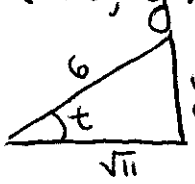
M22 Test #1a Concepts Key

#1 $(\sqrt{11}/6, y)$ in QIV



if x-val is $\frac{x}{r}$

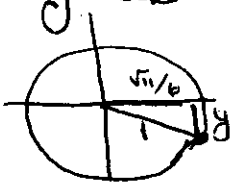
$(\sqrt{11})^2 + y^2 = 6^2 \Rightarrow 11 + y^2 = 36$

$\Rightarrow y^2 = 36 - 11 = 25 \Rightarrow y = \pm\sqrt{25}$

so $y = -5$ since in QIV

$\therefore$ y-coord is $\frac{y}{r} = \boxed{-5/6}$

or



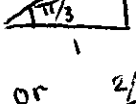
$\left(\frac{\sqrt{11}}{6}\right)^2 + y^2 = 1^2 \Rightarrow y^2 = 1 - \frac{11}{36}$

$\Rightarrow y^2 = \frac{25}{36} \Rightarrow y = \pm\sqrt{\frac{25}{36}} = \pm\frac{5}{6}$

and in QIV $\boxed{y = -5/6}$

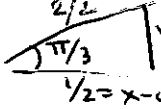
#2 Give P(x, y) for

(a) $\pi/3$ $\boxed{(1/2, \sqrt{3}/2)}$



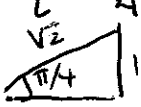
$\cos = \frac{adj}{hyp}$ $\sin = \frac{opp}{hyp}$

or

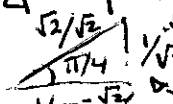


on unit circle ratio to orig.

(b) $\bar{t} = \frac{4\pi}{4} - \frac{3\pi}{4} = \frac{\pi}{4}$ in QII for $t = \frac{3\pi}{4}$



or



$1/\sqrt{2} \cdot \sqrt{2} = \frac{\sqrt{2}}{2}$

unit circle read x & y

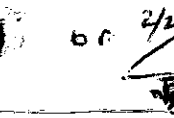
read opp for hyp y-coord

adj for hyp x-coord

$\left(-\sqrt{2}/2, \sqrt{2}/2\right)$

(c) $\bar{t} = \frac{7\pi}{6} - \frac{6\pi}{6} = \frac{\pi}{6}$ in QIII for $t = 7\pi/6$

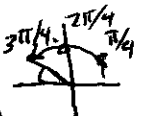
or



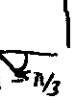
same as before

$\boxed{(-\sqrt{3}/2, -1/2)}$

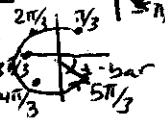
#3 (a) $\bar{t} = \frac{4\pi}{4} - \frac{3\pi}{4} = \frac{\pi}{4}$ in QII



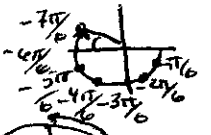
(b) $\bar{t} = 0 - (-\pi/3) = \pi/3$ in QI



(c) $\bar{t} = \frac{6\pi}{3} - \frac{5\pi}{3} = \pi/3$ in QIV

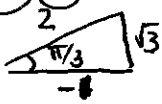


(d) $\bar{t} = \frac{7\pi}{6} - \frac{6\pi}{6} = \frac{\pi}{6}$ in QII



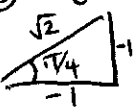
(e) $\bar{t} = \frac{10\pi}{2} - \frac{9\pi}{2} = \frac{\pi}{2}$ between QII & QI

#4 (a) $t = 2\pi/3$ so $\bar{t} = \frac{3\pi}{3} - \frac{2\pi}{3} = \frac{\pi}{3}$ in QII



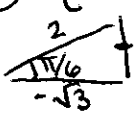
$\sin t = \frac{\sqrt{3}}{2}$	$\csc t = \frac{2\sqrt{3}}{3}$
$\cos t = -\frac{1}{2}$	$\sec t = -2$
$\tan t = -\sqrt{3}$	$\cot t = -\frac{\sqrt{3}}{3}$

(b) $t = -11\pi/4$ so $\bar{t} = \frac{12\pi}{4} - \frac{11\pi}{4} = \frac{\pi}{4}$ in QIII



$\sin t = -\sqrt{2}/2$	$\csc t = -\sqrt{2}$
$\cos t = -\sqrt{2}/2$	$\sec t = -\sqrt{2}$
$\tan t = 1$	$\cot t = 1$

(c) $t = 7\pi/6$ so $\bar{t} = \frac{7\pi}{6} - \frac{6\pi}{6} = \pi/6$ in QIII



$\sin t = -\frac{1}{2}$	$\csc t = -2$
$\cos t = -\frac{\sqrt{3}}{2}$	$\sec t = -\frac{2\sqrt{3}}{3}$
$\tan t = \frac{\sqrt{3}}{3}$	$\cot = \frac{3\sqrt{3}}{3} = \sqrt{3}$

#5 (a) $(-\sqrt{3}/2, 1/2)$ is $(\cos t, \sin t)$

so $\boxed{\sin t = 1/2}$ $\boxed{\cos t = -\sqrt{3}/2}$

and $\tan t = \frac{\sin t}{\cos t} = \frac{1/2}{-\sqrt{3}/2} = \frac{1}{-\sqrt{3}}$

so $\boxed{\tan t = -\frac{\sqrt{3}}{3}}$

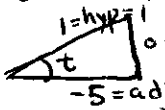
(b) $(3/5, -4/5)$ is $(\cos t, \sin t)$

so $\boxed{\sin t = -4/5}$ $\boxed{\cos t = 3/5}$

and $\tan t = \frac{\sin t}{\cos t} = \frac{-4/5}{3/5} = -4/3$

$\boxed{\tan t = -4/3}$

#6 (a) $\sec t = -5$ in QII

tells us 

opp=y doesn't work

$(-5)^2 + y^2 = 1^2 \Rightarrow y^2 = ?$ ∴

$\sec^2 t = \tan^2 + 1$ so

$25 = \tan^2 + 1 \Rightarrow 25 - 1 = \tan^2$

∴ $\tan = \pm\sqrt{24} = \pm 2\sqrt{6}$ in QII "-"

so $\cot t = \frac{1}{2\sqrt{6}} = \frac{-\sqrt{6}}{12}$

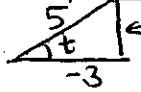
$1 + \cot^2 = \csc^2$ so

$1 + (\sqrt{6}/12)^2 = \csc^2 \Rightarrow \csc^2 = 1 + \frac{6}{144}$

$\csc = \pm\sqrt{\frac{150}{144}} = \pm\frac{5\sqrt{6}}{12}$ "+" in QII

so $\boxed{\cos t = -1/5}$ $\boxed{\sin t = \frac{2\sqrt{6}}{5}}$

(b) $\boxed{\cos t = -3/5}$ $\tan t < 0$ in QII

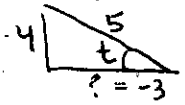


$x^2 + y^2 = r^2$

$(-3)^2 + y^2 = (5)^2 \Rightarrow y = 4$

∴ $\boxed{\sin t = 4/5}$ pos. in QII

#7 P(x, 4/5) in QII



$x^2 + y^2 = r^2 \Rightarrow 16 + x^2 = 25$

$x = 3$

#7 cont'd $\boxed{\sin t = 4/5}$ $\boxed{\cos t = -3/5}$

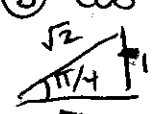
$\boxed{\tan t = -4/3}$

#8 (a) $t = 7\pi/6$ see #2 for sin & cos as they are (cos, sin) on the unit circle & $\tan = \frac{\sin}{\cos}$

$\boxed{\sin 7\pi/6 = -1/2}$ $\boxed{\cos 7\pi/6 = -\sqrt{3}/2}$

$\boxed{\tan 7\pi/6 = \quad = \sqrt{3}/3}$

(b) $\cos 13\pi/4$ need $\bar{t} = \frac{13\pi}{4} - \frac{12\pi}{4} = \frac{\pi}{4}$ in QIII

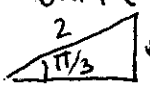


in QIII $\boxed{\sin 13\pi/4 = -\sqrt{2}/2}$

$\boxed{\cos 13\pi/4 = -\sqrt{2}/2}$

$\boxed{\tan 13\pi/4 = +1}$

(c) $\tan(-5\pi/3)$ need $\bar{t} = 6\pi/3 - 5\pi/3 = \pi/3$ in QI



$\tan -5\pi/3$

$= \boxed{\tan \pi/3 = \sqrt{3}}$

(d) $\csc(3\pi/2)$ need $\bar{t} = 2\pi - 3\pi/2 = \frac{\pi}{2}$ between QIII & IV

$\csc(3\pi/2) = \frac{1}{\sin(3\pi/2)} = \frac{1}{-1} = \boxed{-1}$

#9 (a) $\sin 2.3 \doteq 0.745705$

(b) $\cos -1.9 \doteq -0.323290$ rounded from -0.323289[5]

(c) $\sec 1.1 = \frac{1}{\cos 1.1} = \frac{1}{0.453596121} \approx 2.204604$

(d) $\cot 4.9 = \frac{1}{\tan 4.9} = \frac{1}{-5.267493066} \approx -0.189844$

#10 $\tan t = \frac{\sin t}{\cos t} = \frac{\sin t}{\sqrt{1 - \sin^2 t}}$ and it's "-" in QIV

#11 See attached

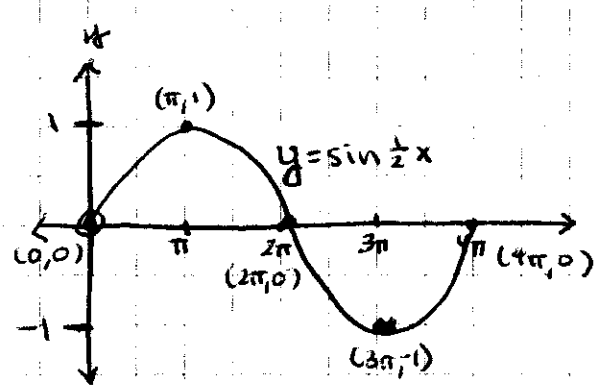
– **#13**

#11, a) $y = \sin \frac{1}{2}x$

x	y	y'
0	0	0
$\pi/2$	1	π
π	0	2π
$3\pi/2$	−1	3π
2π	0	4π

Period = $2\pi / \frac{1}{2} = 4\pi$

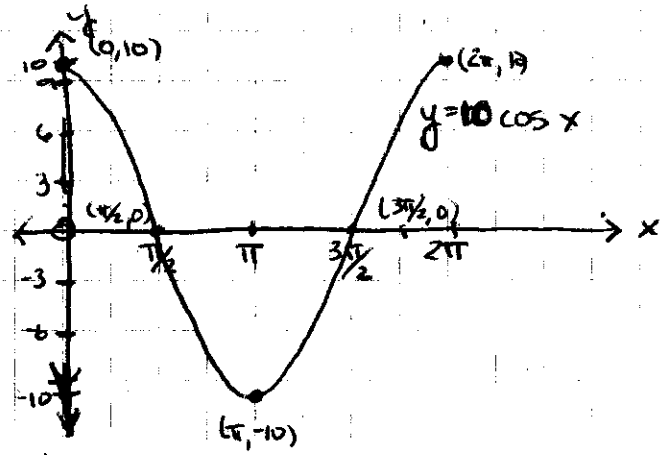
0 to 4π divided into $\frac{1}{4}$'s



b) $y = 10 \cos x$

Amplitude = $|10| = 10$

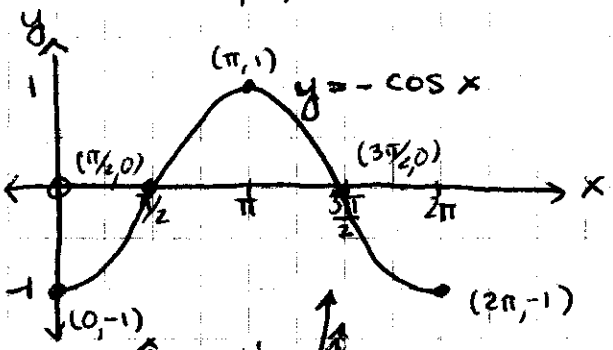
x	y	y'
0	1	10
$\pi/2$	0	0
π	−1	−10
$3\pi/2$	0	0
2π	1	10



c) $y = -\cos x$ ← reflected

Amplitude = $|-1| = 1$

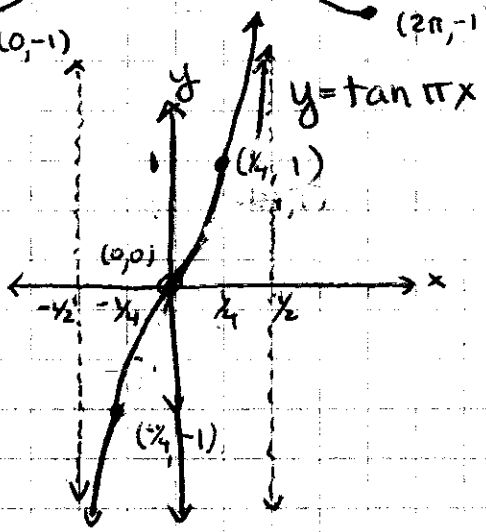
x	y	y'
0	1	−1
$\pi/2$	0	0
π	−1	+1
$3\pi/2$	0	0
2π	1	−1



d) $y = \tan \pi x$

Period = $\pi / \pi = 1$

x	y	x'
$-\pi/2$	undef	$-\frac{1}{2}$
$-\pi/4$	−1	$-\frac{1}{4}$
0	0	0
$\pi/4$	1	$\frac{1}{4}$
$\pi/2$	undef	$\frac{1}{2}$



e) $y = \sin(2x - \pi/2) \Rightarrow y = \sin 2(x - \pi/4)$

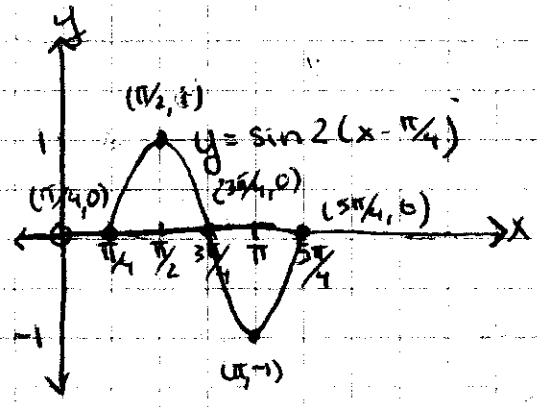
GCF = 2

factor to see correctly!!

Period = $2\pi/2 = \pi$

Phase Shift = $\pi/4$

x	y	x'	x''
0	0	0	$\pi/4$
$\pi/2$	1	$\pi/4$	$\pi/2$
π	0	$\pi/2$	$3\pi/4$
$3\pi/2$	−1	$3\pi/4$	π
2π	0	π	$5\pi/4$



#12 (a) $y = -5\cos 4x$

Period $= 2\pi/4 = \pi/2$

Amplitude $= |-5| = 5$

Phase Shift $=$ None or 0

(b) $y = 2\sin(\frac{1}{2}x - \pi/6)$

factor 1st by "GCF" $= \frac{1}{2}$

$y = 2\sin\frac{1}{2}(x - \pi/3)$

Period $= 2\pi/\frac{1}{2} = 4\pi$

Amplitude $= |2| = 2$

Phase Shift $= \pi/3$

(c) $y = \tan(2x - \pi/2)$

factor 1st by "GCF" $= 2$

$y = \tan 2(x - \pi/4)$

Period $= \pi/2$

Amplitude None Exists

Phase Shift $= \pi/4$

#13 (37) It's a sine

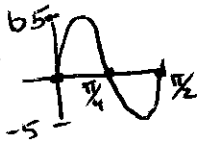
Amplitude $= 5 \leftarrow a$

Period $= \pi/2 \leftarrow k = 4$ since $2\pi/k = \pi/2$

Phase Shift $=$ None $\leftarrow b = 0$

Vertical Shift $=$ None

$y = 5\sin 4x$



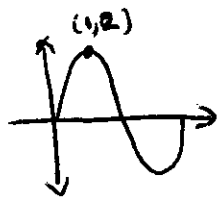
(38) It's a sine

Amplitude $= 2 \rightarrow a$

Period $= 1/\frac{1}{2} = 1$ $\quad k = \frac{1}{2}\pi$ since $\frac{2\pi}{k} = 4$

Phase Shift $=$ None

$y = 2\sin\frac{\pi}{2}x$

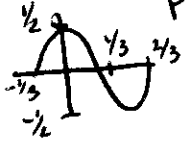


(39) It's a sine

Amplitude $= \frac{1}{2} \rightarrow a$ $\quad \frac{k}{2\pi} = 1 \Rightarrow k = 2\pi$

Period $= 1 \rightarrow k = 2\pi$ since $\frac{2\pi}{k} = 1$

Phase Shift $= -\frac{1}{3} \rightarrow b$



$y = \frac{1}{2}\sin 2\pi(x + \frac{1}{3})$

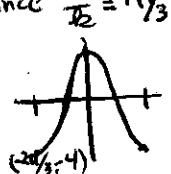
(40) It's a cosine

Amplitude $= 4 \rightarrow a$ $\quad \frac{k}{2\pi} = \frac{3}{4\pi} \Rightarrow k = \frac{3}{2}$

Period $= 4\pi/3$ $\quad k = \frac{3}{2}$ since $\frac{2\pi}{k} = 4\pi/3$

$2(\frac{2\pi}{3}) = 4\pi/3$

Phase Shift $= -\frac{2\pi}{3}$



Reflected so a is -4

$y = -4\cos\frac{3}{2}(x + 2\pi/3)$

btw $y = 4\sin\frac{3}{2}(x + \pi/3)$

is the same graph & what the book chose as an answer