§6.1 Average Value

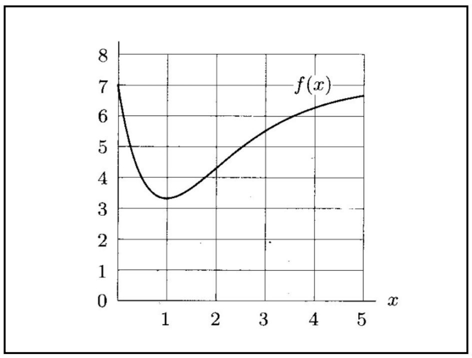
An average ins the sum divided by the total number, so it makes sense that our **average value** would be the Riemann Sum divided by the length of the interval which when taken to the limit as $\Delta t \rightarrow 0$ gives:

Average Value of f(x) on $a \leq x \leq b$

$$\frac{1}{b - a}\int_a^b f(x)\,dx$$

***Note:** that the units are those of f(x).

Example: Use the graph to visually estimate the average height between $x = 0$ & $x = 5$.* (#2 p. 278)



***Note**: Visualize the area outside the curve equaling the area above the curve as you draw a rectangle. This will allow you to visualize the average value which is an average height (a rectangular area) over an interval.

Example: Using the last example use the technique of counting the boxes combined with the definition above to find the average value.

Example: Finally, use Riemann Sums to compute the average value for the function above. Draw a table to express the values of the function for Δt=1 and then compute both the left and right-hand Riemann Sum. Use the area under the curve with the definition above to find the average value of the function.

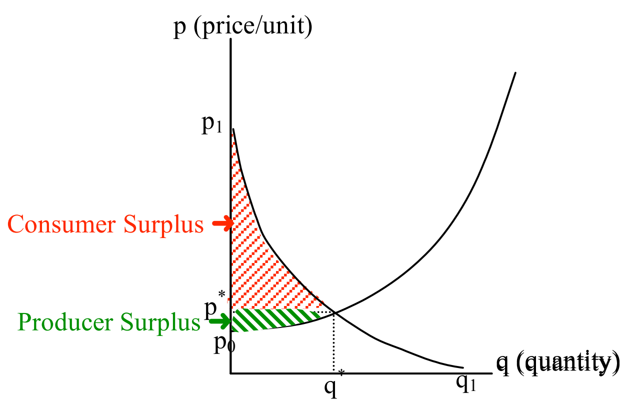
x	0	1	2	3	4	5
f(X)						

*Applied Calculus, Hughes-Hallet et al, Ed. 4

Example: If t is measured in days since June 1 the inventory I(t) for an item in a warehouse is given by $I(t) = 50000(0.9)^t$. Find the average inventory in the warehouse during the 90 days after June 1.
*(#12 p. 279)

***Note:** This is the same technique that we applied in Ch. 5 for finding the area under a curve when the formula is known.*

§6.2 Consumer & Producer Surplus



Recall From Ch. 1: (q^*, p^*) is equilibrium quantity and price

Meaning:
- Some consumer would willingly have paid more (up to p_1)
- Some producers would willingly have supplied at a lower price (as low as p_0)

End Result: Both are richer for having "traded" at a price different from their original willingness to pay/sell.

2 New Definitions

Consumer Surplus – The total amount gained by the consumer by buying at p^* rather than p_1

***Note:** Area under the demand curve and above the line $p = p^*$ (equilibrium price)*
(Total Money Could Have Put Out at p_1) – (Actual Money Out at p^*)

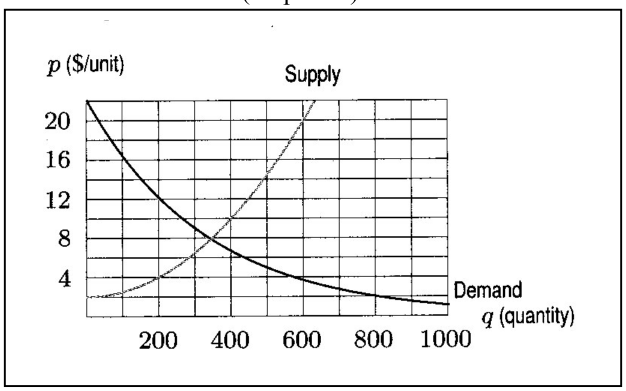
Producer Surplus – The total amount gained by producers by selling at p^* rather than p_0
***Note:** *Area above the supply curve and under the line $p = p^*$ (equilibrium price)*
(Total Money Made at p^*) – (Total Money Would Have Made at p_0)

Note #1: This is an application of finding the area between 2 curves that we discussed in §5.3.

Note #2: This is in the absences of price controls, so current price is assumed to be equilibrium price.

Example: Use the following graph to estimate equilibrium price and quantity. shade the surplus for each. To stay consistent with the above, shade producer surplus in green and consumer surplus in red.

- (#2 p. 284)



Example: If the demand curve is given by $p = 35 - q^2$ &
the supply curve is given by $p = 3 + q^2$
Find the producer surplus at equilibrium. * (#4 p. 285)

***Note:** This is using your algebra skills to find the equilibrium point and then your skills from chapter 5 to find the area between two curves given formulas (you will have to find the equation of the one curve – hint, hint).

Example: Use the following tables to answer the questions. * (#12 p. 285)

Table 1:

q	0	100	200	300	400	500	600
p	60	50	41	32	25	20	17

Table 2:

q	0	100	200	300	400	500	600
p	10	14	18	22	25	28	34

a) Which table is the supply and which is the demand? How do you know?

b) Estimate the equilibrium price and quantity? How do you know?

c) Estimate consumer and producer surplus.
*Hint: Use Riemann Sums

This section also discusses what happens when price controls are applied, but we are going to skip this portion of the chapter.

<u>§6.3 Present & Future Value</u>

In §1.7 the concept of present (P) and future value (B) were introduced.

$B = P(1 + r)^t$ for interest compounded yearly & solving for P we saw

$$P = B(1 + r)^{-t}$$

and

$B = Pe^{rt}$ for interest compounded continuously & solving for P we saw

$$P = Be^{-rt}$$

We can think of money coming in as income for a company as being a **<u>continuous stream of income</u>**:

$S(t)$ in \$/year @ t = # of years from present

Applying our knowledge of Riemann Sums to the interval of 0 to **M** years and combining with the formula for present value we see (for more detailed information see p. 287 of your text)

$$\mathbf{P} = \int_0^M \underbrace{S(t)\,dt}_{\text{Amt. Paid per interval.}}\ e^{-rt}$$

Amt. Paid per interval.
B ➜ Future Value

$$\underbrace{\text{B} \to \text{Future Value}}_{P = B\,e^{-rt}}$$

*M is # years in future

This leads to:

$$\textbf{Future Value} = \mathbf{FV} = \mathbf{B} = \mathbf{P\,e^{rM}}$$

$$\int_0^M S(t)\,e^{-rt}\,dt \bullet e^{rM}$$

*M is # years in future

Example: Find the present and future values of an income stream of \$12,000 a year for 20 years if the interest rate is 6% compounded continuously. * (#4 p. 288)

Note: *This is a calculator exercise using a formula that you will derive. Even though it has been said, it may not be clear to you that S(t) is a rate of increase per year and in this problem it is \$12,000.*

Here is a way that the last example could be applied to a real world situation.

Example: A company expects to earn \$50,000 at a continuous rate for 8 years. You can invest these earnings at 7% compounded continuously. You have a chance to buy the rights to the earnings now for \$350,000. Should you buy? Explain with Calculus why you should or should not buy. * (#10 p. 288)

Note: *You need to find out what the present value of the company's earnings will be and compare this with the money that you will pay. If the present value of the expected earnings is more than the money that you will pay now, then the investment is a good one.*

Here is an example that makes you think backward from area under the curve using a little guess and check.

Example: An oil company has a reserve of 100×10^6 barrels of oil. For $t > 0$, in years, the company's extraction plan is a linear declining function of time.

$$q(t) = 10 - 0.1t \qquad \times 10^6 \text{ barrels per year.}$$

How long will it take (approximately) to exhaust the reserve?

Hint: *Use a picture of the function to find where the area under the curve is approximately 100 and then use that value to guess and check using the function and your calculator.*

*Applied Calculus, Hughes-Hallet et al, Ed. 4