M61 Practice Test 3 Sp11

① $BA = \begin{bmatrix} 1\cdot0+\overset{0}{0}\cdot0 & 1\cdot\overset{-1}{-1+}0\cdot0 & 1\cdot\overset{-1}{-1}+0\cdot1 \\ 0\cdot\overset{0}{0}+5\cdot0 & 0\cdot\overset{0}{-1}+5\cdot0 & 0\cdot\overset{5}{-1+5\cdot1} \end{bmatrix}$ $AC = \begin{bmatrix} 0\cdot1+\overset{-1}{-1\cdot1}+-1\cdot0 & 0 & 0\cdot0+\overset{-5}{-1\cdot2+-1\cdot3} \\ 0\cdot1+\overset{0}{0\cdot1}+1\cdot0 & 0 & 0\cdot0+\overset{3}{0\cdot2}+1\cdot3 \end{bmatrix}$

$$3BA+4AC = \begin{bmatrix} 0 & -3 & -3 \\ 0 & 0 & 15 \end{bmatrix} + \begin{bmatrix} -4 & 0 & -20 \\ 0 & 0 & 12 \end{bmatrix} = \boxed{\begin{bmatrix} -4 & -3 & -23 \\ 0 & 0 & 27 \end{bmatrix}}$$

② $$\left[\begin{array}{ccc|ccc} \overbrace{1 \quad\quad 1}^{M} & & 0 & 1 & 0 & 0 \\ -1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 3 & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_1+R_2\to R_2} \sim \left[\begin{array}{ccc|ccc} \underset{-2}{1} & \underset{-2}{1} & \underset{0}{0} & \underset{-2}{1} & \underset{0}{0} & \underset{0}{0} \\ \underset{0}{0} & \underset{-6}{2} & \underset{-3}{1} & \underset{-3}{1} & \underset{-3}{1} & \underset{0}{0} \\ \overset{0}{0} & \overset{6}{3} & \overset{2}{1} & \overset{0}{0} & 8 & \overset{2}{1} \end{array}\right]$$

$-2R_1+R_2\to R_1$
$-3R_2+2R_3\to R_3$

$$\sim \left[\begin{array}{ccc|ccc} -2 & 0 & 1 & -1 & 1 & 0 \\ 0 & 2 & 1 & 1 & 1 & 0 \\ 0 & 0 & -1 & -3 & -3 & 2 \end{array}\right] \xrightarrow[R_3+R_1\to R_1]{R_3+R_2\to R_2} \sim \left[\begin{array}{ccc|ccc} -2 & 0 & 0 & -4 & -2 & 2 \\ 0 & 2 & 0 & -2 & -2 & 2 \\ 0 & 0 & -1 & -3 & -3 & 2 \end{array}\right]$$

$-\frac{1}{2}R_1\to R_1 \quad \frac{1}{2}R_2\to R_2 \quad -R_3\to R_3$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & 1 & -1 \\ 0 & 1 & 0 & -1 & -1 & 1 \\ 0 & 0 & 1 & 3 & 3 & -2 \end{array}\right] \quad \boxed{M^{-1} = \begin{bmatrix} 2 & 1 & -1 \\ -1 & -1 & 1 \\ 3 & 3 & -2 \end{bmatrix}}$$

③ Row by Columns yield constant matrix

$\boxed{5x - y = -1 \quad \& \quad 3x + 2y = 5}$

④ Write the system

$x + 8y = -36$
$3x + 5y = -13$

$$\boxed{\begin{bmatrix} 1 & 8 \\ 3 & 5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -36 \\ -13 \end{bmatrix}}$$

Practice Test 3 p. 2

⑤ Tie-Dye =
Cheraw-Tech =
Handpainted =

$$[50\ \ 40\ \ 30]\begin{bmatrix}15\\10\\12\end{bmatrix} = [\underset{750}{50\cdot15}+\underset{400}{40\cdot10}+\underset{360}{30\cdot12}] = [1510]$$

1510
760

$\boxed{\text{ⓒ } \$1510}$

This wasn't quite the problem I wanted!

⑥ Rather than solve the system, it is better to pay attention to the fact that the solution to $AX = B$ is $A^{-1}AX = A^{-1}B$ & the fact that $A^{-1}A = I$

A & D are not possible b/c solution is $A^{-1}B$ not BA^{-1}
(also B is the constant matrix for the system in answer A)

B's $\begin{bmatrix}2 & 1\\5 & 3\end{bmatrix}\begin{bmatrix}2 & 1\\5 & 3\end{bmatrix} \neq \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$ C's $\begin{bmatrix}3 & -1\\-5 & 2\end{bmatrix}\begin{bmatrix}2 & 1\\5 & 3\end{bmatrix} = \begin{bmatrix}\overset{1}{6-5} & \overset{0}{3-3}\\ \overset{0}{-10+10} & \overset{1}{-5+6}\end{bmatrix}$

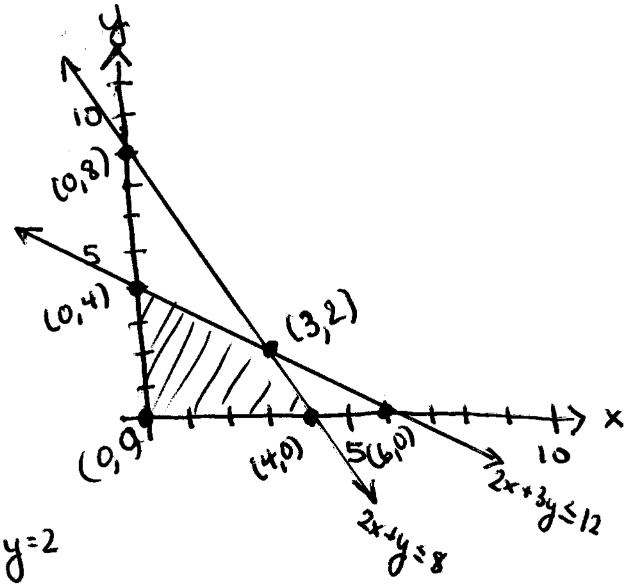
$$\boxed{\text{ⓒ } \begin{bmatrix}x_1\\x_2\end{bmatrix} = \begin{bmatrix}3 & -1\\-5 & 2\end{bmatrix}\begin{bmatrix}2\\13\end{bmatrix}}$$

⑦ $Z = 6x + 7y$

$2x + 3y \leq 12$ } $x=0$ $y=4$; $y=0$ $x=6$

$2x + y \leq 8$ } $x=0$ $y=8$; $y=0$ $x=4$

$x, y \geq 0$



$y = 8-2x \rightarrow 2x + 3(\overset{24\ -6x}{8-2x}) = 12$

$-4x + \underset{-24}{24} = \underset{-24}{12} \Rightarrow -4x = -12$

$x = 3$

$\underset{-6}{2(3)} + 3y = \underset{-6}{12} \Rightarrow 3y = 6 \Rightarrow y = 2$

Possible Optimal Solutions

(0,0)	$Z = 6(0) + 7(0) = 0$	
(0,4)	$Z = 6(0) + 7(4) = 28$	
(3,2)	$Z = 6(3) + 7(2) = 18+14 = 32$	Maximum
(4,0)	$Z = 6(4) + 7(0) = 24$	

$\boxed{\text{Ⓒ Maximum of 32 when } x = 3 \text{ \& } y = 2}$

Practice Test #3 p.3

⑧ Let $x_1=x_2=0$ ∴ $2(0)+(0)+s_1=5\Rightarrow\boxed{s_1=5}$ & $(0)+2(0)+s_2=4\Rightarrow\boxed{s_2=4}$

Let $x_1=s_1=0$ ∴ $2(0)+x_1+(0)=5\Rightarrow\boxed{x_1=5}$ & $(0)+\underset{-10}{2(5)}+s_2=\underset{-10}{4}\Rightarrow\boxed{s_2=-6}$

Let $x_1=s_2=0$ ∴ $2(0)+x_2+s_1=5\Rightarrow s_1=5-x_2$ & $(0)+2x_2+(0)=4\Rightarrow\boxed{x_2=2}$

$\Rightarrow\boxed{s_1=5-2=3}$

Let $x_2=s_1=0$ ∴ $2x_1+(0)+(0)=5\Rightarrow\boxed{x_1=5/2}$ & $x_1+2(0)+s_2=4\Rightarrow s_2=4-x_1$

$\boxed{s_2=\frac{8}{2}-\frac{5}{2}=\frac{3}{2}}$

Let $x_2=s_2=0$ ∴ $2x_1+(0)+s_1=5\Rightarrow s_1=5-2x_1$ & $x_1+2(0)+(0)=4\Rightarrow\boxed{x_1=4}$

$\boxed{s_1=5-2(4)=-3}$

Let $s_1=s_2=0$ ∴ $2x_1+x_2+(0)=5\Rightarrow x_2=5-2x_1$ & $x_1+2(\overset{10}{5}-\overset{-4x_1}{\cancel{2}x_1})+(0)=4$

$\boxed{x_2=5-2(2)=1}$ $\Rightarrow -3x_1=-6\Rightarrow\boxed{x_1=2}$

	x_1	x_2	s_1	s_2	Basic Feasible?	
A)	0	0	$\boxed{5}$	$\boxed{4}$	$\boxed{\text{Yes}}$	no negatives
B)	0	$\boxed{5}$	0	$\boxed{-6}$	$\boxed{\text{No}}$	a negative
C)	0	$\boxed{2}$	$\boxed{3}$	0	$\boxed{\text{Yes}}$	no negatives
D)	$\boxed{5/2}$	0	0	$\boxed{3/2}$	$\boxed{\text{Yes}}$	no negatives
E)	$\boxed{4}$	0	$\boxed{-3}$	0	$\boxed{\text{No}}$	a negative
F)	$\boxed{2}$	$\boxed{1}$	0	0	$\boxed{\text{Yes}}$	no negatives

⑨ $3x_1+x_2+s_1\le 30$ $\left\{\begin{array}{ll} x_1=0 & x_2=30 \\ x_2=0 & x_1=10 \end{array}\right.$

$x_1+x_2+s_2\le 12$ $\left\{\begin{array}{ll} x_1=0 & x_2=12 \\ x_2=0 & x_1=12 \end{array}\right.$

$x_1+3x_2+s_3\le 21$ $\left\{\begin{array}{ll} x_1=0 & x_2=7 \\ x_2=0 & x_1=21 \end{array}\right.$

$x_1,x_2,s_1,s_2,s_3\ge 0$

Ⓑ is the graph since all lines are the same & intersection of all is the irregular pentagon

Practice Test #3 p.4

⑨ Continued

✓Let $x_1 = s_1 = 0$ ∴ $3(0) + x_2 + (0) = 30 \Rightarrow \boxed{x_2 = 30}$ & $(0) + (30) + s_2 = 12 \Rightarrow \boxed{s_2 = 18}$ & $(0) + \overset{90}{3(30)} + s_3 = 21 \Rightarrow \boxed{s_3 = 69}$

✓Let $x_1 = s_2 = 0$ ∴ $3(0) + x_2 + s_1 = 30 \Rightarrow x_2 = 30 - s_1$ & $(0) + (30 - s_1) + (0) = 12 \Rightarrow \boxed{s_1 = 18}$ & $(0) + 3(12) + s_3 = 21$; $-36 \quad -36$ $\Rightarrow \boxed{s_3 = -15}$

$\boxed{x_2 = 30 - 18 = 12}$

✓Let $x_1 = s_3 = 0$ ∴ $3(0) + x_2 + s_1 = 30 \Rightarrow x_2 = 30 - s_1$ & $(0) + (7) + s_2 = 12 \Rightarrow \boxed{s_2 = 5}$ & $(0) + \overset{90\ -3s_1}{3(30 - s_1)} + 0 = 21$ $\Rightarrow 3s_1 = 69 \Rightarrow \boxed{s_1 = 23}$

$\boxed{x_2 = 30 - 23 = 7}$

✓Let $x_2 = s_1 = 0$ ∴ $3x_1 + (0) + (0) = 30 \Rightarrow \boxed{x_1 = 10}$ & $(10) + (0) + s_2 = 12 \Rightarrow \boxed{s_2 = 2}$ & $(10) + 3(0) + s_3 = 21$ $\Rightarrow \boxed{s_3 = 11}$

✓Let $x_2 = s_2 = 0$ ∴ $3x_1 + (0) + s_1 = 30 \Rightarrow 3x_1 = 30 - s_1$ & $(10 - s_1/3) + (0) + (0) = 12$ & $(12) + 3(0) + s_3 = 21$ $\boxed{s_3 = 9}$

$x_1 = 10 - s_1/3$ $\Rightarrow -\frac{s_1}{3} = 2 \Rightarrow \boxed{s_1 = -6}$

$\boxed{x_1 = 10 + 2 = 12}$

✓Let $x_2 = s_3 = 0$ ∴ $3x_1 + (0) + s_1 = 30 \Rightarrow x_1 = 10 - s_1/3$ & $(21) + (0) + s_2 = 12$ $\boxed{s_2 = -9}$ & $(10 - s_1/3) + 3(0) + (0) = 21$ $\Rightarrow -\frac{s_1}{3} = 11 \Rightarrow \boxed{s_1 = -33}$

$\boxed{x_1 = 10 + 11 = 21}$

✓Let $s_1 = s_2 = 0$ ∴ $3x_1 + x_2 + (0) = 30 \Rightarrow x_2 = 30 - 3x_1$ & $x_1 + (30 - 3x_1) + (0) = 12$ & $(9) + 3(3) + s_3 = 21$ $\boxed{s_3 = 21 - 18 = 3}$

$\boxed{x_2 = 30 - 27 = 3}$ $-2x_1 = -18 \Rightarrow \boxed{x_1 = 9}$

✓Let $s_1 = s_3 = 0$ ∴ $3x_1 + x_2 + (0) = 30 \Rightarrow x_2 = 30 - 3x_1$ & $\overset{102}{\frac{69}{8} + \frac{33}{8}} + s_2 = \frac{96}{8}$ & $x_1 + \overset{90\ -9x_1}{3(30 - 3x_1)} + (0) = 21$

$\boxed{x_2 = \frac{240}{8} - \frac{207}{8} = \frac{33}{8}}$ $\boxed{s_2 = -\frac{6}{8} = \frac{-3}{4}}$ $-8x_1 = -69 \Rightarrow \boxed{x_1 = \frac{69}{8}}$

✓Let $s_2 = s_3 = 0$ ∴ $\overset{45/2}{3\left(\frac{15}{2}\right)} + \left(\frac{9}{2}\right) + s_1 = 30 \Rightarrow s_1 = \frac{60}{2} - \frac{54}{2}$ & $x_1 + x_2 + (0) = 12 \Rightarrow x_2 = 12 - x_1$ & $x_1 + \overset{36\ -3x_1}{3(12 - x_1)} + (0) = 21$

$\boxed{s_1 = \frac{6}{2} = 3}$ $\boxed{x_2 = \frac{24}{2} - \frac{15}{2} = \frac{9}{2}}$ $-2x_1 = -15$ $x_1 = 15/2$

x_1	x_2	s_1	s_2	s_3	Intersection Point	Feasible?
0	0	30	12	21	(0,0)	Yes
0	30	0	−18	−69	(0,30)	No
0	12	18	0	−15	(0,12)	No
0	7	23	5	0	(0,7)	Yes
10	0	0	2	11	(10,0)	Yes
12	0	−6	0	9	(12,0)	No
21	0	−33	−9	0	(21,0)	No
9	3	0	0	3	(9,3)	Yes
69/8	33/8	0	−3/4	0	(69/8, 33/8)	No
15/2	9/2	3	0	0	(15/2, 9/2)	Yes

Practice Test #3 p.5

⑩ A) Basic variables have a 1 & 0's in their columns

$\ast\boxed{x_1, S_1 \text{ \& } P}$ *this is choice 4

B) All other variables are non-basic

$\ast\boxed{x_2, S_2}$ *this is choice 4

c) From the tableau $\boxed{x_1 = 11,\ x_2 = 0,\ S_1 = 13,\ S_2 = 0,\ P = 12}$

D) * An additional pivot is needed since the bottom row has a negative number. *This is A)

⑪ $S_2 = 20,\ x_3 = 28,\ P = 20 \quad x_1 = x_2 = S_1 = 0$

This is choice B

⑫ Maximize $P = 30x_1 + 40x_2$

$$2x_1 + x_2 \le 50$$
$$x_1 + x_2 \le 35$$
$$x_1 + 2x_2 \le 60$$
$$x_1, x_2 \ge 0$$

$-2R_1 + R_3 \to R_1 \quad 20R_3 + R_4 \to R_4$

$-2R_2 + R_3 \to R_2$

$$\begin{array}{c|cccccc|c|l} & x_1 & x_2 & S_1 & S_2 & S_3 & P & & \\ \hline S_1 & 2_{-4} & 1_{-2} & 1_{-2} & 0_0 & 0_0 & 0_0 & 50_{-100} & 50 \\ S_2 & 1_{-2} & 1_{-2} & 0_0 & 1_{-2} & 0_0 & 0_0 & 35_{-70} & 35 \\ S_3 & 1^{20} & \textcircled{2}^{40} & 0^0 & 0^0 & 1^{20} & 0^0 & 60^{1200} & 30 \leftarrow \text{pivot row smallest} \\ \hline & -30 & -40 & 0 & 0 & 0 & 1 & 0 & \end{array}$$

↑ pivot (smallest) column

$\frac{1}{2}R_3 \to R_3 \quad -\frac{1}{2}R_1 \to R_1$

$+R_1 \overset{-}{+} 3R_2 \to R_1 \quad R_2 + R_3 \to R_3$

$-R_2 \to R_2 \quad -10R_2 + R_4 \to R_4$

$$\sim \begin{array}{c|cccccc|c|l} & x_1 & x_2 & S_1 & S_2 & S_3 & P & & \\ \hline S_1 & -3 & 0 & -2 & 0 & 1 & 0 & -40 & 40/3 = 13\tfrac{1}{3} \\ S_2 & \textcircled{-1}^{3}_{10} & 0_0 & 0^0_0 & -2^6_{20} & 1^{-3}_{-10} & 0^0_0 & -10^{30}_{100} & 10 \leftarrow \text{pivot} \\ x_2 & 1 & 2 & 0 & 0 & 1 & 0 & 60 & 60 \\ \hline & -10 & 0 & 0 & 0 & 20 & 1 & 1200 & \end{array}$$

↑ pivot column

$$\sim \begin{array}{c|cccccc|c} & x_1 & x_2 & S_1 & S_2 & S_3 & P & \\ \hline S_1 & 0 & 0 & -2 & 6 & 0 & 0 & -10 \\ x_1 & 1 & 0 & 0 & 2 & -1 & 0 & 10 \\ x_2 & 0 & 2 & 0 & -2 & 2 & 0 & 50 \\ \hline & 0 & 0 & 0 & 20 & 10 & 1 & 1300 \end{array}$$

$$\sim \begin{array}{c|cccccc|c} & x_1 & x_2 & S_1 & S_2 & S_3 & P & \\ \hline S_1 & 0 & 0 & 1 & -3 & 0 & 0 & 5 \\ x_1 & 1 & 0 & 0 & 2 & -1 & 0 & 10 \\ x_2 & 0 & 1 & 0 & -1 & 1 & 0 & 25 \\ \hline & 0 & 0 & 0 & 20 & 10 & 1 & 1300 \end{array}$$

$$\boxed{\begin{array}{c} x_1 = 10 \\ x_2 = 25 \\ P = 1300 \end{array}}$$

Practice Test #3 p.6

⑬ \$ in Gov't Bonds $= x_1$

\$ in Mutual Fund $= x_2$

\$ in Money Market $= x_3$

Constraints:

Total Money $x_1 + x_2 + x_3 \leq \$80{,}000$

Types $x_2 + x_3 \leq x_1 \Rightarrow -x_1 + x_2 + x_3 \leq 0$

Maximize: $P = 0.06x_1 + 0.11x_2 + 0.12x_3$

$$\begin{array}{c|cccccc|c} & x_1 & x_2 & x_3 & s_1 & s_2 & P & \\ s_1 & 1 & 1 & 1 & 1 & 0 & 0 & 80000 \\ s_2 & -1 & 1 & 1 & 0 & 1 & 0 & 0 \\ \hline & -0.05 & -0.11 & -0.12 & 0 & 0 & 1 & 0 \end{array}$$

$$\sim \begin{array}{c|cccccc|c} & x_1 & x_2 & x_3 & s_1 & s_2 & P & \\ s_1 & 2 & 0 & 0 & 1 & -1 & 0 & 80000 \\ x_3 & -1 & 01 & 1 & 0 & 1 & 0 & 0 \\ \hline & -0.17 & 0.01 & 0 & 0 & 0.12 & 1 & 0 \end{array}$$

$$\sim \begin{array}{c|cccccc|c} & x_1 & x_2 & x_3 & s_1 & s_2 & P & \\ x_1 & 1 & 0 & 0 & 0.5 & -0.5 & 0 & 40000 \\ x_3 & 0 & 1 & 1 & 0.5 & 0.5 & 0 & 40000 \\ \hline & 0 & 0.01 & 0 & 0.085 & 0.035 & 1 & 6800 \end{array}$$

$$\boxed{\begin{array}{l} x_1 = \$40{,}000 \\ x_2 = \$0 \\ x_3 = \$40{,}000 \\ P = \$6800 \end{array}}$$

⑭ Minimize $C = 2x_1 + 8x_2$

subject to $8x_1 + 2x_2 \geq 7$

$8x_1 + 5x_2 \geq 7$

$x_1, x_2 \geq 0$

$$A = \left[\begin{array}{cc|c} x_1 & x_2 & \\ 8 & 2 & 7 \\ 8 & 5 & 7 \\ \hline 2 & 8 & 1 \end{array}\right] \qquad A^T = \left[\begin{array}{cc|c} y_1 & y_2 & \\ 8 & 8 & 2 \\ 2 & 5 & 8 \\ \hline 7 & 7 & 1 \end{array}\right]$$

A) Dual Problem: $\boxed{P = 7y_1 + 7y_2 \text{ subject to } 8y_1 + 8y_2 \leq 2 \text{ \& } 2y_1 + 5y_2 \leq 8}$

B) Intial System: $\boxed{8y_1 + 8y_2 + x_1 \leq 2,\ 2y_1 + 5y_2 + x_2 \leq 8,\ -7y_1 + -7y_2 + P = 0}$

c) Simplex Tableau:

$$\begin{array}{c|ccccc|c} & y_1 & y_2 & x_1 & x_2 & P & \\ x_1 & 8 & 8 & 1 & 0 & 0 & 2 \\ x_2 & 2 & 5 & 0 & 1 & 0 & 8 \\ \hline & -7 & -7 & 0 & 0 & 1 & 0 \end{array}$$

Practice Test #3 p. 7

⑮ For Minimize $C = 15x_1 + 36x_2$ subject to $2x_1 + 5x_2 \geq 36$ & $3x_1 + 7x_2 \geq 51$ & $x_1, x_2 \geq 0$

Maximize $P = 36y_1 + 51y_2$ subject to $2y_1 + 3y_2 \leq 15$ & $5y_1 + 7y_2 \leq 36$ & $y_1 y_2 \geq 0$

w/ Tableau

$$\begin{array}{c|ccccc|c} & y_1 & y_2 & x_1 & x_2 & P & \\ \hline y_2 & 0 & 1 & 5 & -2 & 0 & 3 \\ y_1 & 1 & 0 & -7 & 3 & 0 & 3 \\ \hline & 0 & 0 & 3 & 6 & 1 & 261 \end{array}$$

A) Optimal Solution of Dual Problem: Maximum of $P = \boxed{261}$

$y_1 = \boxed{3}$

$y_2 = \boxed{3}$

B) Optimal Solution of Minimization Minimum $C = \boxed{261}$

$x_1 = \boxed{3}$

$x_2 = \boxed{16}$

⑯ Minimize $C = 19x_1 + 2x_2$ subject to $4x_1 + x_2 \geq 33$ & $3x_1 + x_2 \geq 8$, $x_1 x_2 \geq 0$

$$A = \begin{array}{cc|c} x_1 & x_2 & \\ 4 & 1 & 33 \\ 3 & 1 & 8 \\ \hline 19 & 2 & 1 \end{array} \qquad A^T = \begin{array}{cc|c} y_1 & y_2 & \\ 4 & 3 & 19 \\ 1 & 1 & 2 \\ \hline 33 & 8 & 1 \end{array}$$

A) Form Dual Problem

Maximize $P = 33y_1 + 8y_2$

subject to $4y_1 + 3y_2 \leq 19$

$y_1 + y_2 \leq 2$

$y_1, y_2 \geq 0$

B)

$$\begin{array}{c|ccccc|c} & y_1 & y_2 & x_1 & x_2 & P & \\ \hline x_1 & 4 & 3 & 1 & 0 & 0 & 19 \\ x_2 & 1 & 1 & 0 & 1 & 0 & 2 \\ \hline & -33 & -8 & 0 & 0 & 1 & 0 \end{array} \sim \begin{array}{c|ccccc|c} & y_1 & y_2 & x_1 & x_2 & P & \\ \hline x_1 & 0 & -1 & 1 & -4 & 0 & 11 \\ y_1 & 1 & 1 & 0 & 1 & 0 & 2 \\ \hline & 0 & 25 & 0 & 33 & 1 & 66 \end{array}$$

$\boxed{\text{Min } C = 66 \text{ at } x_1 = 0 \text{ \& } x_2 = 33}$