

Collab #4-1 Problem #2: $f(x) = 2x^2 - x + 5$

$$\lim_{h \rightarrow 0} \frac{2(x+h)^2 - (x+h) + 5 - 2x^2 + x - 5}{h} = \lim_{h \rightarrow 0} \frac{4xh + 2h^2 - h}{h}$$

$$= \boxed{4x - 1}$$

Collab #4-2 Problem #2: $f(x) = x - \frac{1}{x}$ or $\frac{x^2 - 1}{x}$

$$\lim_{h \rightarrow 0} \frac{(x+h) - \frac{1}{x+h} - x + \frac{1}{x}}{h} = \lim_{h \rightarrow 0} \frac{h - \frac{1}{x+h} + \frac{1}{x}}{h}$$

$$\lim_{h \rightarrow 0} \frac{\frac{xh+h^2}{x(x+h)} - x + (x+h)}{h} = \lim_{h \rightarrow 0} \frac{h(h+x+1)}{x(x+h)} = \lim_{h \rightarrow 0} \frac{h+x+1}{x(x+h)}$$

$$= \boxed{\frac{x+1}{x^2}}$$

Collab #4-3 Problem #2: $f(x) = \frac{1}{2x+1}$

$$\lim_{h \rightarrow 0} \frac{\frac{1}{2(x+h)+1} - \frac{1}{2x+1}}{h} = \lim_{h \rightarrow 0} \frac{\frac{2x+1 - 2x-2h-1}{(2x+2h+1)(2x+1)}}{h} = \lim_{h \rightarrow 0} \frac{-2h}{h(2x+2h+1)(2x+1)}$$

$$= \boxed{\frac{-2}{(2x+1)^2}}$$

Collab #4-4 Problem #2: $f(x) = 2x^3 + 3$

$$\lim_{h \rightarrow 0} \frac{2(x+h)^3 + 3 - 2x^3 - 3}{h} = \lim_{h \rightarrow 0} \frac{6x^2h + 6h^2x + 2h^3}{h} = \lim_{h \rightarrow 0} \frac{h(6x^2 + 6hx + 2h)}{h}$$

$$= \boxed{6x^2}$$

$$\begin{array}{r} x^2 + 2xh + h^2 \\ 2x + 2h \\ \hline 2x^2h + 4h^2x + 2h^3 \\ + 4x^2h + 2h^2x \\ 2x^3 \\ \hline 2x^3 + 6x^2h + 6h^2x + 2h^3 + 3 \\ - 2x^3 \\ \hline -3 \end{array}$$

Collab #4-5 Problem #2: $f(x) = \sqrt{2x+3}$

$$\lim_{h \rightarrow 0} \frac{\sqrt{2(x+h)+3} - \sqrt{2x+3}}{h} = \frac{\sqrt{2(x+h)+3} + \sqrt{2x+3}}{\sqrt{2x+2h+3} + \sqrt{2x+3}} = \lim_{h \rightarrow 0} \frac{2h}{h(\sqrt{2x+2h+3} + \sqrt{2x+3})}$$

$$= \lim_{h \rightarrow 0} \frac{2}{\sqrt{2x+2h+3} + \sqrt{2x+3}} = \frac{2}{\sqrt{2x+3} + \sqrt{2x+3}} = \frac{2}{2\sqrt{2x+3}} = \boxed{\frac{1}{\sqrt{2x+3}}}$$

Collab #4-6 Problem #2: $f(x) = \frac{2x+1}{x+3}$

$$\lim_{h \rightarrow 0} \frac{\frac{2(x+h)+1}{(x+h)+3} - \frac{2x+1}{x+3}}{h} = \lim_{h \rightarrow 0} \frac{(2x+2h+1)(x+3) - (2x+1)(x+h+3)}{h(x+h+3)(x+3)}$$

$$\lim_{h \rightarrow 0} \frac{2x^2+7x+2xh+6h+3 - 2x^2-7x-2xh-h-3}{h(x+h+3)(x+3)} = \lim_{h \rightarrow 0} \frac{5h}{h(x+h+3)(x+3)}$$

$$\lim_{h \rightarrow 0} \frac{5}{(x+h+3)(x+3)} = \boxed{\frac{5}{(x+3)^2}}$$

$$\frac{2x+2h+1}{x+3}$$

$$\frac{2x^2+2xh+x}{2x^2+7x+2xh+6h+3}$$

$$\frac{2x^2+7x+2xh+6h+3}{2x^2+7x+2xh+6h+3}$$

$$\frac{x+h+3}{x+h+3}$$

$$\frac{2x^2+2xh+x}{2x^2+7x+2xh+6h+3}$$

$$\frac{2x^2+7x+2xh+6h+3}{2x^2+7x+2xh+6h+3}$$

Collab #4-7 Problem #2: $f(x) = 2x+5$

$$\lim_{h \rightarrow 0} \frac{2(x+h)+5 - 2x-5}{h} = \lim_{h \rightarrow 0} \frac{2h}{h} = \boxed{2}$$

Collab #4-8 Problem #2: $f(x) = x^4$

$$\lim_{h \rightarrow 0} \frac{(x+h)^4 - x^4}{h} = \lim_{h \rightarrow 0} \frac{4x^3h + 6x^2h^2 + 4xh^3 + h^4}{h}$$

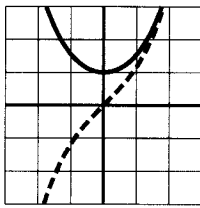
$$= \lim_{h \rightarrow 0} \frac{4x^3 + 6x^2h + 4xh^2 + h^3}{1}$$

$$= \boxed{4x^3}$$

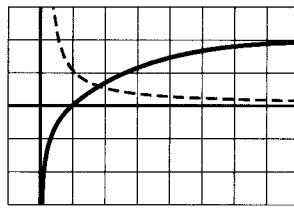
$$\frac{x^2+2xh+h^2}{x^2+2xh+h^2}$$

$$\frac{x^4+2x^3h+x^2h^2+2xh^3+h^4}{x^4}$$

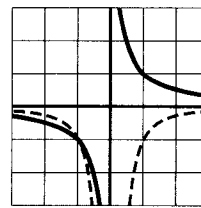
Each of the graphs ^{in problem #1} are shown on the next page the graph # corresponds to collab #4-?. The solid is the original; dotted is the derivative



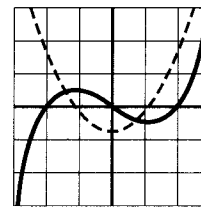
Graph 1



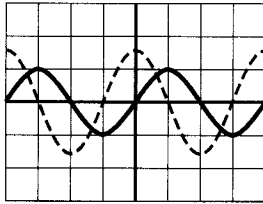
Graph 2



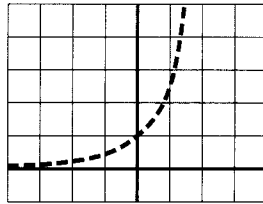
Graph 3



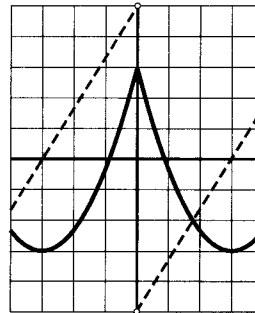
Graph 4



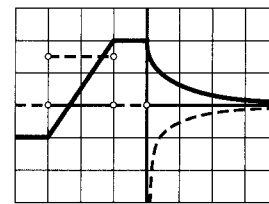
Graph 5



Graph 6



Graph 7



Graph 8