Example #d p. 10 Ch. 11

Given a random equation can you tell if it is a hyperbola, ellipse, parabola or degenerate.

$$\frac{3y^2}{16} + 1 = \frac{x(x-8)}{4} + 4$$

- Start by expanding out the right side. ¼x(x – 8)

$$^3/_{16}\, y^2 + 1 = \tfrac{1}{4}x^2 - \tfrac{1}{4} \bullet 8x + 4$$

So, $^3/_{16}\, y^2 + 1 = \tfrac{1}{4}x^2 - 2x + 4$

- Now clear the equation of fractions by multiplying every term by the LCD = 16

$$16 \bullet {}^3/_{16}\, y^2 + 1 \bullet 16 = 16 \bullet \tfrac{1}{4}x^2 - 16 \bullet 2x + 4 \bullet 16$$

Now With: $3y^2 + 16 = 4x^2 - 32x + 64$

- Bring all x's and y's to the left and the constants to the right to prepare for the next step of completing the square on the x and y's

$3y^2 - 4x^2 + 32x + 16 - 16 = 4x^2 - 4x^2 - 32x + 32x + 64 - 16$

So, $3y^2 - 4x^2 + 32x = 48$

$$3y^2 - 4x^2 + 32x = 48$$

Step 1: Factor out x's numeric coefficient

$$3y^2 - 4(x^2 - 8x\ ) = 48$$

Step 2: Complete the square

$$(1/2 \bullet 8)^2 = (4)^2 = 16$$

$$3y^2 - 4(x^2 - 8x + 16) = 48 - 64$$

Remember -4•16 = -64 must be added to the right side too!

Step 2: Rewrite

$$3y^2 - 4(x - 4)^2 = -16$$

Finish By Making Constant +1

- Divide all terms by -16

$$\frac{3y^2}{-16} - \frac{4(x-4)^2}{-16} = \frac{-16}{-16}$$

$$\frac{(x-4)^2}{4} - \frac{y^2}{16/3} = 1$$

- This is a shifted hyperbola since the x & y are squared but it's a difference.