$n = 21$ (+14 curve) +11
$\bar{x} = 56.5$ Except > 70.5
$s = 23.5$
$\min = 22.5$ $Q_3 = 70$
$Q_1 = 38$
$\tilde{x} = 58$ $\max = 95$

Name: Key

Test #1b – Math 61
Spring 2011

Instructions: *All work must be shown at all times, including intermediate steps in order to receive full credit for any problem. Lack of work can result in little or no credit even if the answer is correct. All answers should contain units where appropriate and be boxed. The use of a calculator is acceptable. You may have a 5x8 note card meeting the prerequisite stipulations. The note card must be surrendered with the test. Staple it to the back of the test upon completion. You have the entire class period! Good luck!*

1. Find the future value of $5000 invested at 4.3% simple annual interest for 36 months if:

   a) (+3) The money earns simple interest

   $A = ? = P + PRT \Rightarrow A = 5000(1 + 0.043(3))$ } Plug in correctly +1
   $P = \$5000$
   $r = 0.043$ +½
   $t = 36\text{mo.} = 36/12 = 3\text{yrs.}$ +½
   $= 5000(1.129)$
   $= \$5645$ +1 ½ units

   b) (+2½) The money earns interest compounded ~~quarterly~~ monthly

   $A = P(1 + r/m)^{mt} \Rightarrow A = 5000\left(1 + \frac{0.043}{12}\right)^{12 \cdot 3}$ } Plug in correctly +1
   $P = \$5000$
   $r = 0.043$
   $m = 12$ +½
   $t = 36\text{mo.}/12 = 3\text{yr}$
   $= 5000(1.0\ldots)^{36}$
   $= 15687.\ldots$ $5687.\ldots$ +½ +½

   c) (+2) The money earns interest compounded continuously

   $A = Pe^{rt} \Rightarrow A = 5000e^{0.043(3)}$ } Plug in correctly +1
   $P = \$5000$
   $r = 0.043$
   $t = 36\text{mo.}/12 = 3\text{yrs.}$
   $= \$5\ldots$
   $\approx \$5\ldots$ +1

2. Which is a better deal and explain your choice using math: 9% compounded weekly ~~quarterly~~ or 9.25% ~~compounded annually~~? or 8.95% compounded continuously

   Compounded m units
   compounded continuously
   annual

   $APY = (1 + r/m)^m - 1 \Rightarrow (1 + 0.09/52)^{52} - 1 = 0.0940\ldots$
   $APY = e^{0.895} - 1 = 0.093627733$
   $APY = 9.25\%$ Explain +1

   compounded weekly. +½

3. What would a car costing $23,000 today have been worth 5 years ago if the inflation rate over that period had averaged 5% annual interest compounded annually?

   $A = \$23000 = P(1 + r/m)^{mt}$
   $P = ?$
   $t = 5\text{yr.}$
   $m = 1$ +½
   $r = 0.05$ +½

   $23000 = P(1 + 0.05/1)^{1(5)}$ } Plugs in correctly +1
   $\frac{23000}{(1.05)^5} = P$
   $18021.10183 = P$ including round off
   $P \approx \$18021.10$ +1

+12

4. You just sold a stock for $17,388.17 (net) that cost you $12,903.28 (net) 3 years ago, what annual compound rate of return did you make on your investment?

$A = 17388.17$ +½ → $17388.17 = 12903.28(1+r)^3$ +1

$P = 12903.28$ +½

$t = 3$ yrs. +½

$m = 1$ +½

$r = ?$ ← finding

$A = P(1 + r/m)^{mt}$ +½ uses

$\frac{17388.17}{12903.28} = (1+r)^3$ +1

$\sqrt[3]{\frac{17388.17}{12903.28}} - 1 = r$ +1 Solving

$r \approx 0.0455$

4.55% ← +½

+4

5. You have $5000 toward the purchase of a boat that will cost $6000. How long will it take (to the next highest quarter, if not exact), for the $5000 to grown to $6000 if it is invested at 9% compounded quarterly? Hint: You will need to take the natural log of each side of the equation to solve it.

$P = 5000$ +½ → $6000 = 5000(1 + \frac{0.09}{4})^{4t}$ +1 Solving

$A = \$6000$ +½

$t = ?$ ← Finding

$r = 9\%$ +½

$m = 4$ +½

$A = P(1 + r/m)^{mt}$ ← using +½

$\frac{6}{5} = (1 + 0.0225)^{4t}$

$\ln(6/5) = \ln(1+0.0225)^{4t}$

$\ln 6/5 = 4t \ln(1+0.0225)$

$\frac{\ln 6/5}{4\ln(1+0.0225)} = t$

$t = 2.0485$

$t \approx 2\frac{1}{4}$ yrs. +½

+4

6. Solve and graph the solution on a number line and give the solution in interval notation: I will be picky about the # lines.

a) $1 \le \frac{6 - 4x}{8} < 3$ +½

$8 \le 6 - 4x < 24$ +1

$-6 \quad -6 \quad -6$

$2 \le -4x < 18$ +1

$\div -4 \quad \div -4 \quad \div -4$

$-\frac{1}{2} \ge x > -4\frac{1}{2}$ +2

+11

#line +½

+1

$-5 \quad -4\frac{1}{2} \quad -4 \quad -3 \quad -2 \quad -1 \quad -\frac{1}{2} \quad 0$

$(-4\frac{1}{2}, -\frac{1}{2}]$ +1

b) $2(x+3) \ge -6$ +½

$x + 3 \ge -3$ +1

$-3 \quad -3$

$x \ge -6$ +1

#line +½

+1

$-10 \quad -6 \quad -5 \quad 0 \quad 5 \quad 10$

$[-6, \infty)$ +1

+19

7. The fixed costs for a publisher of a new novel are determined to be \$54,000, while the variable cost is determined to be \$2.10 per novel. The novels will be sold for \$16 each. Using this story, answer the following questions.

a) +1 Give the cost equation.

$C(q) = \$2.10q + 54000$

b) +1 Give the revenue equation.

$R(q) = \$16q$

+4 ½

c) +2½ How many books must be sold in order for the publisher to break-even?

set equal +1

$C(q) = R(q) \Rightarrow 2.10q + 54000 = 16q$

solve +1

$54000 = 13.9q$

$q = 3885$ books +½

$q = 3884.89$

8. A farmer buys a combine (a large tractor for harvesting grain) for his farm and 5 years after buying it is worth \$115,000. At year 10, it is only worth \$75,000. Answer the following questions based on this story.

a) +4 Create a linear equation that shows the combine's value t years after it's purchase. Show your work.

m +2

+2

$y = -8000x + 155,000$

+6

b) +1 What was the purchase price of the combine?

\$155,000

c) +1 What is the rate of depreciation?

Rate of Depreciation: $\frac{\$@5 - \$@10}{5 \quad 10}$

$= \frac{115000 - 75000}{5}$

$= 40,000$

$= 8000$ /yr.

+10½

9. The following graph represents the cost for the repair of a radiator on a car based upon the cost for the parts and the number of hours for the repair. Each car is different in terms of time to repair the radiator but the parts used in the repair are always the same.

Using the graph and the story:

a) (+1) identify the independent variable **by label**

time (hrs.)

b) (+1) identity the dependent variable **by label**

cost ($)

c) (+1) find the cost of the parts

+150 — the y-intercept

d) (+2) find the hourly rate the shop charges for repairs (use fractions)

$\frac{650-150}{9-0} = \frac{500}{9} = \$55\frac{5}{9}$/hr

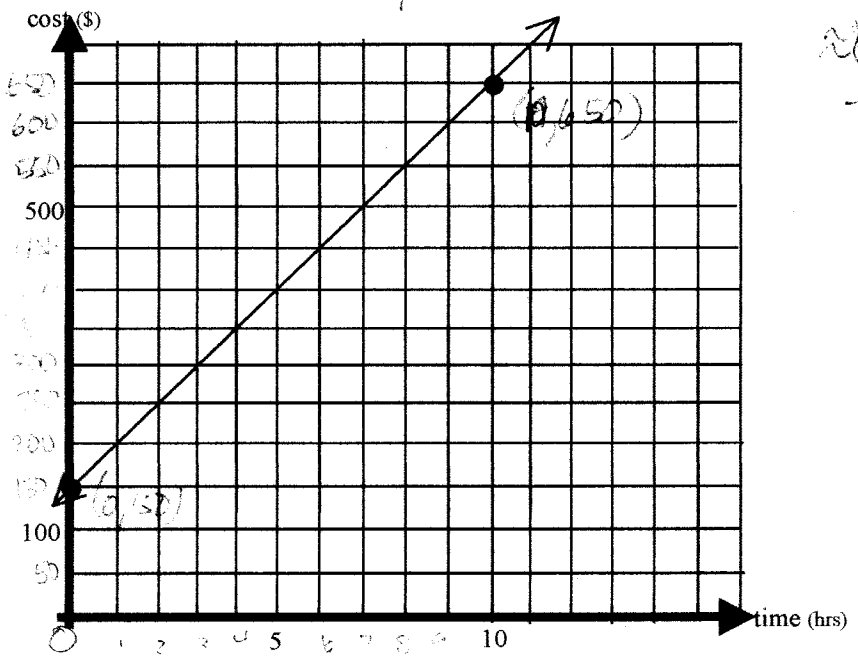
e) (+1) give a linear equation that a customer could use to predict their total repair cost if they are given an estimate of the time required to repair their car's radiator

$C(t) = \frac{500}{9}t + 150$

f) (+1½) predict the total cost for a 6 hour radiator repair (to the nearest $)

$C(6) = \frac{500}{9}(6) + 150 = 333\frac{1}{3} + 150 = 483\frac{1}{3}$

≈ $483



+7½

$n = 13$
$\bar{x} = 75.5$
$s = 19$
$min = 45$
$Q_1 = 61.5$
$\tilde{x} = 81.5$
$Q_3 = 89.5$ max = 100

50.5

Name: Key

Test #2a – Spring 2011
Math 61

Instructions: Show all work. Give your answer using appropriate units and with appropriate rounding for full credit. Box your final answer. You may use a calculator and your 5x8 note card. Note card must conform to specifications on the syllabus. Good luck!

1. E-Loan, an online lending service, recently offered 48-month auto loans at 3.6% compounded monthly to applicants with good credit. If you have good credit and can afford monthly payments of \$396, how much can you borrow from E-Loan? What amount of ~~interest will you~~ pay over the life of the loan?

Present Value of Annuity

$PV = PMT \dfrac{1-(1+i)^{-n}}{i} = ? \Rightarrow PV = 396\left[\dfrac{1-\left(1+\frac{0.036}{12}\right)^{-48}}{\frac{0.036}{12}}\right]$ Equation correct +2; substitute & simplify +1

$PMT = \$396$

$r = 0.036$ ← Decimal +½

$m = 12$

$i = r/m = \frac{0.036}{12} = 0.003$ +½

$t = 4$ yr.

$n = 48$ ← +½

$PV = \$17678.18$ +½

$Interest = FV - PV$ +1

$= (396)(48) - PV$

$= 19008 - 17678.18 = \$1329.82$ +½

+6½

2. More Cash 4 U offered an annuity that pays 5.7% compounded monthly. If you make monthly deposits of \$2201, how much money will you have in 8 years? How much interest will you make on your investment?

Future Value of an Annuity

$FV = PMT \dfrac{(1+i)^n - 1}{i} = ? \Rightarrow FV = 2201\left[\dfrac{\left(1+\frac{0.057}{12}\right)^{96}-1}{(0.057/12)}\right]$ Equation correct +2; substitute & simplify +1

$PMT = \$2201$

$r = 5.7\%$ ← decimal +½

$m = 12$

$i = \frac{0.057}{12} = 0.00475$ +½

$t = 8$ yr

$n = 8(12) = 96$ +½

$= \$266922.43$ +½

$Interest = FV - Total$

$= FV - (PMT)(n)$ +1

$= 266922.43 - 211796 =$

$\$55626.43$ +½

+6½

3. A company needs \$130,000 in 17 years to replace a very expensive piece of equipment. If it plans on putting aside money each month into an account that pays 5.7% compounded monthly, how much should be put into the account each month?

Sinking Fund

$PMT = FV \dfrac{i}{(1+i)^n - 1} = ? \Rightarrow PMT = 130{,}000\left[\dfrac{\frac{0.057}{12}}{\left(1+\frac{0.057}{12}\right)^{(12)(17)}-1}\right]$ Equation correct +2; substitute & simplify +1

$FV = \$130{,}000$

$r = 5.7\%$ ← decimal +½

$m = n$

$i = r/m = \frac{0.057}{12} = 0.00475$ +½

$t = 17$ yr

$n = 17 \cdot 12 = 204$ +½

$= \$379.00$ +½

+5

+18

4. A family has a $98,725, 15 year mortgage at 6.9% compounded monthly. Find the monthly payments and the total interest that they will pay over the life of the mortgage.

Amortization

$PMT = PV\dfrac{i}{1-(1+i)^{-n}} = ?$ (Correct Equation) +2 $\Rightarrow PMT = 98725\left[\dfrac{0.069/12}{1-\left(1+\frac{0.069}{12}\right)^{-12(15)}}\right]$ Substitute; simplify +1

$PV = \$98725$

$r = 6.9\% \leftarrow$ decimal +½

$m = 12$

$i = r/m = 0.069/12 = 0.00575$ +½

$t = 15$ yr.

$n = t \cdot m = 180$ +½

$= \$881.86$ +2

Interest = Total − PV

$= (881.86)(15)(12) - 98725$ +1

$= 158734.8 - 98725$

$= \$60,009.80$ +½

5. An IRA has $9000 and the owner doesn't add any money except the interest of 4% compounded daily. How much money will be in the account 37 years from now? (Use a 365-day year to compute.)

Compound Interest

$A = P(1+i)^n = ?$ (Correct Equation) +2 $\Rightarrow A = 9000\left(1+\frac{0.04}{365}\right)^{37(365)}$ Substitutes; simplify +1

$P = 9000$

$r = 4\% = 0.04$ +½ decimal

$m = 365$

$i = 0.04/365$ +½

$t = 37$ yr.

$n = 365 \cdot 37 = 13505$ +½

$= \$39533.31$

Interest = A − P

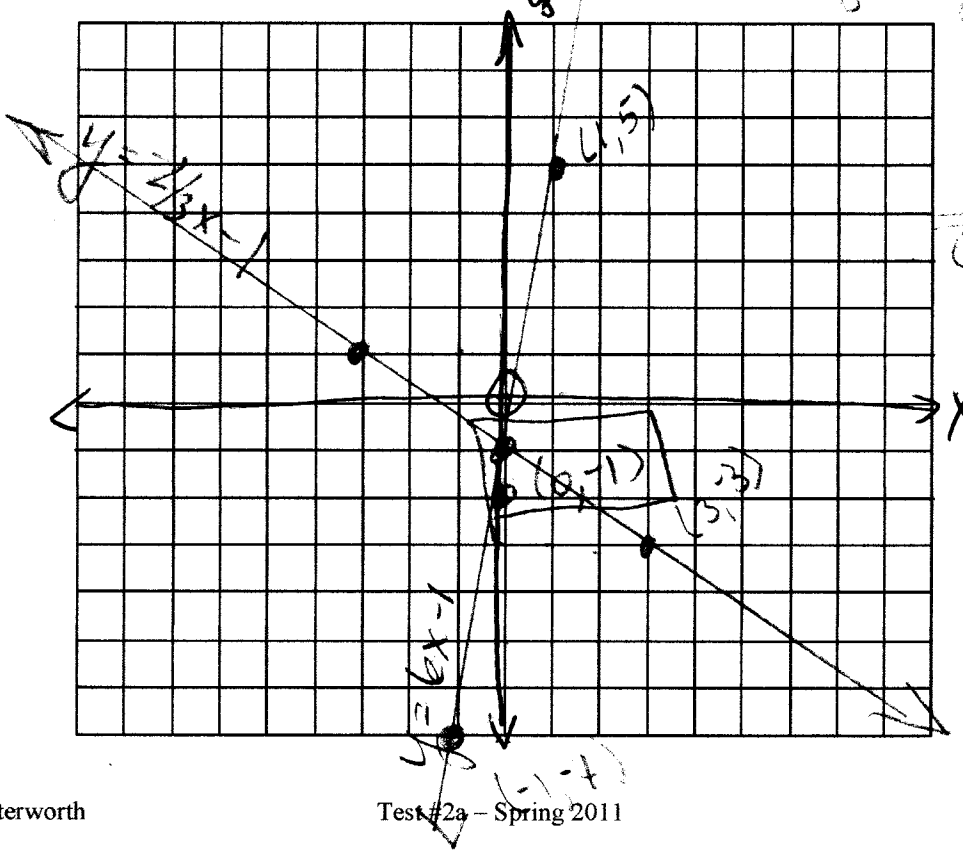
$= 39533.31 - 9000$

$= \$30533.31$ +1 Not asked

6. Solve by graphing. $2x + 3y = -3$

$3x - \frac{1}{2}y = \frac{1}{2}$

$3x - \frac{1}{2} = \frac{1}{2}y \Rightarrow y = 6x - 1$



Each Line Correctly Graphed +1 each

Boxed Intersection & Labeled +1

7. Solve by substitution: $3x + y = 3$
$2x + 4y = 7$

$y = -3x + 3$ +1

substitute: $2x + 4(-3x + 3) = 7$

$3(\frac{1}{2}) + y = 3$

$y = \frac{3}{2}$

$2x - 12x + 12 = 7$

$-10x = -5$

$x = 1/2$

$(\frac{1}{2}, \frac{3}{2})$ +3½

8. Solve by addition: $2x - 5y = -8$
$3x + 2y = 7$

$4x - 10y = -16$

$15x + 10y = 35$

$19x = 19$

$x = 1$

$3(1) + 2y = 7 \Rightarrow 2y = 4$

$y = 2$

$(1, 2)$ +2½

9. Solve using matrices: $x + y + z = 6$
$2x - y + z = 3$
$x + 2y - 3z = -4$

$\begin{bmatrix} 1 & 1 & 1 & 6 \\ 2 & -1 & 1 & 3 \\ 1 & 2 & -3 & -4 \end{bmatrix}$ Augmented +1

$\sim \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & -3 & -1 & -9 \\ 0 & 1 & -4 & -10 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & -4 & -10 \\ 0 & 3 & 1 & 9 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 1 \end{bmatrix}$

$x = 2, y = 3, z = 1$

$(2, 3, 1)$ solution +1

+5

All the way to 1's on diagonal +3

10. Using the following matrices, answer the questions:

$A = \begin{bmatrix} 2 & 4 \\ -1 & 0 \end{bmatrix}$ $B = \begin{bmatrix} 5 & -3 \\ 4 & 1 \end{bmatrix}$ $C = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$ $D = \begin{bmatrix} 5 \\ -8 \end{bmatrix}$

+6

(+2) a) A + B

$\begin{bmatrix} 2 & 4 \\ -1 & 0 \end{bmatrix} + \begin{bmatrix} 5 & -3 \\ 4 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 1 \\ 3 & 1 \end{bmatrix}$

b) (+1) A + C

not defined

not equivalent matrices

c) (+2) 2B

$2\begin{bmatrix} 5 & -3 \\ 4 & 1 \end{bmatrix} = \begin{bmatrix} 10 & -6 \\ 8 & 2 \end{bmatrix}$

d) (+1) C − D

$\begin{bmatrix} 3 \\ 2 \end{bmatrix} + \begin{bmatrix} -5 \\ 8 \end{bmatrix} = \begin{bmatrix} -2 \\ 10 \end{bmatrix}$

+18

7. Solve by substitution: $3x + y = 4$
$2x - 4y = -9$

$y = -3x + 4$ +1

$2x - 4(-3x + 4) = -9$ substitutes +½

$2x + 12x - 16 = -9$

$14x = 7$ 16 − 9 = 7

$x = 7/14$ +1

$x = 1/2$

$\frac{3}{2} + y = 4$ $\times 2$, $-3/2$

$y = 5/2$ +½

$(1/2, 5/2)$ +3½

8. Solve by addition: $2x + 5y = -8$ ×2
$3x - 2y = 7$ ×5

$4x + 10y = -16$ +1

$15x - 10y = 35$ +1

$19x = 19$

$x = 1$

$3(1) - 2y = 7$

$-3 \quad -3$

$-2y = 4$

$y = -2$ +½

$(1, -2)$ +3½

9. Solve using matrices:

$x + y + z = 6$
$2x - y + z = 3$
$x + 2y - 3z = -4$

$\begin{bmatrix} 1 & 1 & 1 & 6 \\ 2 & -1 & 1 & 3 \\ 1 & 2 & -3 & -4 \end{bmatrix} \sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & -3 & -1 & -9 \\ 0 & 1 & -4 & -10 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & -4 & -10 \\ 0 & 3 & 1 & 9 \end{array}\right] \sim \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & -4 & -10 \\ 0 & 0 & 13 & 39 \end{bmatrix}$

+5 Augmented +1

$\hookrightarrow \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & -4 & -10 \\ 0 & 0 & 1 & 3 \end{bmatrix} \sim \left[\begin{array}{ccc|c} 1 & 1 & 0 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right]$

All the way to 1's on diagonal

$(1, 2, 3)$ Solution +1

10. Using the following matrices, answer the questions: +3

$A = \begin{bmatrix} 4 & 3 \\ -5 & -1 \end{bmatrix}$ $B = \begin{bmatrix} 2 & 4 \\ -1 & 0 \end{bmatrix}$ $C = \begin{bmatrix} -8 \\ 2 \end{bmatrix}$ $D = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$

+6

b) +2 $A + B$

$\begin{bmatrix} 4 & 3 \\ -5 & -1 \end{bmatrix} + \begin{bmatrix} 2 & 4 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 6 & 7 \\ -6 & -1 \end{bmatrix}$

a) +1 $A + C$

$\begin{bmatrix} 4 & 3 \\ -5 & -1 \end{bmatrix} + \begin{bmatrix} -8 \\ 2 \end{bmatrix}$ not defined

c) +2 $2A$

$2\begin{bmatrix} 4 & 3 \\ -5 & -1 \end{bmatrix} = \begin{bmatrix} 8 & 6 \\ -10 & -2 \end{bmatrix}$

d) +1 $D + C$

$\begin{bmatrix} 5 \\ 3 \end{bmatrix} + \begin{bmatrix} 8 \\ -2 \end{bmatrix} = \begin{bmatrix} 13 \\ 1 \end{bmatrix}$

Name: Key

Test #2b – Spring 2011
Math 61

Instructions: Show all work. Give your answer using appropriate units and with appropriate rounding for full credit. Box your final answer. You may use a calculator and your 5x8 note card. Note card must conform to specifications on the syllabus. Good luck!

Compound Interest

1. An IRA has \$9000 and the owner doesn't add any money except the interest of 4% compounded daily. How much money will be in the account 37 years from now? (Use a 365-day year to compute.) See #5 on 2a

Amortization

2. A family has a \$98,725, 15 year mortgage at 6.9% compounded monthly. Find the monthly payments and the total interest that they will pay over the life of the mortgage. See #4 on 2a

Present Value of Annuity

3. E-Loan, an online lending service, recently offered 48-month auto loans at 3.6% compounded monthly to applicants with good credit. If you have good credit and can afford monthly payments of \$396, how much can you borrow from E-Loan? What amount of interest will you pay over the life of the loan? See #1 on 2a

Future Value of Annuity

4. More Cash 4 U offered an annuity that pays 5.7% compounded monthly. If you make monthly deposits of $2201, how much money will you have in 8 years? How much interest will you make on your investment? See #2 on 2a

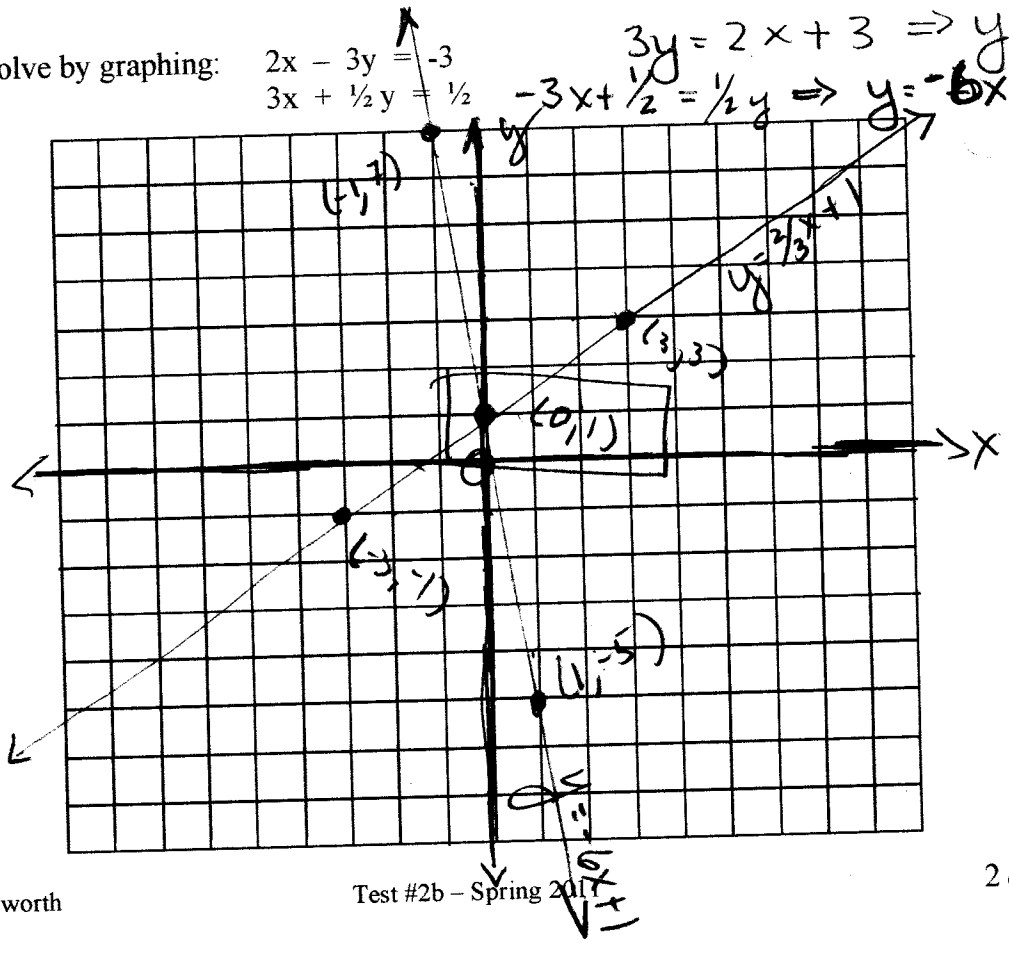
Sinking Fund

5. A company needs $130,000 in 17 years to replace a very expensive piece of equipment. If it plans on putting aside money each month into an account that pays 5.7% compounded monthly, how much should be put into the account each month? See #3 on 2a

6. Solve by graphing: $2x - 3y = -3$
$3x + \frac{1}{2}y = \frac{1}{2}$

$3y = 2x + 3 \Rightarrow y = \frac{2}{3}x + 1$

$-3x + \frac{1}{2} = \frac{1}{2}y \Rightarrow y = -6x + 1$



$\bar{X} = 63.5 + 10$ (all but 10c which make 100)

$s =$

$n =$

min

max = 100

435

Name: Key

Test #3a – Spring 2011
Math 61

Instructions: Show all work. Some problems are multiple choice. Read each problem carefully. Box your final answer. You may use a calculator and your 5x8 note card. Note card must conform to specifications on the syllabus. Good luck!

1. Using the following matrices find ACD. You must show work in at least 2 places for full credit

$$A = \begin{bmatrix} 1 & -2 & 3 \\ 0 & 4 & -2 \end{bmatrix} \quad C = \begin{bmatrix} -2 & 0 & 1 \\ 4 & -3 & 1 \\ -2 & 1 & 2 \end{bmatrix} \quad D = \begin{bmatrix} 1 & -2 \\ 0 & -3 \\ 3 & 1 \end{bmatrix}$$

$$ACD = \begin{bmatrix} -1 & 10 \\ 20 & 2 \end{bmatrix}$$

+5

$$AC = \begin{bmatrix} 1\cdot-2 + -2\cdot4 + 3\cdot-2 & 1\cdot0 + -2\cdot-3 + 3\cdot1 & 1\cdot1 + -2\cdot1 + 3\cdot2 \\ 0\cdot-2 + 4\cdot4 + -2\cdot-2 & 0\cdot0 + 4\cdot-3 + -2\cdot1 & 0\cdot1 + 4\cdot1 + -2\cdot2 \end{bmatrix} \cdot \begin{bmatrix} 1 & -2 \\ 0 & -3 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} -16\cdot1 + 9\cdot0 + 5\cdot3 & -16\cdot-2 + 9\cdot-3 + 5\cdot1 \\ 20\cdot1 + -14\cdot0 + 0\cdot3 & 20\cdot-2 + -14\cdot-3 + 0\cdot1 \end{bmatrix}$$

2. Find the inverse of the following matrix, if it exists. You must show the work for full credit.

$$A = \begin{bmatrix} 1 & -1 & 1 \\ 0 & -2 & 1 \\ -6 & -7 & 0 \end{bmatrix}$$

$6R_1 + R_3 \to R_3$ Setup +1

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 1 & 1 & 0 & 0 \\ 0 & -2 & 1 & 0 & 1 & 0 \\ -6 & -7 & 0 & 0 & 0 & 1 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} 7 & -7 & 1 \\ -6 & 6 & -1 \\ -12 & 13 & -2 \end{bmatrix}$$

+5

$-2R_1 + R_2 \to R_2$, $\frac{1}{2}R_2 \to R_2$, $-R_2 + R_3 \to R$, $R_1 + R_3 \to R_1$, $-\frac{1}{2}R_1 \to R_1$, $\frac{1}{2}R_2 \to R_2$

$$\sim \left[\begin{array}{ccc|ccc} 1 & -1 & 1 & 1 & 0 & 0 \\ 0 & -2 & 1 & 0 & 1 & 0 \\ 0 & -13 & 6 & -6 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} -2 & 0 & -1 & -2 & 1 & 0 \\ 0 & -2 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & -12 & 13 & -2 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} -2 & 0 & 0 & -14 & 14 & -2 \\ 0 & 2 & 0 & -12 & 12 & -2 \\ 0 & 0 & 1 & -12 & 13 & -2 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 7 & -7 & 1 \\ 0 & 1 & 0 & -6 & 6 & -1 \\ 0 & 0 & 1 & -12 & 13 & -2 \end{array}\right]$$

Row Transformations done w/ goal in mind +1

3. Write the matrix equation as a system of linear equations without matrices.

$$\begin{bmatrix} 7 & 2 & -8 \\ 8 & 3 & -7 \\ 4 & 5 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ -8 \\ -1 \end{bmatrix}$$

+3

$$7x + 2y - 8z = 6$$
$$8x + 3y - 7z = -8$$
$$4x + 5y - 2z = -1$$

If the problem is done w/ diagonals 1

$R_2 + R_1 \to R_2$, $13R_2 + R_3 \to R_3$, $R_1 + R_3 \to R_1$, $-R_3 + R_2 \to R_2$, $-2R_3 \to R_3$

$$\sim \left[\begin{array}{ccc|ccc} 1 & -1 & 1 & 1 & 0 & 0 \\ 0 & 1 & -\frac{1}{2} & 0 & -\frac{1}{2} & 0 \\ 0 & -13 & 6 & 6 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & 0 & \frac{1}{2} & 1 & -\frac{1}{2} & 0 \\ 0 & 1 & -\frac{1}{2} & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & -\frac{1}{2} & 6 & -13\frac{1}{2} & 1 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 7 & -7 & 1 \\ 0 & 1 & 0 & -6 & 6 & -1 \\ 0 & 0 & 1 & -12 & 13 & -2 \end{array}\right]$$

+13

4. Determine which of the following matrix equations represents the solution to the system. Circle the answer.

$$2x + y = 2$$
$$5x + 3y = 13$$

$$\begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 13 \end{bmatrix}$$

+2

i) $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 5 & -3 \end{bmatrix}\begin{bmatrix} 2 \\ 13 \end{bmatrix}$

ii) $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 13 \end{bmatrix}\begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}$ } Not Possible Can't multiply

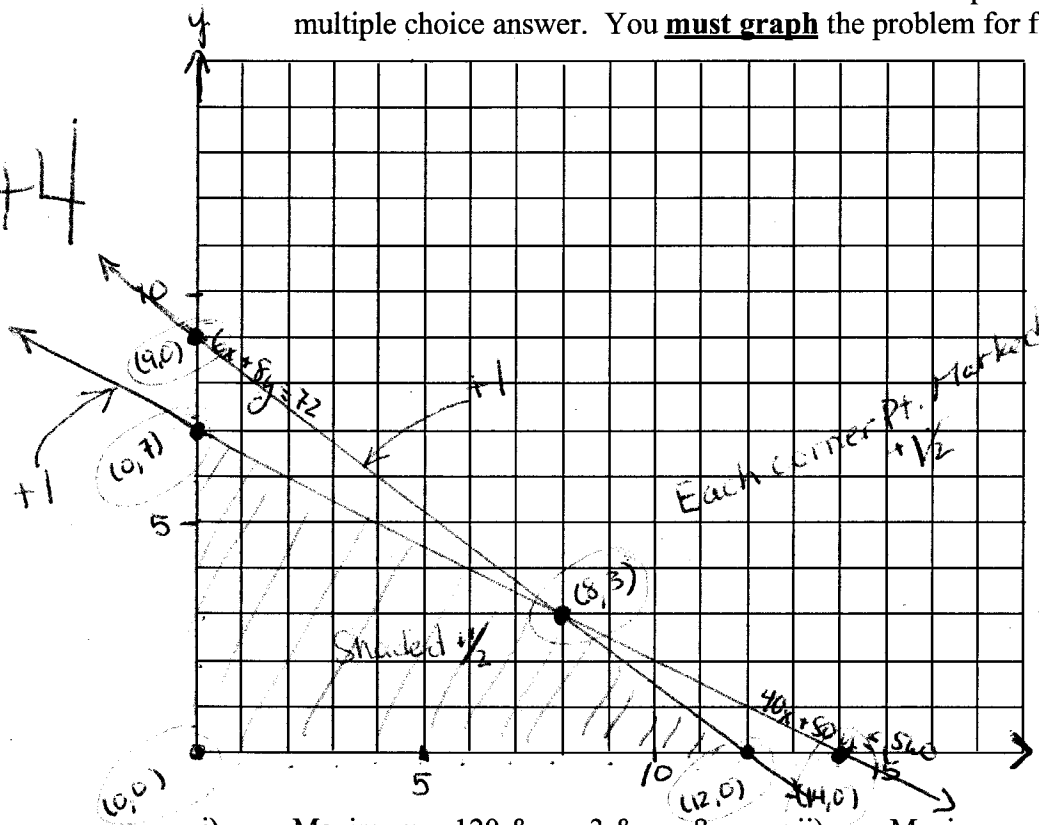
iii) $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}\begin{bmatrix} 2 \\ 13 \end{bmatrix}$

iv) $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 13 \end{bmatrix}\begin{bmatrix} -2 & -1 \\ -5 & -3 \end{bmatrix}$

b/c $A^{-1}AX = A^{-1}B = X$

5. Graph the system of linear inequalities in the graph shown and then use it to solve the linear programming problem by circling one of the multiple choice selections below. You will not receive more than one point for just circling a multiple choice answer. You **must graph** the problem for full credit.

+4



$z = 8x + 12y$

$40x + 80y \le 560$ — $x=14$ $y=0$; $y=7$ $x=0$

$6x + 8y \le 72$ — $x=12$ $y=0$; $y=9$ $x=0$

$x, y \ge 0$

Corner Pts Plot

$40x + 80y = 560$
$-60x - 80y = -720$
$-20x = -160$
$x = 8$

$6(8) + 8y = 72$
$-48 \quad -48$
$8y = 24 \Rightarrow y = 3$

$z = 8(0) + 12(0) = 0$
$z = 8(0) + 12(7) = 84$
$z = 8(8) + 12(3) = 100$
$z = 8(12) + 12(0) = 96$

i) Maximum = 120 & x = 3 & y = 8

ii) Maximum = 92 & x = 4 & y = 5

+2

iii) Maximum = 96 & x = 9 & y = 2

iv) Maximum = 100 & x = 8 & y = 3

+8

7. Pivot one about the circled element in the simplex tableau and read the solution from the result. Show your work. Make sure to give the values of x_1, x_2, x_3, s_1, s_2 & z

$\frac{1}{2}R_1 \to R_1$

$-R_1 + R_3 \to R_3$

$-2R_1 + R_2 \to R_2$

	x_1	x_2	x_3	s_1	s_2	z	
x_1	-1	0	(2)	0	-4	0	-10
x_2	0	1	4	3	2	0	13
	0	0	2	-4	0	1	0

$\sim$

	x_1	x_2	x_3	s_1	s_2	z	
x_3	$\frac{1}{2}$	0	1	0	2	0	5
x_2	-2	1	0	3	-6	0	-7
	-1	0	0	-4	-4	1	-10

$x_3 = 5$, $x_2 = -7$, $x_1 = s_1 = s_2 = 0$ | $z = -10$

8. Set up the system of equations that would be needed to solve for the maximum number of interviews. Don't forget non-negative constraints. Be sure to clearly indicate what your variables represent. Call the equation that you are maximizing P for consistency.

> The budget for a project on voting trends includes $4200 for hiring undergraduate students, graduate students and faculty members to conduct interviews the day before an election. Each undergraduate student will conduct 23 interviews for $100. Each graduate student will conduct 28 interviews for $150. Each faculty member will conduct 31 interviews for $200. No more than 23 interviewers can be hired. How many of each type of interviewer should be hired in order to maximize the number of interviews?

x = # undergrad
y = # graduate
z = # faculty

$P = 23x + 28y + 31z$ Maximize # of interviews

$100x + 150y + 200z \leq 4200$ Constraint on cost to hire

$x + y + z \leq 23$ Constraint on # of hires

$x, y, z \geq 0$

9. For problem #8 give the simplex tableau. Put the variables along the top and the basic variables along the side. You don't have to do anything more than create the tableau.

	x	y	z	s_1	s_2	P	
s_1	100	150	200	1	0	0	4200
s_2	1	1	1	0	1	0	23
	-23	-28	-31	0	0	1	0

10. Formulate the dual problem for the linear programming problem. Show the A matrix and A^T. You don't have to do any more than write the system of equations for the dual problem after giving A and A^T.

Minimize $C = 3x_1 + x_2$ subject to $2x_1 + 3x_2 \geq 60,\ x_1 + 4x_2 \geq 40$
with $x_1, x_2 \geq 0$

$$A = \left[\begin{array}{cc|c} x_1 & x_2 & \\ 2 & 3 & 60 \\ 1 & 4 & 40 \\ \hline 3 & 1 & 1 \end{array}\right] \quad +1$$

$$A^T = \left[\begin{array}{cc|c} y_1 & y_2 & \\ 2 & 1 & 3 \\ 3 & 4 & 1 \\ \hline 60 & 40 & 1 \end{array}\right] \quad +1$$

$P = 60y_1 + 40y_2$ +1

$2y_1 + y_2 \leq 3$ +½

$3y_1 + 4y_2 \leq 1$ +½

$y_1, y_2 \geq 0$

+4½

11. In the following problem, a minimization problem, the corresponding dual problem and the final simplex tableau in the solution of the dual problem are given. Find the optimal solution for the minimization and maximization problems and list the answers as prompted.

Minimize	$C = 21x_1 + 50x_2$	Maximize	$P = 24y_1 + 34y_2$
subject to	$2x_1 + 5x_2 \geq 24$	subject to	$2y_1 + 3y_2 \leq 21$
	$3x_1 + 7x_2 \geq 34$		$5y_1 + 7y_2 \leq 50$
	$x_1, x_2 \geq 0$		$y_1, y_2 \geq 0$

$$\begin{array}{c} \\ y_2 \\ y_1 \end{array}\begin{array}{c} \begin{array}{ccccc} y_1 & y_2 & x_1 & x_2 & P \end{array} \\ \left[\begin{array}{ccccc|c} 0 & 1 & 5 & -2 & 0 & 5 \\ 1 & 0 & -7 & 3 & 0 & 3 \\ \hline 0 & 0 & 2 & 4 & 1 & 242 \end{array}\right] \end{array}$$

a) The optimal solution for the dual problem is:

Maximum of $P =$ 242

$y_1 =$ 3

$y_2 =$ 5

+1½

b) The optimal solution for the minimization problem is:

Minimum of $C =$ 242

$x_1 =$ 2

$x_2 =$ 4

+1½

+7½