

Example 1 Ellipses p. 6 Ch 11

$$4x^2 + y^2 = 16$$

- Starting from where we left off in Example #a on page 5

$$\frac{x^2}{4} + \frac{y^2}{16} = 1$$

$$\frac{x^2}{4} + \frac{y^2}{16} = 1$$

- Get a, b & c
- a^2 is the larger denominator
so, $a^2 = 16$ so, $a = 4$
- b^2 is the smaller denominator
so, $b^2 = 4$ so, $b = 2$
- $c^2 = a^2 - b^2$
so, $c = \sqrt{c^2} = \pm\sqrt{16 - 4} = \pm\sqrt{12} = \pm\sqrt{4 \cdot 3}$
so, $c = \pm 2\sqrt{3}$

a) Give the Vertices

- The vertices are $(0, \pm a)$ since this ellipse has a major axis that is vertical

$$V_1(0, -4) \quad \& \quad V_2(0, 4)$$

b) Find the Foci

- Use c to give the foci. For an ellipse which a vertical major axis (y^2 denominator $>$ x^2 denominator) the foci will be $(0, c)$ & $(0, -c)$

So, $F_1(0, 2\sqrt{3})$ & $F_2(0, -2\sqrt{3})$

c) Find the Eccentricity

- The eccentricity tells us how “squashed” the ellipse is around its major axis. $e = c/a$

So,

$$e = \frac{2\sqrt{3}}{4} = \frac{\sqrt{3}}{2}$$

Note: This is looking less like a circle because it is fairly close to 1.

d) Find the Major Axis length

- The major axis is vertical since the larger denominator is on the y^2 . That is a^2 & $a = 4$

$$\text{Major Axis Length: } 2(4) = 8$$

So, we see that the vertices being at $(0, 4)$ & $(0, -4)$ puts them 8 units apart which is the length of the major axis.

e) Find the Minor Axis length

- The minor axis is horizontal since the smaller denominator is on the x^2 . That is b^2 & $b = 2$

$$\text{Minor Axis Length: } 2(2) = 4$$

So, we see that two points on a horizontal line through the center are at $(-2, 0)$ & $(2, 0)$ putting them 4 units apart which is the length of the minor axis.

e) Sketch the graph

- 1st Place the vertices
- 2nd Place the foci
- 3rd Place the 2 points on the minor axis
- 4th Draw the ellipse

