This chapter is an investigation into an area of statistics known as hypothesis testing. Hypothesis testing is an area of study in inferential statistics. Using hypothesis testing statisticians conduct statistical testing to support claims made about populations.

Let's begin with a n example before we discuss 2 important concepts in hypothesis testing.

Example: A manufacturer of a new medication is testing it against an old medication that has a success rate of 60%. The manufacturer will not accept the new medication <u>unless it has a success rate better than the old.</u> If the manufacturer is not very sure about his results the manufacturer will lose money and possibly eve be investigated for a false claim.

Discussion 1 – Hypotheses
Hypotheses
H_0 – The Null Hypothesis (This always contains the equal sign – Triola will use the equal sign only, but I will use the opposite inequality with the equal sign where appropriate.)
H_a or H_1 – The Alternative or Research Hypothesis (This is the claim that we wish to make or what we wish to disprove)

Three Types of Hypotheses
H_0: $\mu \leq$ H_A: $\mu >$ One Tail Test (RR in Right or Upper Tail)
H_0: $\mu \geq$ H_A: $\mu <$ One Tail Test (RR in Left or LowerTail)
H_0: $\mu =$ H_A: $\mu \neq$ Two Tail Test (RR in both tails)

For Our Example:
We wish to prove that the new is **better** than the old. Since the old had a 60% rate then the new must be more than 60%. This is what we will make our research hypothesis. Note that this is a proportion, so instead of μ we will use the population proportion, p.
H_0: p $\leq 60\%$
H_A: p $> 60\%$

Discussion 2 – Rejection Region

The rejection region (RR) also called a critical region is the region in which the alternative hypothesis is true, the region beyond the critical value. The critical value is determined with a z or t value based upon the hypothesis μ.

When the test statistic (the z or t computed based upon the sample data and the value of μ) is found to be within the rejection region we **reject the null hypothesis and accept the alternative**. This wording is **very** important!!! We do not prove our alternative we only accept that it is true. If the test statistic is not in the rejection region then it leads to a **failure to reject the null hypothesis**. Again this wording is **very** important!!!

Fail to reject H_0

Rejection Region
Probability equal to α
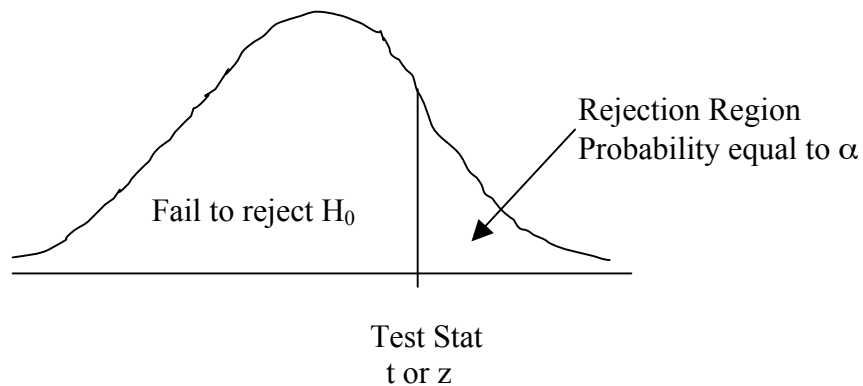
Test Stat
t or z

There are two criteria for rejecting. One, the traditional method is based upon comparison of the test statistic to the z or t value for α and the other is based upon the p-value, the probability of the computed test statistic to the desired α. The traditional method is more straight forward in terms of gauging whether to reject or fail to reject, but more advanced texts may base their rejection or failure to reject solely upon p-value.

Discussion 3 – Error Discussion

If we reject the null hypothesis when it is true then we are making a serious error!!

For our example, if we reject H_0 and accept H_A when H_0 was actually true, then the manufacturer could lose money and/or be investigated. Remember, that the manufacturer is wanting to replace a current med with a new one and will possibly take the old one off the market if it is shown that the new one will do a much better job! This is an error that we want to be able to control. This error is called a **Type I Error**. We control error by setting the probability of its occurrence. This is our α, called the **significance level**.

Our Example: $\alpha = 0.01$ would be an acceptable probability of making a Type I error if the outcome of making such an error can have serious repercussions.

$1 - \alpha$ gives the **level of confidence in rejecting** our null hypothesis.

There is another type of error that can be made as well, it is called a **Type II Error**. It is not as serious and can't be controlled, but it is dependent upon α and the population under H_A. The probability of making a Type II Error is symbolized by β (beta). A Type II Error is a failure to reject the null hypothesis when the null is actually false. The penalty for such an error in the real world is not as severe as for making a Type I Error.

     Our Example: Type II Error is failing to reject the claim that the population proportion is less than 60% (the new med isn't better than the old) when in actuality the new med is better. No harm is done except that the manufacturer may start development or trials over again to make a better medicine – this probably isn't a bad thing – it may cost a little more money for the consumer and manufacturer in the long run but the result will probably be a better med still!

The following table summarizes error and can also be found on p. 381 of Triola's ninth edition.

	H_0 is True	H_0 is False
Reject H_0	Type I Error α	Correct Decision
Fail to Reject H_0	Correct Decision	Type II Error β

     Example: A corporation maintains a large fleet of company cars for its sales people. To check the average number of miles driven per month per car, a random sample of 40 cars is examined. The mean x-bar = 2752 miles with std. dev., s = 350 miles. Records for the previous year indicate the average number of miles driven per car per month was 2600. For $\alpha = 0.05$ is there a difference between this year's average mileage and last? Answer the following:

a)   Express the claim that there is no difference in symbols

b)   ID both H_0 & H_A

c)   ID test as a 2 tail, left or right tailed test

d)   ID Type I Error (in words)

e)   ID Type II Error (in words)

f)   Assume you reject the null hypothesis. State the conclusion.

g)   Assume you fail to reject the null hypothesis. State the conclusion.

Example: The average live weight of farmer's steers prior to slaughter was 380 lbs. in past years. This year his 50 steers were fed a new diet. Suppose that they have a mean weight of x-bar = 390 lbs. and std dev., s = 35.2 lbs. he thinks that his steers weigh more this year than in the past. Is he correct in his thoughts with an α = 0.01? Answer the same questions as for the above example.

Example: the administrator of a nursing home would like to do a time-in-motion study of staff time spent per day performing non-emergency type chores. In particular, she would like to test to see if efficiency is up (spending less time on chores) from a prior study that showed that an average of 16 hours were spent on such chores. Answer the same question as for the prior examples.

Discussion 4 – Test Statistics
The test statistics used in this chapter are as follows can be found on p. 374 Triola:

Population Proportions	Population Means	Population Variance
$Z = \dfrac{(\text{p-hat} - p_0)}{(\sqrt{p_0 q_0 / n})}$	For n > 30 & σ known $Z = \dfrac{(\text{x-bar} - \mu_0)}{\sigma/\sqrt{n}}$ For n ≤ 30 & σ unknown $T = \dfrac{(\text{x-bar} - \mu_0)}{s/\sqrt{n}}$	$\chi^2 = \dfrac{(n - 1)s^2}{\sigma_0{}^2}$

Note: The p, μ or σ^2 is the value of the null hypothesis!! P-hat, x-bar and s^2 come from sample data used to substantiate your claim. There are 2 methods of looking at this as previously stated:

Traditional
Test Stat
  Vs.
Z_{critical}
From P(Z>Z_{critical})= α

P-Value
P(Z > Test Stat) = Prob
    vs.
α

§7.3 Testing a Claim About a Proportion

Some information that we need to know is that we are that the following conditions must be met:

1) Meets the conditions of a binomial distribution (recall ch. 4)
2) Sample must be large enough and p small enough such that $np > 5$ and $nq > 5$, so that the normal approximation to the binomial can be assumed

In this section our test statistic will be the z:

$$Z = \frac{\text{p-hat} - p_0}{\sqrt{p_0 \cdot q_0} / n}$$

Example: A 5 year-old census recorded 20% of families in large communities lived below poverty level. To determine if this percentage has changed, a random sample of 400 families is studied and 70 are found to be living below the poverty level. Does this finding indicate that the current percentage of families earning incomes below the poeverty level has changed from 5 years ago? Use $\alpha = 0.1$

Step 1:   State the hypotheses

Step 2:   Find the critical values & draw a diagram
$\alpha =$ 0.1 and we want a two tailed test so 0.05 is in each tail
The Z-value that we may have memorized by now from confidence intervals where there is 0.05 probability above it is 1.645

Step 3:   Find the test statistic (for a binomial see the chart above)

Step 4:   using the traditional approach compare the Test Stat & Critical Values. If test stat is in critical region then reject H_0 and accept H_A or if it is not in critical region then fail to reject H_0.

Example: A concerned group of citizens wishes to show that less than half of the voters support the President's handling of a recent crisis. If in a random sample of 500 voters 228 are in support of the President, at an $\alpha = 0.05$ is the claim of the citizens upheld?

Step 1:   State the hypotheses

Step 2:   Find the critical value & draw a picture

Step 3:   Find the test statistic

Step 4:   State the conclusion

For the last example let's focus on the P-Value method.

Example: A buyer from an outing center wants to know if he should carry more backpacks. He figured if more than ½ of the students at the local university use backpacks then he could sell more. A random sample of 78, reveals that 49 carry backpacks. At an $\alpha = 0.01$, do you believe he should carry more backpacks?

Step 1:   State the hypotheses

Step 2:   Calculate the test statistic & draw a picture to find $P(z > z_{test})$

Step 3:   Compare the $P(z > z_{test})$ and α
              If the value of $P(z > z_{test})$ is less than α then reject null

§7.4 Testing a Claim about a Mean: σ Known

Assumptions in this section are the same as those assumptions made for confidence intervals for populations where σ is known.

1)   The sample constitutes a simple random sample
2)   The value of σ is known (recall that this rarely happens)
3)   Either the population is known to be Normally distributed or n>30. If both are true great!

In this section our test statistic will be the z:

$$Z = \frac{\text{x-bar} - \mu_0}{\sigma / \sqrt{n}}$$

In this section Triola introduces a 3rd method for testing a hypothesis – the confidence interval method. This is in addition to the two other methods used in §7.3, the traditional method using critical values and the P-value method using the probability of being beyond the critical value.

Example: In §7.2 we had an example that involved company cars. Recall that x-bar = 2752 and σ = 350 & n = 40. At an α = 0.05 test the claim that last year the average was not the same as last year's average of 2600 .

Step1: State the hypotheses   **Step2:** Find the critical value & draw the picture

Step3: Find the test statistic & locate it on the diagram in 2.

Step 4: State the conclusion

Example: Recall the example involving the live weights of steers where x-bar= 390, σ = 35.2 and n = 50. Test the claim at α = 0.01 that this year the average weight is more than in past years where the average weight was 380 lbs.

Step1: State the hypotheses   **Step2:** Find the critical value &
                                                      draw the picture

Step3: Find the test statistic & locate it on the diagram in 2.

Step 4: State the conclusion

Example: Recall the administrator data where x-bar = 12, σ = 7.64 and n = 54. For an α = 0.05 test the claim that the efficiency of the employees has increased (spending less time on chores) than in the past when the average time on chores has been 16 hours.

Step1: State the hypotheses   **Step2:** Find the critical value &
                                                      draw the picture

Step3: Find the test statistic & locate it on the diagram in 2.

Step 4: State the conclusion

Now let's use the P-Value method to do the company car problem.

P-Value Method
1.   State hypotheses
2.   Draw a diagram and put in value(s) of α
3.   Find the test statistic & find it's probability using the z or t table
4.   If the P-value of the test statistic is less than α then reject the null

Example: In §7.2 we had an example that involved company cars. Recall that x-bar = 2752 and σ = 350 & n = 40. At an α = 0.05 test the claim that last year the average was not the same as last year's average of 2600 .

Step1: State the hypotheses   **Step2:** Draw the picture of
                                                                            α- value(s)

Step3: Find the test statistic & it's probability in z table

Step 4: State the conclusion

Now let's do the confidence interval (CI) method on the same data.

CI Method
1. State the hypotheses
2. Construct a confidence interval of $1 - \alpha$ for a two tailed test and $1 - 2\alpha$ for a one tailed test. CI is about the x-bar not about the H_0
3. If the CI contains the claim in the hypotheses then fail to reject the null

Example: In §7.2 we had an example that involved company cars. Recall that x-bar = 2752 and $\sigma = 350$ & n = 40. At an $\alpha = 0.05$ test the claim that last year the average was not the same as last year's average of 2600 .

Step1: State the hypotheses

Step2: Construct the confidence interval for a two tailed test

Step 3: State the conclusion (reject if μ_0 is not in the interval)

Example: Recall the administrator data where x-bar = 12,
σ = 7.64 and n = 54. For an α = 0.05 test the
claim that the efficiency of the employees has
increased (spending less time on chores) than in the past
when the average time on chores has been 16
hours.

Step1: State the hypotheses

Step2: Construct the confidence interval for a one-tailed test
(1 – 2α, 90% this time)

Step 3: State the conclusion (reject if μ_0 is not in the interval)

§7.5 Testing a Claim About a Mean: σ unknown

Assumptions in this section are the same as those assumptions made for confidence intervals for populations where σ is unknown.

1)   The sample constitutes a simple random sample
2)   The value of σ is unknown (recall that this is usually the case)
3)   Either the population is known to be normally distributed or n>30. If both are true great!

In this section our test statistic will be the t:

$$t = \frac{\text{x-bar} - \mu_0}{s / \sqrt{n}}$$

Example: Measurements of the amounts of suspended solids in river water on 15 Monday mornings yilds x-bar =47 and s = 9.4. The water quality is acceptable if the mean amount of suspended solids is less than 49. Construct an α = 0.05 test to establish that the quality is acceptable.

Step1: State the hypotheses

Step2: Find the critical value & draw the picture

Step3: Find the test statistic & locate it on the diagram in 2.

Step 4: State the conclusion

Example: In a lake pollution study, the concentration of lead in the upper sedimentary layer of a lake bottom is normally distributed. In 25 random samples x-bar = 0.38 and s = 0.06. A previous study showed that the mean amount of lead was 0.34. At an $\alpha = 0.01$, show that the last study was incorrect.

Step1: State the hypotheses   **Step2:** Find the critical value & draw the picture

Step3: Find the test statistic & locate it on the diagram in 2.

Step 4: State the conclusion

Example: An accounting firm wishes to set a standard time, μ, required for employees to complete a certain audit operation. The average times from 18 employees yields x-bar = 4.1 with s = 1.6. The firm believes that it will take longer than 3.5 hours to complete the audits. Test their hypotheses at the $\alpha = 0.05$ level.

Step1: State the hypotheses

Step2: Find the critical value & draw the picture

Step3: Find the test statistic & locate it on the diagram in 2.

Step 4: State the conclusion

§7.6 Testing a Claim About a Standard Deviation or Variance

The assumptions are the same made for building a confidence interval about the standard deviation.
  1)   Simple random sample
  2)   The population must have a normal distribution

The test statistic is the Chi-Square statistic that we are familiar with from CI's as well:
$$\chi^2 = \frac{(n - 1) s^2}{\sigma_0^2}$$
Because the Chi-Square distribution is not symmetric we will still need to pay attention to the right and left χ^2 values when finding the critical values.

Example: The claim is that peanut M&M's have weights that vary more than the weights of plain M&M's. The weights of plain M&M's has $\sigma = 0.04$g. A sample of 40 peanut M&M's has weights with s = 0.31 g. (#5 p. 424 Triola)

Step1: State the hypotheses   **Step2:** Find the critical value(s) & draw the picture

Step3: Find the test statistic & locate it on the diagram in 2.

Step 4: State the conclusion

Example: The Stewart Aviation Products Company uses a new production method to manufacture aircraft altimeters. A simple random sample of 81 altimeters is tested in a pressure chamber, and the errors in altitude are recorded as positive values or negative values. The sample has s = 52.3 ft. At α = 0.05 level, test the claim that the new production line has errors with a standard deviation different from the old line which had σ = 43.7 ft.

Step1: State the hypotheses

Step2: Find the critical value(s) & draw the picture

Step3: Find the test statistic & locate it on the diagram in 2.

Step 4: State the conclusion