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Instructions: You may not receive any human help for completion of this quiz. You may use your book, notes and the internet or computer. Please show work. The quiz is due on Monday during the 1st 15 minutes of class. Reduction of points will result if the quiz is late as previously outlined.

Name: Key

Quiz #3 – Summer 2008
Math 5

1. I usually give my Intermediate Algebra students a multiple choice question that tests their order of operation skills. In a random sample of 55 students that took the exam, 23 got the question correct. Based upon this sample, answer the following questions.

+2

a) The random variable represents the number of students that correctly answered the question. What is the distribution of the random variable? Support you distribution with the characteristics and the exact manner in which they are satisfied? Binomially Distributed

+½ (1) Fixed # of trials – yes there are 55
+½ (2) Success/Failure – yes correct or incorrect
+½ (3) Constant prob. – yes, if guessing 1/4 each time
+½ (4) Independence – Each does not effect the other (no cheating)

+1

b) The possible answers for the question were a) – d). If my students were guessing what would be their probability of getting the correct answer?

a, b, c, d

$\boxed{p = 1/4 = 0.25}$ under classic prob.

+1½

c) Using your calculator, and assuming that my students were guessing (see b's answer) when they answered the exam question, find the probability at least 23 students answered my test question correctly. (Hint: This needs the binomcdf(n,p,x), which gives you the probability of being less than or equal to the x. If you don't have a calculator that will do this you can use Stat Disk or Excel or Minitab.)

$$P(X_B \geq 23) = \underbrace{1 - P(X_B < 23)}_{+1} = 1 - P(X_B \leq 22) = 1 - 0.9954$$

Rounded from 0.9953741842

$$\approx \boxed{0.0046} \quad +½$$

+1

d) Based upon the answer to part c), do you think it is likely that my students were guessing? Why or why not?

+½ No, +½ the probability in c) is less than 0.05, & ∴ it would be a rare occurance that more than 23 guessed correctly.

+1½

e) Now let's assume that my students really define the population of students who will answer this question, what is the probability that a person belonging to this population will answer the question correctly? (Hint: Use x/n from the original information given.)

$$p = \hat{p} = \frac{23}{55} = 0.41\overline{81} \approx \boxed{0.418}$$

+½ +½ +½

+4½

f) Now, find the **approximate** probability that in a sample of 55, more than 20 people will answer the question correctly. (Hint: Use my Alg. Students' sample proportion as the proportion for calculation.) $0.\overline{675675}$

Note for future: use # in class.

Continuity +1

$$P(X_B > 20) = P(X_N \geq 20.5) = P\left(\frac{x-\mu}{\sigma} \geq \frac{20.5-23}{3.7}\right) = P(Z > -0.68)$$

$$= 1 - P(Z < -0.68) = 1 - 0.2496 = \boxed{0.7504}$$

+½ +½ +½

+1 $\mu = 55 \cdot \frac{23}{55} = 23$

+1 $\sigma = \sqrt{55\left(\frac{23}{55}\right)\left(\frac{32}{55}\right)} = \sqrt{13.3\overline{818}} = 3.658116753 \approx 3.7$

+11.5

+1½

g) What would the **expected number** of correct answers be in a sample of 55, if my Algebra students really represent the population proportion of students that will answer the question correctly?

+½ +½ +½

$$E(x) = np = 55 \cdot \frac{23}{55} = 23$$

+2

+½ +½

h) Between 15.6 and 30.4 students in a sample of 55 would **usually** answer the question correctly, if the Algebra students' proportion is assumed to represent the population proportion. Show the work for giving the answers to this question.

$\mu \pm 2\sigma$

$23 \pm 2(3.7)$

$23 + 7.4 = 30.4$

$23 - 7.4 = 15.6$

+1 See work on f) for answers

+3½

i) Give me a 95% confidence interval for the true population proportion of students that will answer the question correctly. (Hint: Use my Alg. Students as your sample information.) Show all work for E and for obtaining the interval as well as the interval. +½

$z_{\alpha/2} = 1.96$ +1

$\boxed{(0.288,\ 0.548)}$ +½ +½

$$\hat{p} \pm z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}} \Rightarrow \frac{23}{55} \pm 1.96\sqrt{\frac{(23)(32)}{55^3}} \Rightarrow 0.418 \pm 0.130$$

0.1303619788 +1

+2

j) If I wanted to test the hypothesis that the proportion of students that get the answer correct is over 50%, write the null and alternative hypotheses and indicate whether this is a right/left or two-tail test.

$H_0: p \le 0.5$ or $p = 0.5$

$H_A: p > 0.5$ +1

+1 $\boxed{\text{Right/Upper Tail}}$

EC

k) OK, here's your chance to see if my Algebra students are representative of the population! Oh, and to test your knowledge!! HAHAHA! BTW: 1 pt EC!! ☺

For the following circle the most appropriate answer: $\dfrac{-3\,|\,3 - 18\,| \div 9 + 2}{5 + 2[-6 - (-2)] + -2^2} = \dfrac{-3}{-7} = \dfrac{3}{7}$

-45; $|-15| = 15$; -5; -4; -8; $-(2 \cdot 2) = -4$

a) $\frac{3}{-1} = -3$ b) $\frac{7}{-7} = -1$ $\boxed{\text{c) } \frac{-3}{-7} = \frac{3}{7}}$ d) $\frac{45}{11}$

2. The following data represents the number of grams of fat in 22 randomly sampled McDonald's breakfast meals. Compute the following for this data.

2, 8, 11, 15, 16, 23, 23, 23, 31, 33, 35

1, 8, 11, 12, 16, 17, 23, 28, 28, 33, 40

+2

a) Find the mean and standard deviation of the sample. No work is necessary, your calculator is allowed to do the work for you this time. Just list the answers here.

+1 $\bar{x} = 19.9$ Exact was $19.86\overline{36}$

+1 $S = 10.9$ ⟵ Mark wrong if wrong b/c they use σ not s
10.68 or 10.7 is this #

+11

b) If we assume that the data comes from an approximately normal distribution, with the calculated mean and standard deviation, find the probability that a randomly chosen breakfast meal has more than 28 grams of fat.

+1½

$$P(X>28) = P\left(\frac{x-\mu}{\sigma} > \frac{28-19.9}{10.9}\right) = P(Z>0.74) = 1-P(Z<0.74)$$

$$= 1-0.7704 = \boxed{0.2294}$$

+½ +½ +½

Using s=10.7 or 10.68 would give P(Z>0.76) = 1-0.7764 = 0.2236

c) How does the answer in b) compare with the probability that we expect in being above the 3rd quartile of a distribution? First calculate the 3rd quartile using an indicator/locator function and show all work here. $L_{25} = \frac{1}{4}\cdot 22 = 5\frac{1}{2} \uparrow 6$ or $L_{75} = \frac{3}{4}\cdot 22 = \frac{66}{4} = 16\frac{1}{2} \uparrow 17$

+½

$Q_3 = 28$

$$P(X>Q_3) = 0.25$$

$\boxed{\text{b)'s answer is close to 25\%}}$

d) If we assume that the data comes from an approximately normal distribution, with the calculated mean and standard deviation, find the probability that a randomly chosen breakfast meal has less than 11 grams of fat.

+1½

$$P(X<11) = P\left(\frac{x-\mu}{\sigma} < \frac{11-19.9}{10.9}\right) = P(Z<-0.82) = 0.2061$$

+½ +½ +½

Using 10.7 or 10.68 would give P(Z<-0.83) = 0.2033

e) If we assume that the data comes from an approximately normal distribution, with the calculated mean and standard deviation, find the probability that a randomly chosen breakfast meal has between 11 and 28 grams of fat.

+2

$$P(11<X<28) = P\left(\frac{11-19.9}{10.9} < \frac{x-\mu}{\sigma} < \frac{28-19.9}{10.9}\right) = P(-0.82<Z<0.74)$$

$$= P(Z<0.74) - P(Z<-0.82) = \boxed{0.7704-0.2061}$$

$$= \boxed{0.5643}$$

+½ +½ +½ +½

(TI yields 0.5642 or wrong s=10.7 yields 0.7764-0.2033 = 0.5731)

f) Find the probability that in 22 samples of breakfast meals the average number of grams of fat would be above 25.

CLT +1

+2½

$$P(\bar{X}>25) = P\left(\frac{\bar{x}-\mu_{\bar{x}}}{\sigma_{\bar{x}}} > \frac{25-19.9}{10.9/\sqrt{22}}\right) = P(Z>2.19)$$

$$= 1-P(Z<2.19) = 1-0.9857 = \boxed{0.0143}$$

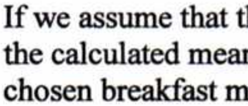
+½ +½ +½

Wrong s=10.68: $\frac{25-19.9}{10.7/\sqrt{22}} = 2.24$, $1-0.9875 = 0.0125$

g) If you conducted the sampling in f), and got averages above 25, what would you say about the data which you based your assumptions on?

+½

The original sample wasn't that good to app. pop b/c it is unusual to get $\bar{x}$ above 25.
↳ ≈0.01 < 0.05



← 0.0901 as 0.0885 1.35

+1

h) Give a 82% confidence interval for the true population mean for grams of fat in breakfast meals. Show your work for critical values as well as for margin of error, calculation of the interval and the explicit interval.

+3

$$\bar{X} \pm Z_{\alpha/2}\, \sigma/\sqrt{n}$$

$$19.9 \pm 1.34(10.9/\sqrt{22})$$

Really should use t, but this requires more than book gives us so must assume σ = s & use z going with all other parts too

+1 +½ +½

E = 3.114009663 ≈ 3.1 $\boxed{16.8,\ 23}$

+11.5

$\chi^2_{21,0.95} = 11.591$ +½ $\chi^2_{21,0.05} = 32.671$ +½

0.1/2 = 0.05

+4

i) Find a 90% confidence interval for the true population standard deviation.

$$\sqrt{\frac{(10.9)^2 21}{32.671}} < \sigma < \sqrt{\frac{(10.9)^2 21}{11.591}}$$

+½ +½

$\sqrt{76.36712673}$ $\sqrt{215.2540764}$

Roots +1

(8.7 , 14.7)

Range +1

j) Your client, competitor of McDonald's, claims that their breakfast meals have less fat than McDonald's. State a null and alternative hypothesis for this claim and indicate whether this a right/left or two-tail test.

+2

$H_0: \mu \geq 19.9$

$H_A: \mu < 19.9$

$\boxed{\text{Left Tail/Lower Tail}}$

+6