

Example #c p. 10 Ch. 11

Given a random equation can you tell if it is a hyperbola, ellipse, parabola or degenerate.

$$x^2 + 4y^2 + 20x = 40y - 300$$

- Notice both x^2 & y^2 's so 1st move the y 's left & leave the constant on the right

$$x^2 + 20x + 4y^2 - 40y = 40y - 40y - 300$$

So, $x^2 + 20x + 4y^2 - 40y = -300$

Complete the Square for x's

$$x^2 + 20x + 4y^2 - 40y = -300$$

Step 1: Complete the square

$$\left(\frac{1}{2} \cdot 20\right)^2 = (10)^2 = 100$$

$$(x^2 + 20x + 100) + 4y^2 - 40y = -300 + 100$$

Step 2: Rewrite

$$(x + 10)^2 + 4y^2 - 40y = -200$$

Complete the Square for y's

$$(x + 10)^2 + 4y^2 - 40y = -200$$

Step 1: Remove the numeric coefficient

$$(x + 10)^2 + 4(y^2 - 10y) = -200$$

Step 2: Complete the square

$$\left(\frac{1}{2} \cdot 10\right)^2 = (5)^2 = 25$$

$$(x + 10)^2 + 4(y^2 - 10y + 25) = -200 + 100$$

Remember $4 \cdot 25 = 100$ is actually added to the left

Step 3: Rewrite

$$(x + 10)^2 + 4(y - 5)^2 = -100$$

Problem!

- Even if you divide the constant will be negative or both variables will be.

$$\frac{(x + 10)^2}{100} + \frac{(y - 5)^2}{25} = -1$$

or

$$\frac{-(x + 10)^2}{100} - \frac{(y - 5)^2}{25} = 1$$

- This is a **degenerate** conic.