

Example 2 Ellipses p. 7 Ch 11

This is an shifted ellipse

We began this problem in Example b)

$$9y^2 - 36y + 24x = -4(x^2 + 9)$$

- Starting from where we left off in Example #b on page 7

$$\frac{(y - 2)^2}{4} + \frac{(x + 3)^2}{9} = 1$$

$$\frac{(y - 2)^2}{4} + \frac{(x + 3)^2}{9} = 1$$

- Get a, b & c
- a^2 is the larger denominator
so, $a^2 = 9$ so, $a = 3$
- b^2 is the smaller denominator
so, $b^2 = 4$ so, $b = 2$
- $c^2 = a^2 - b^2$
so, $c = \sqrt{c^2} = \pm\sqrt{9 - 4} = \pm\sqrt{5}$
so, $c = \approx\pm 2.25$

a) Give the center

- The center is at (h, k)

$$\frac{(y - 2)^2}{4} + \frac{(x + 3)^2}{9} = 1$$

So, $C(-3, 2)$

b) Find the Foci

- Use c to give the foci. For an ellipse which a horizontal major axis (x^2 denominator $>$ y^2 denominator) the foci will be $(h - c, k)$ & $(h + c, k)$

$$F_1(-3 - \sqrt{5}, 2) \quad \& \quad F_2(-3 + \sqrt{5}, 2)$$

So, $F_1(\approx -5.25, 2)$ & $F_2(\approx -0.75, 2)$

c) Give the Vertices

- The vertices are $(h + a, k)$ & $(h - a, k)$ since this ellipse has a major axis that is horizontal

$$V_1(-3 - 3, 2) \quad \& \quad V_2(3 + 3, 2)$$

So, $V_1(-6, 2)$ & $V_2(0, 2)$

d) Find the Eccentricity

- The eccentricity tells us how “squashed” the ellipse is around its major axis. $e = c/a$

So, $e = \sqrt{5}/3$

Note: This is looking less like a circle because it is closer to 1 than it is to zero.

e) Find the Major Axis length

- The major axis is horizontal since the larger denominator is on the x^2 . That is a^2 & $a = 3$

$$\text{Major Axis Length: } 2(3) = 6$$

So, we see that the vertices being at $(-6, 2)$ & $(0, 2)$ puts them 6 units apart which is the length of the major axis.

f) Find the Minor Axis length

- The minor axis is vertical since the smaller denominator is on the y^2 . That is b^2 & $b = 2$

$$\text{Minor Axis Length: } 2(2) = 4$$

So, we see that two points on a vertical line through the center, and 2 units to the left & right of center are at $(-3, 2 + 2)$ & $(-3, 2 - 2)$ or $(-3, 4)$ & $(-3, 0)$ putting them 4 units apart which is the length of the minor axis.

g) Sketch the graph

- 1st Place the vertices
- 2nd Place the foci
- 3rd Place the 2 points on the minor axis
- 4th Draw the ellipse

