(t) positive & increasing (a(t) positive)

Particle slows down when signs of a(t) & v(t) are opposite

[0,2) v(t) positive & a(t) negative

(4,6) v(t) negative & a(t) positive

(7) The position f(n) of a particle is given by

$s = t^3 - 4.5t^2 - 7t, \quad t \geq 0$

$s' = 3t^2 - 9t - 7$

a) When does particle reach velocity of 5m/s

$3t^2 - 9t - 7 = 5 \Rightarrow 3t^2 - 9t - 12 = 0 \Rightarrow 3(t^2 - 3t - 4) = 0$
$-5 \quad -5$

$\Rightarrow 3(t-4)(t+1) = 0$

$t = 4$ or $t = -1$
extraneous

$\boxed{t = 4 \text{ sec}}$

b) When is acceleration 0?

$s'' = 6t - 9 = 0 \Rightarrow 6t = 9 \Rightarrow \boxed{t = 3/2 \text{ sec}}$

The significance?

This is an inflection point of position. It is the point where it reaches its ~~minimm~~ minimum (absolute) velocity and velocity increases

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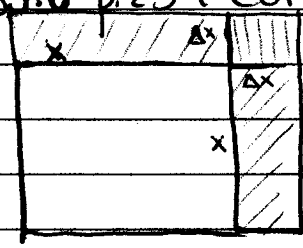


(11) ⓐ $A(x) = x^2 \quad \therefore \quad A'(x) = 2x$

$A'(15) = 2(15) = 30\ mm^2/mm$ The area changes by $30\ mm^2$ wrt to side length, when the side length is 15mm.

ⓑ The $P = 4x$ & $\frac{1}{2} \cdot 4x = 2x \quad \therefore \quad A'(x) = \frac{1}{2} \cdot P(x)$

The area of the figure above changes from A(x) by the increase of the shaded regions shown as x changes by Δx.

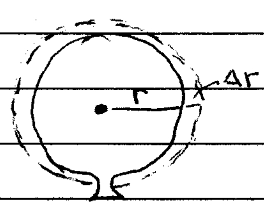
Thus $\underset{\text{old}}{x^2} + \overbrace{2(x \cdot \Delta x) + (\Delta x)^2}^{\text{increase} = \Delta A}$

or another way of looking at it is $A(x) = x^2$ is old area
$A_n(x) = (x + \Delta x)^2$ is new area

$\Delta A = \text{new} - \text{old}$
$= [x^2 + 2x\Delta x + (\Delta x)^2] - \cancel{x^2} = 2x\Delta x + (\Delta x)^2$

$\therefore$ either way
$\Delta A = 2x\Delta x + (\Delta x)^2$
and if Δx is very small $(\Delta x)^2 \approx 0$
so
$\Delta A \approx 2x\,\Delta x$ & solving $\frac{\Delta A}{\Delta x} \approx 2x$

(15)



$A = 4\pi r^2 \qquad \frac{\Delta A}{\Delta r} = 8\pi r$

a) when r = 1ft $\quad \frac{\Delta A}{\Delta r} = 8\pi(1) = 8\pi\ ft^2/ft$

b) when r = 2ft $\quad \frac{\Delta A}{\Delta r} = 8\pi(2) = 16\pi\ ft^2/ft$

c) when r = 3ft $\quad \frac{\Delta A}{\Delta r} = 8\pi(3) = 24\pi\ ft^2/ft$

Notice that the change in Area wrt radius is a linear function, therefore for every foot increase in radius the Area changes (increases) by 8 ft².

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(23) A bacteria population triples every hour and begins with 400 bacteria.

$$P(t) = n_0 r^t$$

$\therefore$ since

$\frac{P(t)}{n_0} = 3$ The ratio of new population to the old is 3 when $t=1$

$3 = r^1 \Rightarrow r = 3$

so $P(t) = 400\, 3^t$

thus

$$P'(t) = 400 3^t \ln 3$$

and therefore the rate of change (growth) in the population after 2.5 hours is

$$P'(2.5) = 400\, 3^{2.5} \ln 3 \approx 6850.268286 \approx \boxed{6850 \frac{\text{batera}}{\text{hr}}}$$