§1.1 Linear Equations and Inequalities

Linear Equation in 1 Variable
Any equation that can be written in the following form:

$$ax + b = 0 \qquad a,b \in \Re,\ a \neq 0 \text{ and x is a variable}$$

Any equation has a solution, sometimes called a root or a zero. The solution to any equation is the number or numbers, which make it a true statement, or satisfies the equation. We typically write the **solution** to an equation as:

$$\text{var} = \# \text{ or } \{\#\}$$

Note: *This is an equivalent form to the original form – ax + b = 0*

You may remember as a child doing missing addend and missing factor problems. It is the skills those problems developed combined with the two properties of additive inverses and addition property of zero (or identity property of addition) and/or the multiplicative inverse and the multiplicative property of 1 (or the identity property of multiplication) that give us the skills to solve equations.

RECALL: Addition Property of Equality
Add the same thing to both sides of the equation and the resulting equation is equivalent to the original. This allows us to move **terms** from one side of the equation to the other.

If $a = b$, then $a + c = b + c$

Example: If $x - 5 = 7$, then $x - 5 + 5 = 7 + 5$

Note: *Usually in printed text you will note the horizontal fashion in which the addition property is presented, but when I do this in class you will see me write it in a columnar form. The columns are preferred because they allow us to see that the same thing is being done to both sides of the equation, however columns are difficult in printed text.*

RECALL: Multiplication Property of Equality
Multiply both sides of the equation by the same number and resulting equation is equivalent to the original. This allows us to isolate the variable (remove the numeric coefficient).

If $a = b$, then $ac = bc$

Example: If $\frac{1}{5}x = 2$, then $\frac{1}{5} \bullet 5x = 2 \bullet 5$

Note: *By your level this is most commonly shown by division, however I will refrain from saying, if not from doing this as a division problem to further concrete the principle in your mind so that when fractions become common place, you do not freeze up and fail to do the problem correctly, simply because you don't know how to think it through and divide by a fraction.*

Solving Algebraic Equations

1) Clear equation of fractions or decimals **(We'll focus our attention here!)**
2) Simplify each side of the equation (distribute and combine like terms)
3) Move the variable to the "left" by applying addition property
4) Move constants to the "right" by applying the addition property
5) Remove the numeric coefficient by applying the multiplication property
6) Write solution as a solution set or variable = # or {#}

Example: Solve the following and give the solution set in roster form.

a) $2(y + 3) = 8 + y$

b) $2(c + 5) - 3 = 2(c - 3) + 2c + 1$

c) $\frac{1}{2}g + \frac{2}{3} = 5$

d) $0.5x + 0.25 = 1.2$

Note c): *Find the LCD and multiply each term by that to remove fractions.*
Note d): *Note the largest number of decimal places and multiply each term by a factor of 10 containing that number of zeros.*

e) $\frac{1}{3}(y - 5) = \frac{1}{4}$

f) $0.25(x - 0.1) = 0.5x + 0.75$

Note e): *In my opinion it is generally easier to simplify any distributive properties before clearing the equation of fractions or decimals.*

Solving for Variables when More Than One Exists

Focus on variable!! (highlight the one that you are solving for)

Focus on isolating the one variable!!!

What is added to (subtracted from) the one variable of focus

Undo this by addition property (adding the opposite)

What is multiplied by (what is it divided by) the one variable of focus

Undo this by using the multiplication property (mult. by reciprocal)

Example: Solve the following for the variable specified

a) $5 + y = 9x$; y

b) $2x - y = 3$; y

c) $2(x - 5) + 2y = 9$; y

Note: *You need to distribute before attempting to solve if the variable being isolated is within the parentheses.*

d) $Ab + Bc = D$; c

e) $G = 9h(a_1 + a_2)$; a_1

First we will learn our last way of writing a set. This is another method of writing a solution set which augments our roster form and set builder notation. **Interval Notation** indicates the solution set of an inequality and the inclusion of the endpoints. This is similar to roster form for the solution set of a linear equality, but unlike a linear equality a linear inequality is all real numbers between two points or beginning at a point and traveling out to negative or positive infinity. **Endpoints** are the beginning or end of the solution set. They are either included or not included in the solution set of an inequality. When graphing the solution set of an inequality we used either open or solid circles or parentheses and brackets. The parentheses and brackets (used by most algebra texts today) are what we need for interval notation. An ***open circle or a parenthesis*** indicates that the number is ***not a part*** of the solution set. A ***solid circle or a bracket*** indicates that the number ***is a part*** of the solution set. When graphing simple inequalities, we can get a solution set that travels out to negative or positive infinity (-∞, ∞). Infinity is an elusive point since you can never reach it; therefore, in interval notation we always use a parenthesis around infinity.

Summary

Endpoint included	[or]
Endpoint not included	(or)
Negative Infinity	(-∞
Positive Infinity	∞)

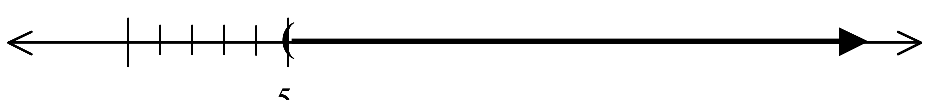
Review of Set Builder Notation

Description of the set using braces, vertical line and simple or compound inequality.

{x | x > # }
{x | x < # }
{x | # < x < #}   It is assumed that $x \in \Re$

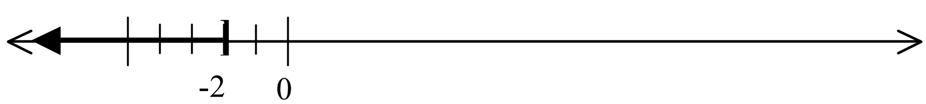
Visually Relating to the Number Graph of an Inequality

x > 5   Will be graphed with an parenthesis (open circle) on 5 and a line to the right with an arrow on the end showing that it continues on to infinity.



Interval Notation   (5, ∞)
Set Builder Notation   {x|xϵ R, x > 5}

-2 ≥ x   Will be graphed with an bracket (closed circle) on 5 and a line to the right with an arrow on the end showing that it continues on to infinity.

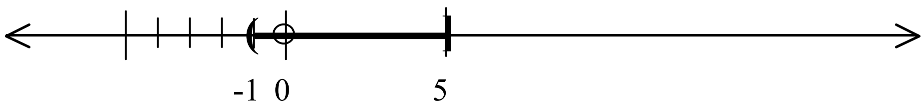


Interval Notation   (-∞, -2]

* Finite Mathematics, Barnett, Ziegler & Byleen, 12^{th} edition

Set Builder Notation $\{x | x \in R, x \le -2\}$

$-1 < x \le 5$ Will be graphed with an parenthesis (open circle) on -1 and a bracket (closed circle) on 5 and a line in between.



Interval Notation $(-1, 5]$

Set Builder Notation $\{x | x \in R, -1 < x \le 5\}$

Solving Simple Linear Inequalities (x ≥ a or x > a)

1) Follow the same procedure for solving a linear equation
   a) Addition Property of Equality
      i) move the variable to one side
      ii) move # to other side
   b) Multiplication Property*
      i) Multiply by the reciprocal of the numeric coefficient

*If multiplying by a negative the **sense of the inequality reverses**, which means that inequality symbol flips around. (a < becomes a >)

2) Graph, give the solution set as indicated

Example: Solve and give the solution set in interval notation

a) $5x - 2 > 3$ b) $2(5 - 3x) \ge 22$

Note: *Be cautious of the inequality reversing. To see why it does, let's solve this problem in a different manner.*

Let’s not forget about fractions in equations and the concept of clearing! This will work with inequalities as well. Find the LCD and multiply all terms in the entire equation by the LCD to clear, just as you did with equations.

Example: Clear and solve. Give your solution in interval notation.

a) $\frac{3A}{10} + 1 > \frac{1}{5} - \frac{A}{10}$ b) $\frac{4k - 3}{6} + 2 \ge \frac{2k - 1}{12}$

Solving Compound Linear Inequalities

1) Move constants from middle to the outsides
   a) Add opposite to right side of the compound inequality
   b) Add opposite to left side of the compound inequality

2) Remove the numeric coefficient*
   a) Multiply the right side of the compound inequality by the reciprocal
   b) Multiply the left side of the compound inequality by the reciprocal

**If multiplying by a negative the sense of the inequality reverses, which means that inequality symbol flips around. (a < becomes a >). Unlike the simple inequality you have no choice but to remember this rule!*

3) Graph and write the solution set as indicated

Example: Solve, graph and write the solution set in both interval notation and set builder notation.

a) $-1 \le x - 5 < 9$

b) $3 < \frac{-x - 5}{3} \le 6$

In this chapter we introduce the application problems that we find in Algebra labeled as linear equations and/or percent increase/decrease problems.

Features of a Linear Equation

Baseline = Known Quantity

Rate of Change = A rate of change

Independent Variable = Unknown which is being chosen

Dependent Variable = Unknown = Baseline + Rate of Change • Independent Variable

The linear equation comes in many forms. Here are some of the linear equations you may already be familiar with:

Simple Interest Problems
Distance Problems
Sales Tax Problems
Discount Problems
Taxi cab/Phone call/Movie Ticket type problems
Cost & Revenue Problems
Break Even Analysis

These are word problems. This is usually student's worst nightmare, but that is what real math is all about – being able to use your math knowledge in the real world. Here is my suggestion.

1) Read the problem from beginning to end. Don't jump into making equations or trying to solve it in your head. Read for understanding, trying to decide what the key information is and how it will mathematically fit together.
2) Look for the indication that you have a linear equation or several linear equations. The key is to look for a rate of change. Do you have a baseline to add or subtract from?
3) Jot down the important information and give the numeric values for each key feature and make a plan for getting a equation (use the linear equation model to help).
4) When you have all information written down, assign a variable for the unknown value(s). Even if there are two, consider that the second may just be a relationship to the other or the exact same variable.
5) Create an equation or equations using substitution and solve.

Recall that the simple interest formula is: $I = PRT$, where P = principle, R=Annual Interest Rate & T= Time in years. This makes the interest earned on a principle amount fit the model of a linear equation with no baseline. For our first example as with many of our equations we will be seeing a two equation, two unknown set up.

Example: You have $500,000 to invest into 2 funds. We will call the funds A & B. Fund A earns 5.2% simple annual interest and Fund B earns 7.7% simple annual interest. How much interest should be invested in each if you want the simple annual interest income form both Fund A & B to amount to $30,000. *(#50p.11)

The CPI (consumer price index) is a introduced in example 10 on p. 10. The price index has can be seen as a ratio of prices from one year to another. This means that problems can be set up as proportion problems of prices of consumer goods from one year to another as the ratio of CPI to the comparable year.

* Finite Mathematics, Barnett, Ziegler & Byleen, 12th edition

Example: If the price change of cars parallels the change in the CPI (see the table bellow), what would a house valued at $200,000 in 2005 be valued at (to the nearest dollar) in 1960? *(#52 p. 11)

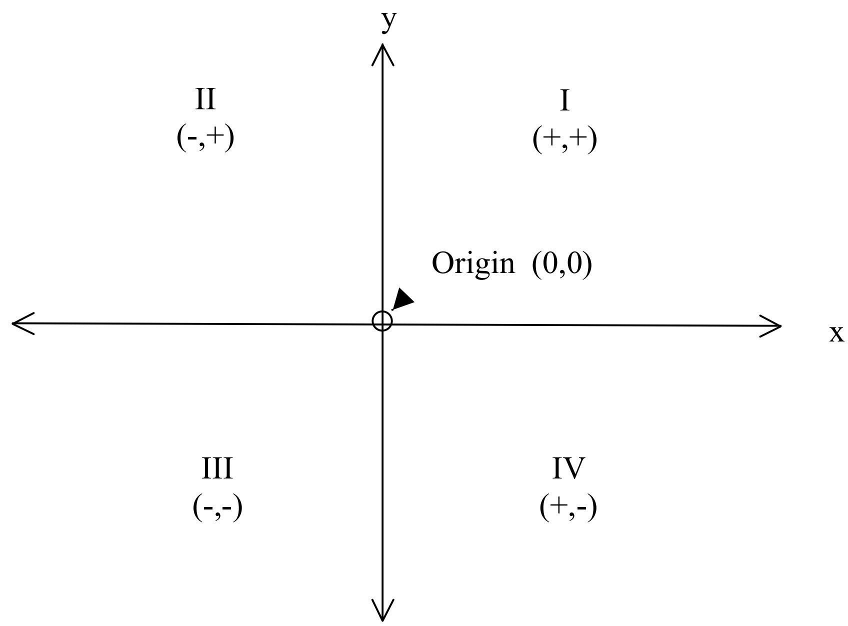
Year	Index
1960	29.6
1975	53.8
190	130.7
2005	195.3

The next example relates to economics and something called Break-Even Analysis. We can create a linear equation to represent the cost of producing q items and another the represent selling those q items. The cost equation has a baseline that we called the fixed cost and a rate of change that we call variable cost. The revenue equation has no baseline and only a rate of change called the revenue per item. A company will break even when the cost is equal to the revenue.

Example: The publisher of a new book figures fixed costs at $92,000 and variable costs at $2.10 each book produced. If the book is sold to distributors for $15 each, how many must be sold for the publisher to break even? *(#60 p. 12)

§1.2 Graphs and Lines

The following is the Rectangular Coordinate System also called the Cartesian Coordinate System (after the founder, Renee Descartes). The x-axis is horizontal and is labeled just like a number line. The y-axis intersects the x-axis perpendicularly and increases in the upward direction and decreases in the negative direction. The two axes usually intersect at zero on each. They form a **plane**, which is a flat surface such as a sheet of paper (anything with only 2 dimensions is a plane).



A **coordinate** is a number associated with the x or y axis.

An **ordered pair** is a pair of coordinates, an x and a y, read in that order. An ordered pair names a specific point in the system. Each point is unique. An ordered pair is written (x,y)

The **origin** is where both the x and the y axis are zero. The ordered pair that describes the origin is (0,0).

The **quadrants** are the 4 sections of the system labeled counterclockwise from the upper right corner. The quadrants are named I, II, III, IV. These are the Roman numerals for one, two, three, and four. It is not acceptable to say One when referring to quadrant I, etc.

A **Linear Equation in Two Variables** is an equation in the form shown below, whose *solutions are ordered pairs.* A linear equation in two variables can visually (graphically) be represented by a straight line.

$$ax + by = c$$

a, b, & c are constants
x, y are variables
x & y both can not = 0

A linear equation in two variables has a solution that is an **ordered pair**. Since the first coordinate of an ordered pair is the x-coordinate and the second the *y*, we know to substitute the first for *x* and the second for *y*. *(If you ever come across an equation that is not written in x and y, and want to check if an ordered pair is a solution for the equation, assume that the variables are alphabetical, as in **x** and **y**. For instance, 5d + 3b = 10 : **b** is equivalent to **x** and **d** is equivalent to **y**, unless otherwise specified.)*

Horizontal, Vertical & Lines Through the Origin

Now, we need to discuss 3 types of special lines. Two of these don't appear to be linear equations in 2 variables because they are written in 1 variable, but this is because the other variable can be anything. The linear equations in question are **vertical** and **horizontal lines**. Horizontal lines have equations that look like ***y* = #.** Vertical lines have equations that look like ***x* = #.** There is a third special type of line that has an x and y-intercept that are the same. This is a line through the origin and it will appear as *ax = by* or *ax + by = 0*. Here is a summary of information that you will eventually need to know about these three types of lines:

Type	**Equation**	**Slope**	**Type of ordered Pairs**	**Intercepts**
Horizontal	y = #	Zero	(#1, y), (#2, y), (#3, y); y agrees with equation & #1, #2 & #3 can be anything	y-intercept: (0,y) no x-intercept
Vertical	x = #	Undefine d	(x, #1), (x, #2), (x, #3); x agrees with equation & #1, #2,& #3	No y-intercept x-intercept: (x, 0)
Through Origin	ax = by or ax + by = 0	$m = {}^{a}/_{b}$ when ax = by		x & y-intercept: (0,0)

A linear equation in standard form can be written in a special form, called **slope-intercept form**. This can be done by solving the equation for y.

y = mx + b

m = slope (the numeric coefficient of x)
b = y-intercept (the y-coordinate of the ordered pair, (0, b))

Finding X-intercept Point (Y-Intercept if not in slope-intercept form)

Step 1: Let y = 0 (for y-intercept let x = 0)
Step 2: Solve the equation for x (solve for y to find the x-intercept)
Step 3: Form the ordered pair **(x, 0)** where x is the solution from step two. [the ordered pair would be **(0, b)**]

Slope is the ratio of vertical change to horizontal change.

$$m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x}$$

Rise is the amount of change on the y-axis and run is the amount of change on the x-axis.

A line with positive slope "climbs up" when viewing from left to right and a line with negative slope "slides down" from left to right.

There are actually **<u>3 methods</u>** for finding a slope.

<u>First Method</u> is by using the equation:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

We will use this method most often under 3 circumstances:

1) When we have a graphed line and we are trying to give its equation (a skill we will come to soon)
2) When we know two points on a line (also generally used to give the equation of a line)
3) When we have an equation and find two points on the line.

To use the equation above you must know that each ordered pair is of the form (x_1, y_1) and (x_2, y_2). The subscripts (the little numbers below and to the right of each coordinate) just help you to keep track of which ordered pair they are coming from. **You must have the coordinate from each ordered pair "lined up over one another" in the formula to be doing it correctly!**

<u>Second Method</u> is a visual/geometric approach:

$$m = \frac{\text{rise}}{\text{run}}$$

We will use this method only for a line that is already graphed.

Note: *You could get the ordered pairs from the graph and then plugging them into the equation as above but that would be a lot of work, when there is an easier way!*

<u>Finding the Slope of a Line Visually/Geometrically</u>

Step 1: Choose 2 points (must be integer ordered pairs) on the line.

Step 2: Draw a right triangle by drawing a line horizontally from the lower point and vertically from the higher point (so they meet at a right angle forming a triangle below the line).

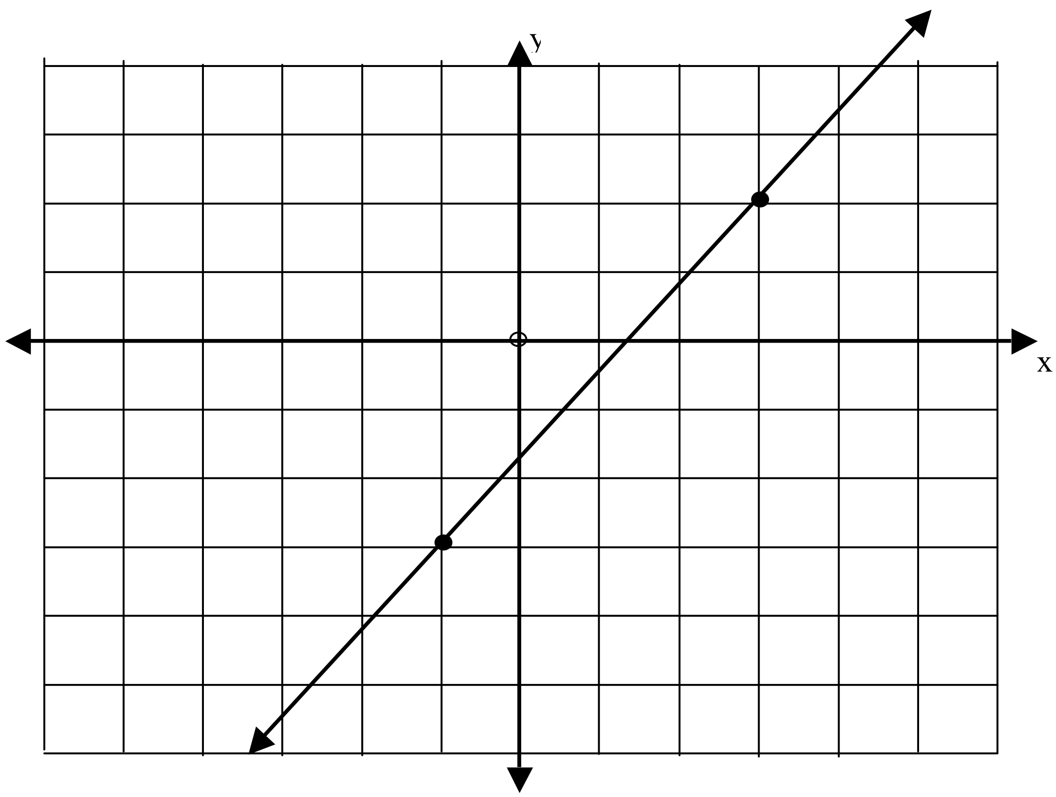
Step 3: Count the number of units from the upper point to the point where the 2 lines meet. This is the **rise**, and if you traveled down it is negative.

Step 4: Count the number of units from your current position on the triangle, horizontally to the line. This is the **run**, and if you traveled to the left it is negative.

Step 5: Use the version of the slope formula that says $m = \frac{\text{rise}}{\text{run}}$, plug in and simplify.

Note: *The slope is always an improper fraction in lowest terms or a whole number. Don't ever make it a mixed number!*

Example: Find the slope of the line below using the visual approach.



<u>Third Method</u> uses the equation of a line in a special form, called **<u>slope-intercept form</u>**.

$y = mx + b$ — $m =$ slope (the numeric coefficient of x)
$b =$ y-intercept (the y-coordinate of the ordered pair, (0, b))

To put the equation of a line in this special form we solve the equation for y. The process is the same each and every time so it should not be difficult, but sometimes we make it difficult by thinking too much.

<u>Solving for y from Standard Form</u>

Step 1: Add the opposite of the x term to both sides (moving the x to the side with the constant)

Step 2: Multiply all terms by the reciprocal of the numeric coefficient of the y term (every term meaning the y, the x and the constant term)

Example: Solve the equation for y (put it into slope-intercept form):

$$-3x + \frac{1}{2}y = -2$$

If there is no y-intercept given (or if it is not an integer ordered pair) then we must use the point-slope form. There are also 3 scenarios here. They are as follows:

<u>Point-Slope Form</u>

Scenario 1: You are given the slope & a point without the y-intercept

Scenario 2: You are given two points neither of which is the y-intercept

Scenario 3: You are given a graph & the y-int. isn't an integer ordered pair.

You need only plug into the **point-slope form**:

$\mathbf{y - y_1 = m(x - x_1)}$ — $m =$ slope
(x_1, y_1) is a point on the line
x & y are variables (don't substitute for those)

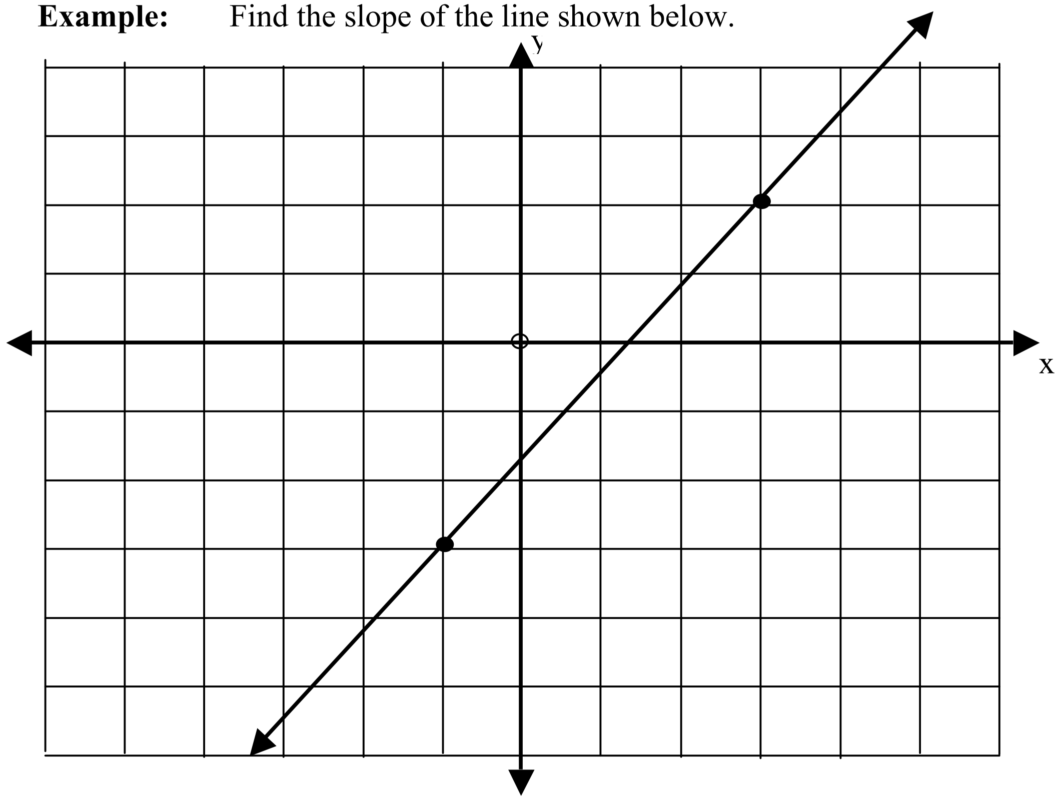
Example: a) Find the equation of the line with slope m = -6 thru (-4,1) *(#46 p. 24)

b) Find the equation of the line thru (2, 3) & (-3, 7)

* Finite Mathematics, Barnett, Ziegler & Byleen, 12th edition

c) Your book does not use any examples of the 3rd type but I will give you an example from another class anyway. Give the equation of the line shown below.

Example: Find the slope of the line shown below.



Your Turn

Example: Find the equation of the line described by the point and slope given.

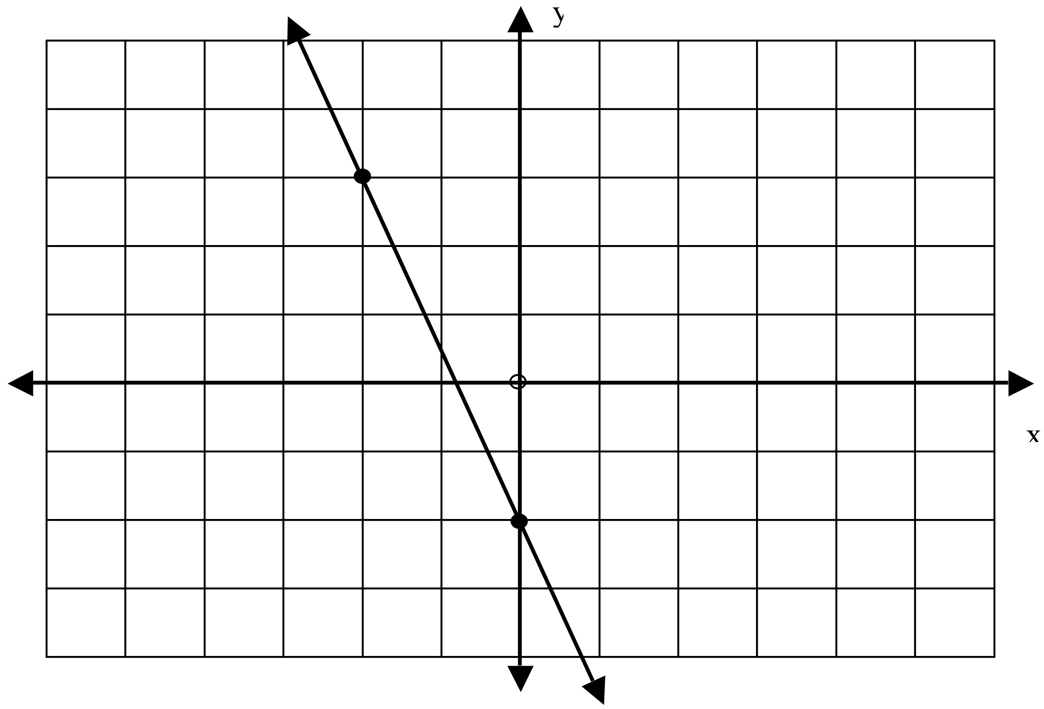
a) (0, 5) m = -1

b) (-3, -1) and (-4, 2)

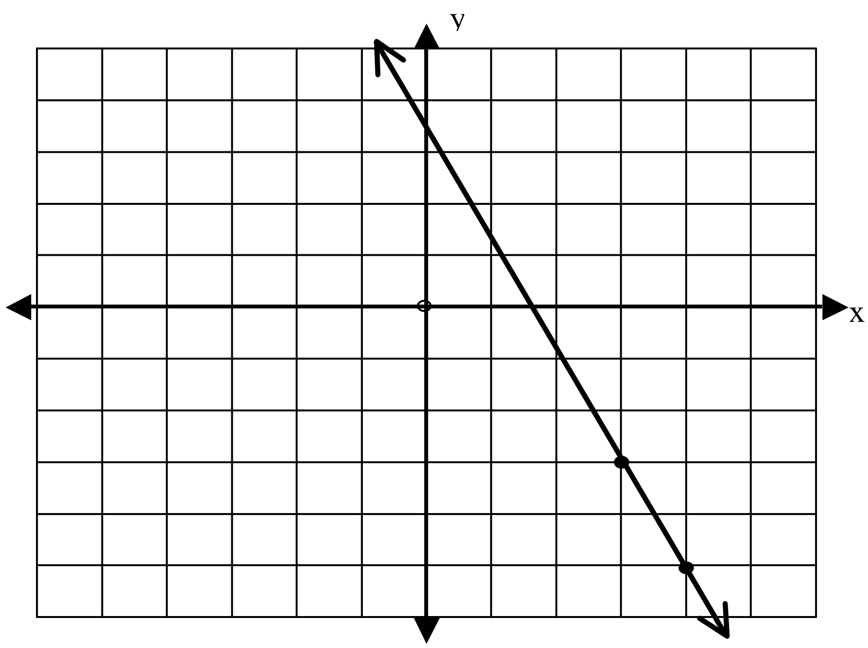
c) m= 5 thru (5, 2)

d) Thru (2,0) and (2, 9)

e)



f)



Example: A plant can manufacture 50 tennis rackets per day for a total daily cost of $3,855 and 60 tennis rackets per day for a total dailey cost of $4245. *(#64 p. 24)

a) Assuming that the daily cost and production are linearly related, find the total daily cost of producing x tennis rackets.

b) Interpret the slope and y-intercept of this cost equation.

Example: The temperature at which water starts to boil is also linearly related to barometric pressure. Water boils at 212°F at a pressure of 29.9 Hg (in. of mercury) and at 191°F at a pressure of 28.4 Hg (biggreenegg.com). * (#70 p. 25)

a) Find a relationship of the form T- mx + b where T is °F and x is pressure in inches of mercury.

b) Find the boiling point at a pressure of 31 in. Hg.

c) Find the pressure if the boiling point is 199°F.

* Finite Mathematics, Barnett, Ziegler & Byleen, 12th edition

Example: At a price $1.94 per bushel, the supply of corn is 9,800 million bushels and the demand is 9,300 million bushels. At a price of $1.82 per bushel, the supply is 9,400 million bushels and the demand is 9,500 million bushels. *(#78 p. 26)

a) Find a supply equation of the form $p = mx + b$

b) Find a demand equation of the form $p = mx + b$

c) Find the equilibrium point

* Finite Mathematics, Barnett, Ziegler & Byleen, 12^{th} edition