Example 1 Hyperbolas p. 9 Ch 11

Shifted Hyperbolas

$$x^2 - y^2 = 10(x - y) + 1$$

- 1st the expand

$$x^2 - y^2 = 10x - 10y + 1$$

- Group all x's & y's leaving the constant on the right

$$x^2 - 10x - y^2 + 10y = 10x - 10x - 10y + 10y + 1$$

$$x^2 - 10x - y^2 + 10y = 1$$

Complete the Square for x's & y's

- Complete the squares for the x's

$$(1/2 \cdot 10)^2 = (5)^2 = 25$$

$$(x^2 - 10x + 25) - y^2 + 10y = 1 + 25$$

$$(x - 5)^2 - y^2 + 10y = 26$$

- Complete the squares for the y's

Step 1: Remove negative coefficient

$$(x - 5)^2 - (y^2 - 10y\ \ ) = 26$$

Step 2: Complete the square

$$(1/2 \cdot 10)^2 = (5)^2 = 25$$

$$(x - 5)^2 - (y^2 - 10y + 25) = 26 - 25$$

Remember that -25 was actually added in the left because of the negative factored out!

$$\frac{(x - 5)^2}{1} - \frac{(y - 5)^2}{1} = 1$$

a) Opens up/down or left/right?

$$\frac{(x-5)^2}{1} - \frac{(y-5)^2}{1} = 1$$

- This Hyperbola opens left/right since the x^2 is positive & y^2 is negative

$$\frac{(x-5)^2}{1} - \frac{(y^2-5)^2}{1} = 1$$

- Get a, b & c
- a^2 is the positive denominator

  so, $a^2 = 1$ so, $a = 1$
- b^2 is the negative denominator

  so, $b^2 = 1$ so, $b = 1$
- $c^2 = a^2 + b^2$

  so, $c = \sqrt{c^2} = \pm\sqrt{1 + 1} = \pm\sqrt{2} \approx \pm 1.4$

  so, $c = \pm\sqrt{2}$

b) Give the Center

- The center of a shifted hyperbola is at (h, k)

C(5, 5)

c) Give the Vertices

- The vertices are $V_1(h - a, k)$ & $V_2(h + a, k)$ since this hyperbola opens left/right

Thus, $V_1(5 - 1, 5)$ & $V_2(5 + 1, 5)$

$V_1(4, 5)$ & $V_2(6, 5)$

d) Find the Foci

- Use c to give the foci. For an hyperbola which opens left/right (x^2 term is positive) the foci will be $F_1(h - c, k)$ & $F_2(h + c, k)$

Thus, $F_1(5 - \sqrt{2}, 5)$ & $F_2(5 + \sqrt{2}, 5)$

$F_1(\approx 3.6, 5)$ & $F_2(\approx 6.4, 5)$

c) Find the Asymptotes

- The asymptotes tells us what values the function will approach but never reach and are given by $y - k = {}^{b}/_{a}\,(x - h)$ and

  $y - k = -{}^{b}/_{a}\,(x - h)$ when the x^2 term is positive.

Thus, $y - 5 = 1(x - 5)$ & $y - 5 = -1(x - 5)$

$y = x$ & $y = -x + 10$

Find the 4 points that Form Central Box

- These 4 points lie on the asymptotes and are (h–b, k+a) & (h–b, k–a) & (h+b, k+a) &

  (h+b, k –a) when the hyperbola opens

  left/right

(4, 6) & (4, 4) & (6, 6) & (6, 4)

e) Sketch the graph

- 1st Place the vertices
- 2nd Place the foci
- 3rd Draw the asymptotes
- 4th Place the 4 points

  that make the central box
- 5th Draw the hyperbola