Example 2 Parabolas p. 4 Ch 11

$$x^2 + 6x + 12y = 9$$

- 1st move the first degree term to the right, away from the squared term

$$x^2 + 6x + 12y - 12y = -12y + 9$$

So, $x^2 + 6x = -12y + 9$

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- Next complete the square for the squared variable – x

$$(½ \bullet 6)^2 = (3)^2 = 9$$

$$x^2 + 6x + 9 = -12y + 9 + 9$$

$$(x - (-3))^2 = -12y + 18$$

$$(x - (-3))^2 = -12(^{-12}/_{-12}y + ^{18}/_{-12})$$

$$(x - (-3))^2 = -12(y - {}^3/_2)$$

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- Find p. You know that the coefficient of (y – k) is 4p, so use algebra

$$4p = -12$$

So, $4p \div 4 = -12 \div 4$

Therefore, $p = -3$

a)Give the Vertex

- The vertex is (h, k); make sure you are reading the form correctly.

$$V(-3, {}^{3}/_{2})$$

b) Find the Focus (h, k + p)

- Use p to give the focus. For a up/down facing parabola the focus will be (h, k + p).

So, $x = -3$ and $y = {}^{3}/_{2} + -3 = {}^{3}/_{2} - {}^{6}/_{2} = {}^{-3}/_{2}$

$$F(-3, -{}^{3}/_{2})$$

c) Find Directrix

- Since this is an downward facing parabola this is a horizontal line p units to the above the vertex, which is (h, k) in this case,

So, $y = k - p = {}^{3}/_{2} - -3 = {}^{3}/_{2} + {}^{6}/_{2} = {}^{9}/_{2} = 4\ {}^{1}/_{2}$

$$y = {}^{9}/_{2}$$

d) Find the Focal Diameter

- The focal diameter is 4 times p's distance (that means absolute value is used) or 2 times p's distance on either side of the focus

$$|4 \bullet -3| = 12$$

and $|2 \bullet -3| = 6$ which is more helpful in finding 2 more points on the parabola

$(-3 - 6, -{}^{3}/_{2})$ & $(-3 + 6, -{}^{3}/_{2})$ are the pts on the parabola equidistant from the focus which are $(-9, -{}^{3}/_{2})$ & $(3, -{}^{3}/_{2})$

Y-Intercept

- We could also make our parabola have an even more exact graph by finding the y - intercept

Let $x = 0$ so, $(0)^2 + 6(0) + 12y = 9$, thus

$12y = 9$ so $y = {}^{9}/_{12} \div {}^{3}/_{3} = {}^{3}/_{4}$

$(0, {}^{3}/_{4})$

X-intercepts

- The x-intercepts can also be found to help make a more accurate picture by letting y = 0

Let y = 0, so $x^2 + 6x + 12(0) = 9$

So, $x^2 + 6x - 9 = 0$ and by the quadratic formula

$$y = \frac{-(6) \pm \sqrt{(6)^2 - 4(1)(-9)}}{2(1)}$$

$$x = \frac{-6 \pm 6\sqrt{2}}{2} = -3 \pm 3\sqrt{2} \qquad (\approx -7.2, 0) \text{ \& } (\approx 1.2, 0)$$

e) Sketch the graph

- 1st Place the vertex
- 2nd Place the directrix
- 3rd Place the 2 points

  on the focal diameter
- 4th Optional to place

  x & y-intercepts
- 5th Draw the parabola