

Example #2 p. 5 Ch. 9

Find the angle between the vectors
 $2i + j$ and $-3i + j$

Understand Component form of unit vectors

- Remember that you “pluck off” i & j and use them as components a & b

so, $2i + j = \langle 2, 1 \rangle$

so, $-3i + j = \langle -3, 1 \rangle$

Find the magnitude of $u = \langle 2, 1 \rangle$,

$$|u|$$

- Magnitude of u is $c = \sqrt{a^2 + b^2}$

So, $c = \sqrt{4 + 1} = \sqrt{5}$

$$|u| = \sqrt{5}$$

Find the magnitude of $v = \langle -3, 1 \rangle$,
 $|v|$

- Magnitude of v is $c = \sqrt{a^2 + b^2}$

So, $c = \sqrt{9 + 1} = \sqrt{10}$

$$|v| = \sqrt{10}$$

Find the dot product of u & v

- Multiply the **vertical** components of u & v

$$u_a \bullet v_a = 2 \bullet -3 = -6$$

- Multiply the **horizontal** components of u & v

$$u_b \bullet v_b = 1 \bullet 1 = 1$$

- The dot product is a **scalar**. Sum vertical & horizontal component products

$$u \text{ dot } v = -6 + 1 = -5$$

Use the dot product formula to solve for θ

- Using the fact that the dot product of u & v is equal to $|u| |v| \cos \theta$, θ can be found as

$$\theta = \cos^{-1} \frac{u \text{ dot } v}{|u| |v|}$$

$$\text{So, } \theta = \cos^{-1} \frac{-5}{\sqrt{5} \cdot \sqrt{10}} = \cos^{-1} \frac{-5}{5\sqrt{2}} = \cos^{-1} \frac{-1}{\sqrt{2}}$$

thus, we know this is 45° in Quadrant II since inverse cosine is defined on $[0, \pi)$, therefore the angle between them is $180^\circ - 45^\circ = 135^\circ$

Thus, θ is

$$\theta = 135^\circ$$