§4.5 Carson's 3rd Edition

More Complex Translation/Number Problems

These number problems involve not one but two unknowns that are defined in terms of one another. This definition of one unknown in terms of the other will involve a translation of a sum. Finally, the two unknowns will sum to a given total, completing the similarity to the simple number problem.

> First Unknown = x
> Second Unknown = First Unknown + #
> Total = First Unknown + Second Unknown

Example: #8 p. 279
One number is eight less than another number. The sum of the numbers is seventy-two. Find the numbers.

Complementary & Supplementary Angle Problems

A **degree** is the basic unit of measure in angles. It is defined as $\frac{1}{360}$th of a rotation in a whole circle. A degree can be denoted by a tiny circle above and to the right of a number (°). A **right angle** (denoted with a square inside the angle; see below) is a 90° angle and a **straight angle** is a 180° angle. Complementary and supplementary angles are related to right and straight angles respectively. Just one more note – the word angle can be written short hand with the symbol ∠.

Complementary angles sum to 90°. (I remember complementary is 90° because it comes before 180° and "C" is comes before than "S" in the alphabet.)

Example: The following angles are complementary.
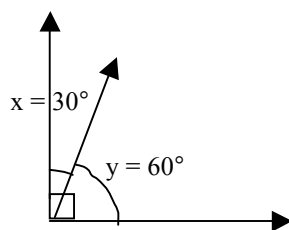
$$x + y = 90°;$$
$$30° + 60° = 90°$$

Supplementary angles sum to 180°. (Likewise, I remember supplementary is 180° because it comes after 90° and "S" is comes after than "C" in the alphabet.)

Sometimes we will be given the information that two angles are either supplementary or complementary, but we will only be given the measure of one angle and we will have to use algebra to find the missing angle.

Example: #23 p. 281
A laser beam hits a flat mirror a some angle and reflects off the mirror at the same angle with respect to the mirror creating an angle between the incoming and outgoing beams that is 10° more than three times the measure of the incoming/outgoing angle. Find the measure of all the angles created.

While we are on the topic of angles let's briefly discuss a topic that comes up in some exercises – the number of **degrees in a triangle**. We have discussed the three-sided polygon before in terms of its perimeter, but we have not discussed any other properties of this figure. Since there are three sides that make up a triangle, each pair meet at an angle, and although each angle in any given triangle is unique there are some basic rules that they must adhere to. The most basic is that the **sum of all angles in a triangle** (**degrees in a triangle**) is always 180°. This means that no angle can ever measure 180° or more. A triangle with 3 equal sides is called an equilateral triangle. An equilateral triangle has three equal angles as well. Another special triangle is an isosceles triangle. An isosceles triangle has 2 equal sides and the angles opposite those sides are also equal.

> **Example:** In a triangle, the second angle is 5° less than the twice the smallest and the largest is 10° more than four times the smallest. Find the largest angle's measure.

Next we'll remind ourselves of the formula for the Perimeter of Rectangle. The distance around the outside of a geometric figure is its perimeter. Here is the formula for perimeter of a rectangle:

$$P = 2\ell + 2w \qquad \text{where } \ell = \text{length } \& \ w = \text{width}$$

> **Example:** #12 p. 279
> Eighty-four feet of wallpaper boarder was used to go around a rectangular room. The width of the room is 4 feet less than the length. What are the length and the width of the room?

HW §4.5 p. 279 #7, 13, 15, 19, 21 & 27

Review From §3.4 of Carson's 3rd Edition
Product Rule for Exponents

$a^r \, a^s \; =$

Example: Use Product Rule of Exponents to simplify each of the
following. Write the answer in exponential form.

a) $\quad x^2 \, x^3$ b) $\quad (-7a^2 \, b)(5ab)$ c) $\quad 3^2 \bullet 3^7$

d) $\quad y^2 \bullet y^3 \bullet y^7 \bullet y^8$

Monomial x Monomial
 Multiply the coefficients
 Add the exponents of like bases

Example: Find each product.

a) $\quad (5a^2)(-6ab^2)$ b) $\quad (-r^2s^2)(3ars)$

Multiplying polynomials is an application of the <u>distributive property.</u> This is also called
<u>expanding</u>.

Review $a \, (\, b \, + \, c \,) \; = \; ab \, + \, ac$

Monomial x Polynomial

Example: Multiply.

a) $\quad 2x \, (x^2 \, + \, 2x \, + \, 3)$ b) $\quad -4x^2 \, y(x^2y \, - \, 2xy \, + \, 3y)$

Binomial x Binomial

Now we'll extend the distributive property further and to help us remember how we will have an acronym called the **FOIL** method.

$$(a + b)(d + c)$$

F = **First**

O = **Outside** ⎱

I = **Inside** ⎰ These add & are like terms if the binomials are alike.

L = **Last**

Example: Multiply. Think about only sum & product of the constants to get the middle & last terms of your polynomial.

a) $(x + 2)(x + 3)$ b) $(x - 5)(x + 9)$

c) $(x - 4)(x - 3)$ d) $(x + 7)(x - 9)$

Note: A binomial of the form (x + c) multiplied by a binomial of the same form will always yield a trinomial. The first terms yield the highest degree term (of degree 2), the inside and outside add to give the middle term and the last will yield the constant.

Example: Multiply. Remember that it is the outside & inside products that will add & the last & first terms multiply to give first and last terms of the trinomial.

a) $(x - 5)(2x + 3)$ b) $(3x + 4)(x + 5)$

c) $(2x - 5)(x - 4)$ d) $(2x - 5)(x + 3)$

e) $(2x + 3)(5x + 4)$ f) $(2x - 4)(3x + 5)$

g) $(2x + 4)(3x - 5)$ h) $(3x + 5)(4x + 1)$

Example: Multiply. $(x^2 + y)(x - y)$

Note: This didn't yield a trinomial because the binomials are not of the same form.

Example: Multiply.
a) $(5x + 1)(3x + 1)$ b) $(4x - 1)(3x - 1)$

c) $(5x - 1)(6x + 1)$ d) $(8x + 1)(9x - 1)$

Note: These are just like multiplying when the numeric coefficient of the x is one. The only differences are that the leading coefficient is the product, not the constants, and you must watch the sign on the middle term a little more carefully. Product of x coefficients gives leading coefficient & sum/difference of the x's coefficients gives the middle term (just watch if positive or neg. since that comes from outside & inside products).

Example: Find the product.
a) $5x(x + 2)(x - 2)$ b) $-3r(5r + 1)(r + 3)$

Note: The 1ˢᵗ example is possibly more easily done by doing the product of the binomials 1ˢᵗ and then multiplying by the monomial. Always remember that multiplication is commutative and associative so you can choose which product you find 1ˢᵗ.

Multiplying Conjugates (the Sum & Difference of 2 terms)

$(a + b)(a - b) =$

1st Term Squared   –   2nd Term Squared

Example: Expand each of the following using the shortcuts
a) $(2x - 4)(2x + 4)$ b) $(a + 2)(a - 2)$ c) $(3a + ¼)(3a - ¼)$

Your Turn
Multiply the conjugates.
a) $(x + y)(x - y)$ b) $(2 - y^2)(2 + y^2)$

c) $(c + \frac{1}{3})(c - \frac{1}{3})$

Polynomial x Polynomial
Multiplying two polynomials is an extension of the distributive property. The first term of the first polynomial gets multiplied by each term of the second, and then the next term of the first gets multiplied by each term of the second and so on and the end result is the sum of all the products.

Of course there is a second way to do the distributive property that takes care of keeping all the degrees of variables in order, and that looks like long multiplication. All polynomials must be ordered for this method to be successful. I will do each of the examples below using both methods.

Example: Multiply using long multiplication.
a) $(2x + 3)(x^2 + 4x + 5)$ b) $(2x - 7)(x^2 - 2x + 1)$

The following example involving fractions may very well best be done using the method of long multiplication.

Example: Find the product.
$$(2x^2 + x - 8)(2x + 3)$$

In this last example we have to simplify by multiplying the first two polynomials out and then multiplying that result by the third polynomial. We'll only do this example one way, but I will use one method on the first 2 and then the other method on the result times the last. It is interesting to point out that sometimes it is useful to use the commutative property to rearrange the order thus deciding the two you will multiply first, as some binomial products are much simpler than others! We will learn about these cases in the next section.

Example: $(x + 1)(x - 2)(2x + 3)$

HW for §3.4 p. 190 #65-77odd, #87,89&91, #95-101odd

§3.6 of Carson's 3rd Edition
Now we are going to un-do the multiplication that we just did.

Recall: The Quotient Rule of Exponents
$$\frac{a^r}{a^s} =$$

Example: Simplify each. Don't leave any negative exponents.

a) $\dfrac{a^{12}}{a^7}$ b) $\dfrac{2b^5}{b^3}$ c) $\dfrac{x^8}{x^2}$

Division of a Polynomial by a Monomial
Step 1: Simplify the numerator as much as possible (combine like terms)
Step 2: Break down as sum of fractions
Step 3: Use exponent rules and division to simplify each term
Step 4: Check by multiplying quotient by divisor to see if it equals dividend

Example: Divide and check

a) $\dfrac{x^2 + 3x}{x}$

b) $\dfrac{15\,x^3\,y + 3\,x^2\,y - 3\,y}{xy}$

c) $(27\,x^5 - 3\,x^3 + 4x) \div 9x^2$

d) $\dfrac{x^2 - 4x + 1}{-x^2}$

Your Turn

Example: Divide and write in the most simplified form (Do not convert improper fractions to mixed numbers.)

a) $\dfrac{15x^2y - 21xy^2 + 39}{3x^2y}$

b) $(20a^5b^3 + 15a^3b^2 - 25ab) \div 5a^2b$

Next we will be using our division of a polynomial by a monomial to do something called **factoring**. First, we want to remember some past skills.

Recall that the greatest common factor(GCF) is the largest number that two or more numbers are divisible by. (This is §3.5 in Carson if you would like to review a little more)

Finding Numeric GCF
Step 1: Factor the numbers.
    a) Write the factors in pairs so that you get all of them starting with 1 • ? = #
Step 2: Find the largest that both have in common.

Example: Find the GCF of the following.

a)   18 & 36

b)   12, 10 & 24

We are going to be extending this idea with algebraic terms. The steps are:
  1) Find the numeric GCF (negatives aren't part of GCF)
  2) Pick the variable(s) raised to the **smallest power** that all have in common (this btw also applies to the prime factors of the numbers)
  3) Multiply number and variable and you get the GCF

  Example: For each of the following find the GCF
  a) x^2, x^5, x b) x^2y, x^3y^2, x^2yz

  c) $8 x^3, 10x^2, -16x^2$ d) $15 x^2y^3, 20 x^5y^2z^2, -10 x^3y^2$

HW §3.5 p. 206 #69-79odd

Now we'll use this concept to **factor** a polynomial. **Factor** in this sense means change from an addition problem to a multiplication problem! This is the opposite of what we did in chapter 5.

Factoring by GCF
Step 1: Find the GCF of all terms
Step 2: Rewrite as GCF times the sum of the quotients of the original terms divided by the GCF

Let's start by practicing the second step of this process. Dividing a polynomial that is being factored by its GCF.

  Example: Divide the polynomial on the left by the monomial (the GCF) on the right to get what goes in the parentheses on the right.

  a) $2x^2 + 10 = 2($ $)$

  b) $15a^2b + 18a - 21 = 3($ $)$

  c) $y - x = -1($ $)$

Note: This makes the binomial $y - x$ look like its opposite $x - y$. This is important in factoring by grouping!

Example: For each of the following factor using the GCF

a) $8x^3 - 4x^2 + 12x$

b) $27a^2b + 3ab - 9ab^2$

c) $18a - 9b$

d) $108x^2y^2 - 12x^2y + 36xy^2 + 96xy$

Sometimes the GCF can be a binomial too. This is a little trickier because we have to be able to see the individual terms before we can take out the GCF.

Example: Factor the binomial GCF from this binomial.
$$2x^2(y + 3) + 7(y + 3)$$

Your Turn

1. Find the GCF of the following

a) $28, 70, 56$

b) x^2, x, x^3

c) $3x^3, 15x^2, 27x^4$

d) $x^2y^3, 3xy^2, 2x$

e) $12xy^2, 20x^2y^3, 24x^3y^2$

2. Factor the following by factoring the GCF
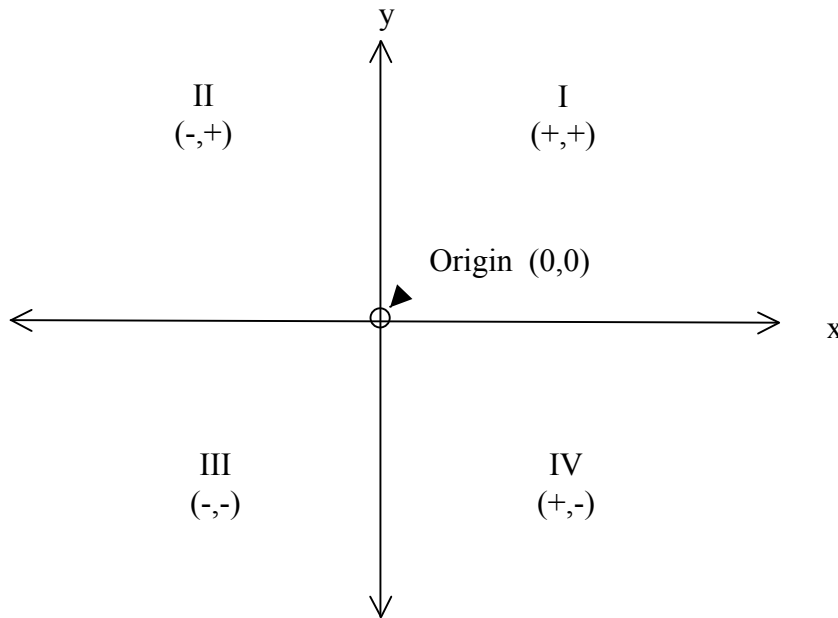
a) $28x^3 - 56x^2 + 70x$

b) $9x^3y^2 - 12xy - 9$

c) $15a^2 - 60b^2$

d) $a^2(a + 1) + b^2(a + 1)$

HW §3.6 p. 214 #31, 37, #63-81 multiples of 3 (add 3 from 63)

§9.2 Carson's 3rd Edition

The following is the Rectangular Coordinate System also called the Cartesian Coordinate System (after the founder, Renee Descartes). The x-axis is horizontal and is labeled just like a number line. The y-axis intersects the x-axis perpendicularly and increases in the upward direction and decreases in the downward (negative) direction. The two axes usually intersect at zero. They form a **plane**, which is a flat surface such as a sheet of paper (anything with only 2 dimensions is a plane).

```
                        y

        II                      I
      (-,+)                   (+,+)


                     Origin  (0,0)


  ←————————————————○———————————————→  x


        III                    IV
      (-,-)                   (+,-)
```

A **coordinate** is a number associated with the x or y axis.

An **ordered pair** is a pair of coordinates, an x and a y, read in that order. An ordered pair names a specific point in the system. Each point is unique. An ordered pair is written (x, y)

The **origin** is where both the x and the y axis are zero. The ordered pair that describes the origin is (0, 0).

The **quadrants** are the 4 sections of the system created by the two axes. They are labeled counterclockwise from the upper right corner. The quadrants are named **I, II, III, IV**. These are the Roman numerals for one, two, three, and four. It is not acceptable to say One when referring to quadrant I, etc.

Plotting Points/Graphing Ordered Pairs

When plotting a point on the axis we first locate the x-coordinate and then the y-coordinate. Once both have been located, we follow them with our finger or our eyes to their intersection as if an imaginary line were being drawn from the coordinates.

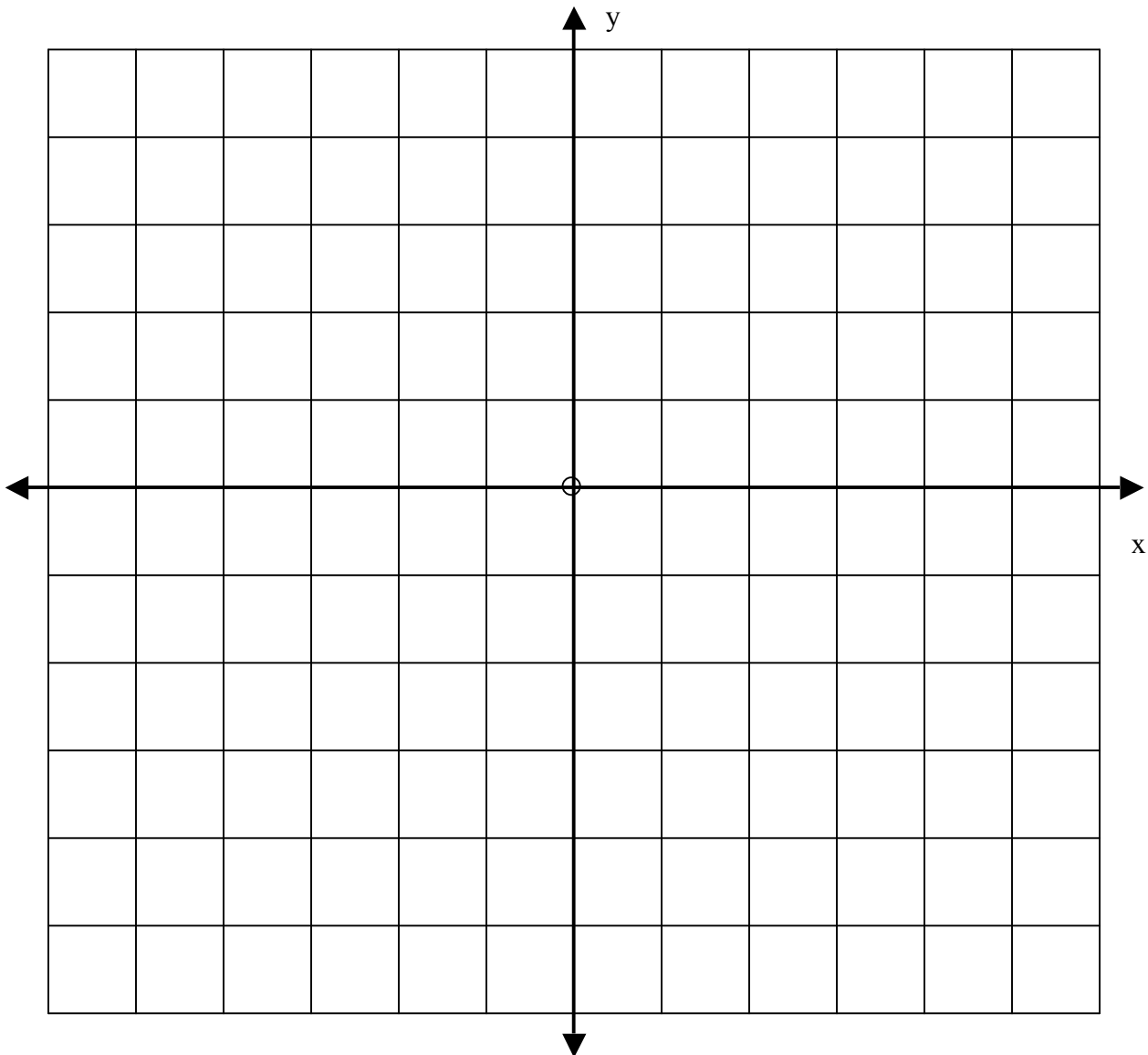**Example:**   Plot the following ordered pairs and note their quadrants

a) (2,-2)   b) (5,3)   c) (-2,-5)

d) (-5,3)   e) (0,-4)   f) (1,0)

Also useful in our studies will be the knowledge of how to label points in the system.

Labeling Points on a Coordinate System

We label the points in the system, by following, with our fingers or eyes, back to the coordinates on the x and y axes. This is doing the reverse of what we just did in plotting points. **When labeling a point, we must do so appropriately! To label a point correctly, we must label it with its ordered pair written in the fashion – (x,y).**

      Example: Label the points on the coordinate system below.
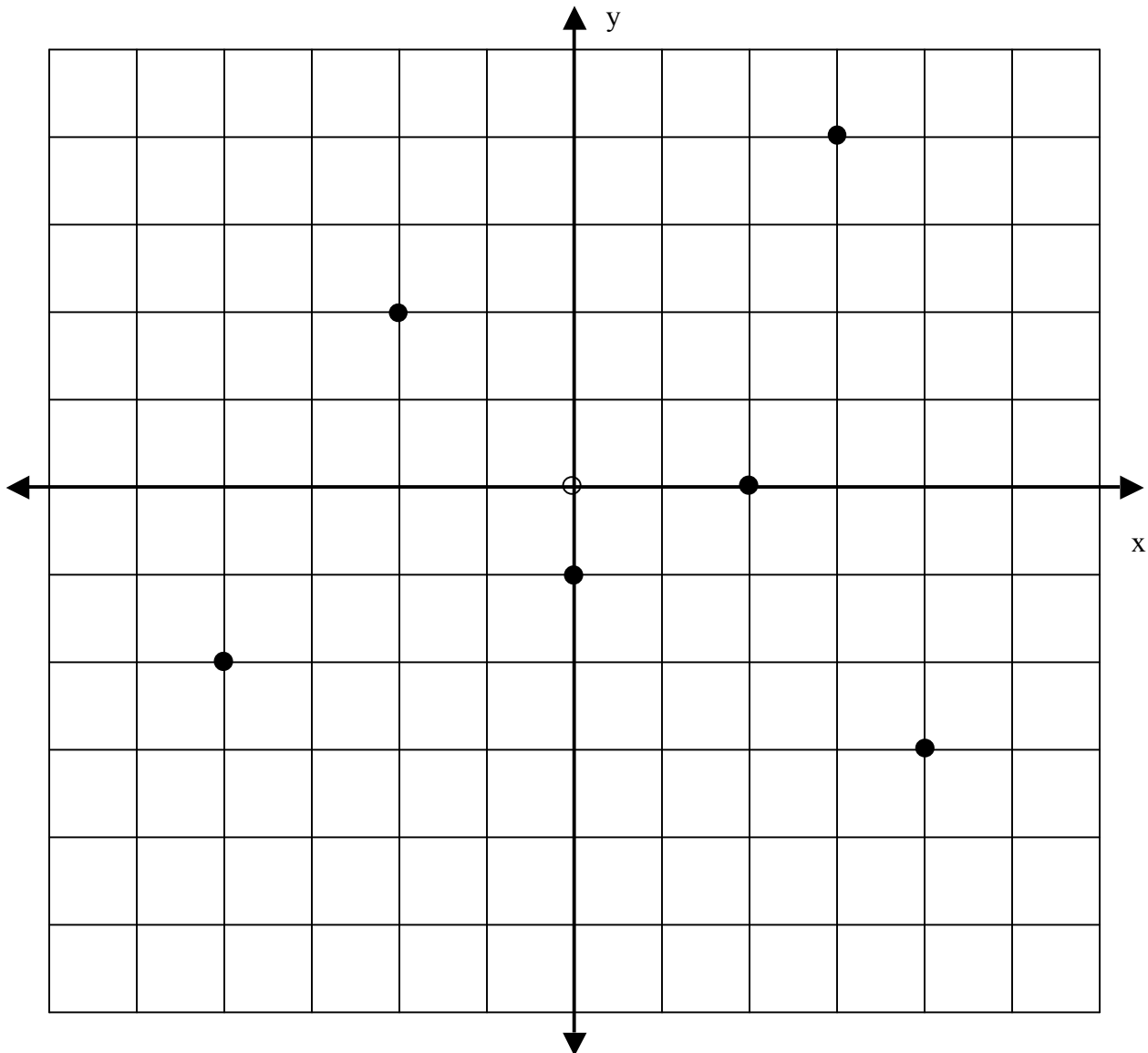
HW §9.2 p.666 #9, 12, 15, 18, 20, 21, 22, 24, & 26

§9.3 Carson's 3rd Edition

A **Linear Equation in Two Variables** is an equation in the form shown below, whose *solutions are ordered pairs*. A linear equation in two variables can visually (graphically) be represented by a straight line (our linear equations from chapter 2).

$$ax + by = c$$

a, **b**, & **c** are real numbers
x, y are variables
x & y both can not = 0

A linear equation in two variables has solutions that are **ordered pairs**. Since the first coordinate of an ordered pair is the x-coordinate and the second the *y*, we know to substitute the first for *x* and the second for *y*. *(If you ever come across an equation that is not written in x and y, and want to check if an ordered pair is a solution for the equation, assume that the variables are alphabetical, as in **x** and **y**. For instance, 5d + 3b = 10 : **b** is equivalent to **x** and **d** is equivalent to **y**, unless otherwise specified.)*

Notice that we said above that the **solutions** to a linear equation in two variables! A linear equation in two variables has an *infinite number of solutions*, since the equation represents a straight line, which stretches to infinity in either direction. An ordered pair can represent each point on a straight line, and there are infinite points on any line, so there are infinite solutions. The trick is that there are only *specific ordered pairs* that are the solutions!

If we wish to check a solution to a linear equation it is a simple evaluation problem. Substitute the x-coordinate for x and the y-coordinate for y and see if the statement created is indeed true. If it is true then the ordered pair is a solution and if it is false then the ordered pair is not a solution. Let's practice.

Is An Ordered Pair a Solution?
Step 1: Substitute the 1st number in the ordered pair for x
Step 2: Substitute the 2nd number in the ordered pair for y
Step 3: Is the resulting equation true?   Yes, then it is **a solution**.
                                          No, then it is **not a solution**.

   Example: Check to see if the following is a solution to the linear equation.
   a) $y = -5x$; (-1, -5)   b) $x + 2y = 9$; (5,2)

   c) $x = 1$; (0,1)   d) $x = 1$; (1, 10)

   e) $y = 5$; (5, 12)

Note: When you are given an equation for a horizontal or vertical line (you may recall from another class that these are y=#, x=# respectively) if the ordered pair does not have the correct y-coordinate (when it's a horizontal line) or x-coordinate (when it's a vertical line) then it is not a solution as we have observed in the last 3 examples.

Along with checking to see if an ordered pair is a solution to an equation, we can also **complete an ordered pair** to make it a solution to a linear equation in two variables. The reason that we can do this is because every line is infinite, so at some point, it will cross each coordinate on each axis. **The way that we will do this is by plugging in the coordinate given and then solving for the missing coordinate.** Problems like this will come in two types – find one solution or find a **table of solutions**. Either way the problem will be solved in the exact same way. This will prepare us for finding our own solutions to linear equations in two variables so that we will be able to graph a line!

Completing An Ordered Pair to Find a Solution
Step 1:   Substitute the given coordinate into the linear equation in 2 variables
Step 2:   Solve for the remaining variable using your skills from Chapter 2
Step 3:   Give the solution as an ordered pair or in a table

> **Example:**   Complete the ordered pairs to make it a solution for the linear equation in two variables.
> $$x - 4y = 4 \; ; \; (\;, -2), (4, \;)$$

> **Example:**   Complete the <u>table of values</u> for the linear equation
> $$-2x + y = -1$$

x	y
-2	
	-3
	5

Now, that you have found solutions when someone gave you a "jump-start" it is time to do it all by yourself. Choose an x and solve for y or choose a y and solve for x.

Graphing Using Three Random Points

**Step 1:** Find three solutions for your equation (make sure they are integer solutions)
Step 2: Graph the 3 solutions found in step 1 and label with ordered pairs
Step 3: Draw a straight line through the ordered pairs
(if it's not a straight line, you've incorrectly found at least one solution.)
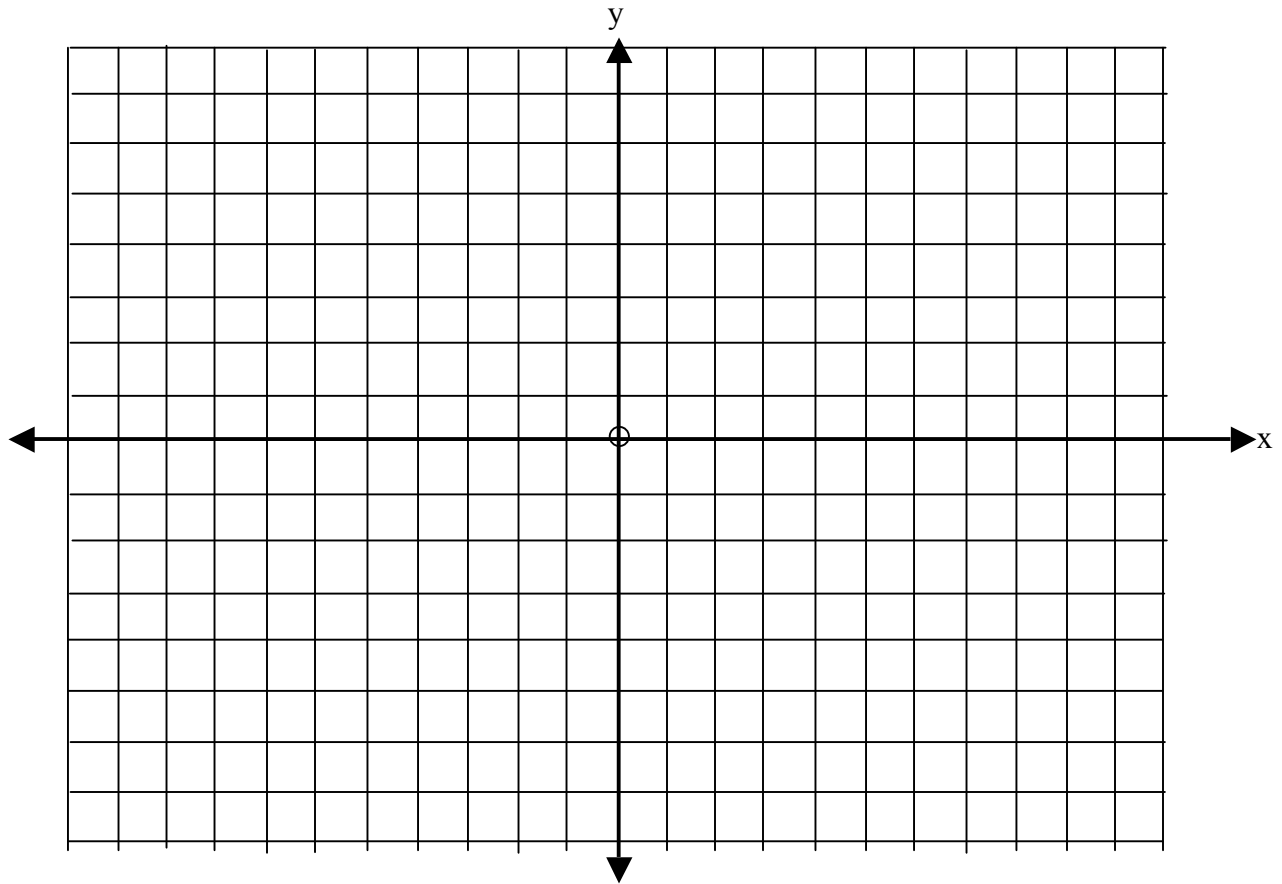**Step 4:** Put arrows on the ends of the line, label it with its equation.

We're only doing this once, because this really isn't the best way to graph a line as we will see in the following sections. Each section will succeed in bettering our method. In section 3 we will use this method, but with 2 special points. In section 4 we will learn about using the slope and y-intercept which can be used to graph any equation quickly and easily.

Example: Graph $2x + 5y = 10$ using the method above.
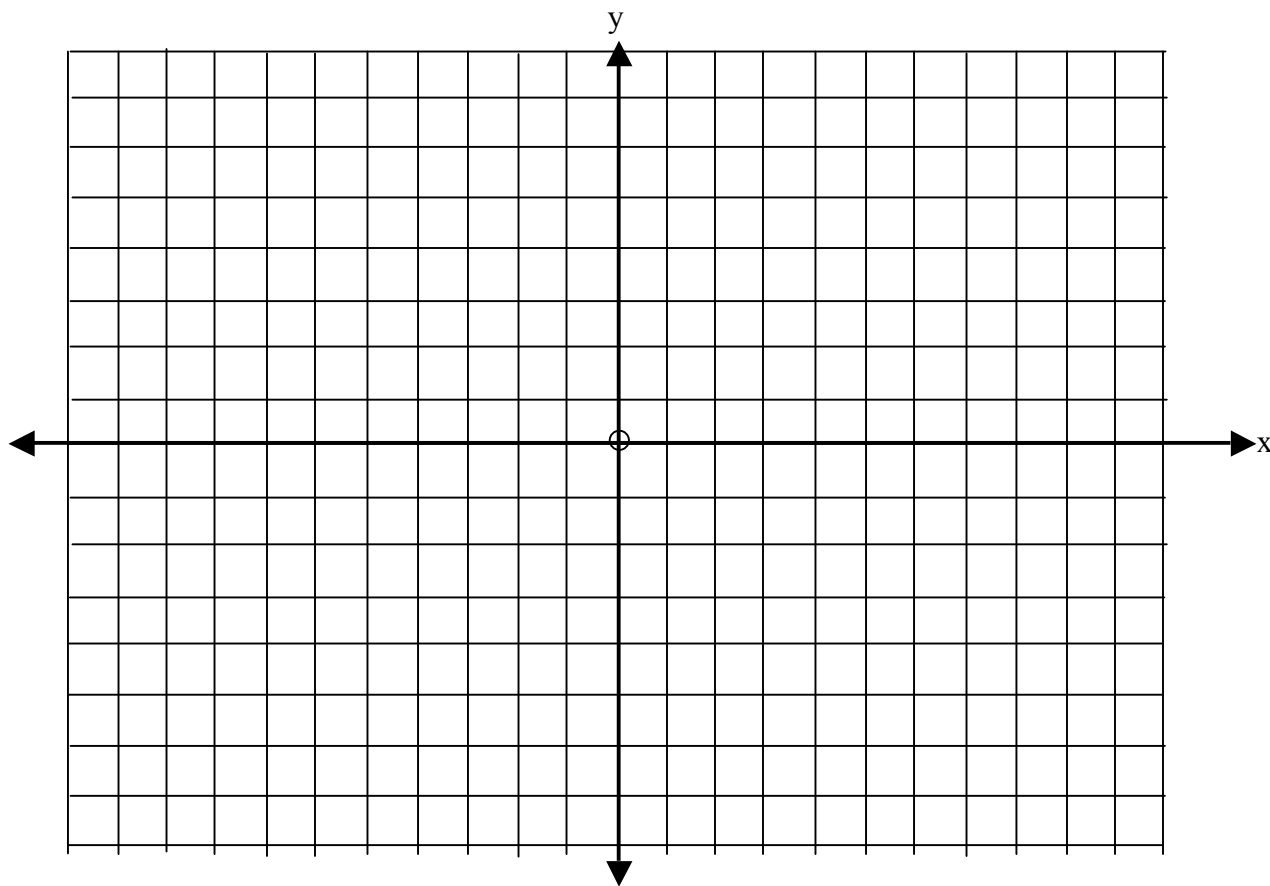
Your Turn

      Example: Graph $3y = 2x - 12$ using the method above.

HW §9.3 p. 676 #15, 18, 21 & 24