Pre Chapter 9 Coverage

Quadratic (2^{nd} Degree)

Form a type of graph called a parabola

Form of equation we'll be dealing with in this chapter: $y = ax^2 + c$

Sign of '**a**' determines opens up or down

"+" opens up

"−" opens down

$0 < | \mathbf{a} | < 1$ then wide with not very "steep" sides

$| \mathbf{a} | > 1$ is narrow with "steep" sides

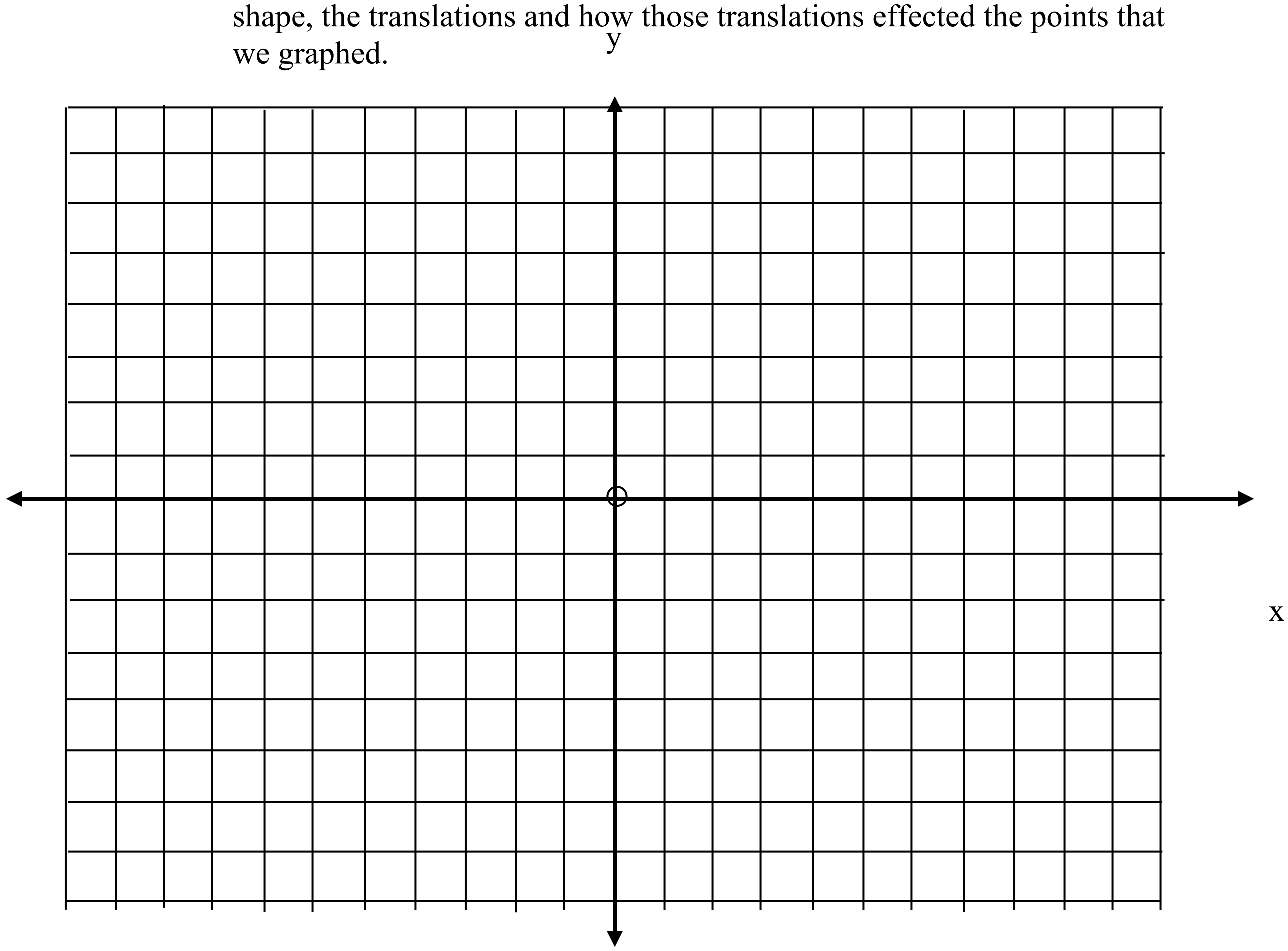
The **vertex** (where the graph changes direction) is at (0, c)

Symmetric around a vertical line called a **line of symmetry**

Goes through the vertex in a line x = 0 for this basic quadratic

Note: The y-intercept is the vertex when there is no **bx term*

Example: Graph $y = x^2$, $y = -x^2$ and $y = -x^2 + 2$ on this graph, by making a t-table of points and while thinking about the vertex, the line of symmetry and 4 points. Use the same points for each graph. After we will discuss the shape, the translations and how those translations effected the points that we graphed.



§9.1 & 9.2 Graphing Quadratic Functions in Vertex Form & Standard Form

Quadratic (2nd Degree) Functions

Form a type of *graph* called a *parabola*

Form of equations we'll be dealing with are: $y = ax^2 + bx + c$ & $\mathbf{y = a(x - h)^2 + k}$

Sign of '**a**' determines opens up or down

"+" opens up

"–" opens down

The **vertex** (where the graph changes direction) is at:

$(-^{b}/_{2a}, f(-^{b}/_{2a}))$ in $y = ax^2 + bx + c$

(h, k) in $\mathbf{y = a(x - h)^2 + k}$

Symmetric around a vertical line called a **line of symmetry**

Goes through the vertex, and has the equation x = "x-coordinate of vertex"

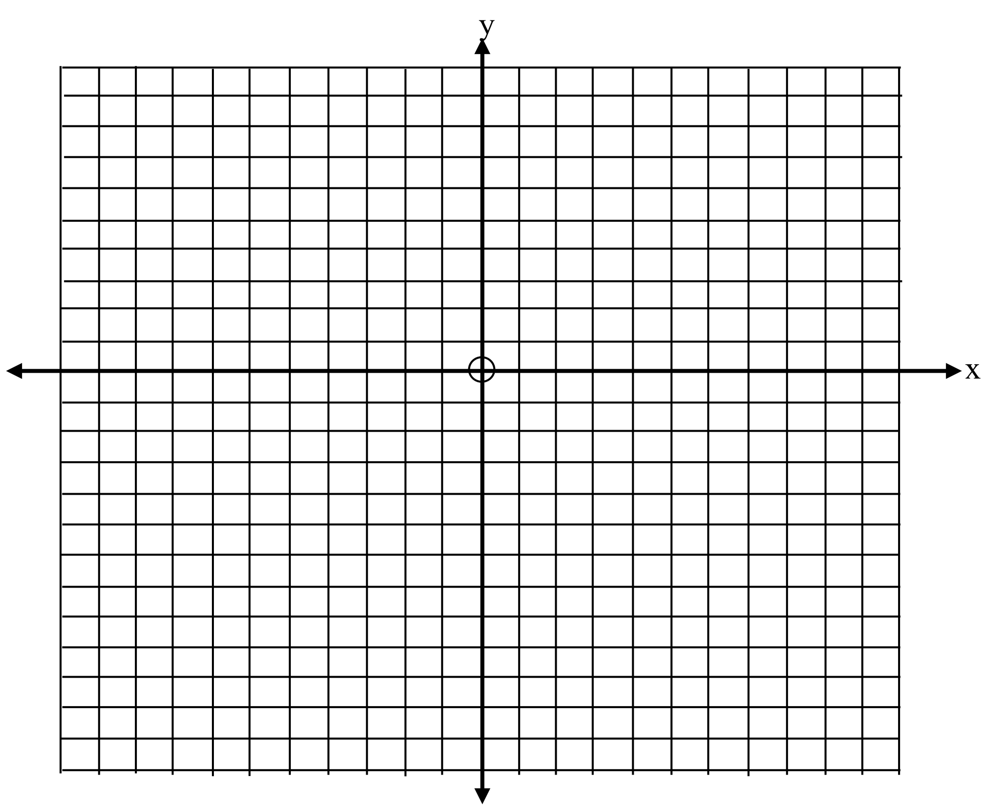
Two points on every parabola are symmetric to the line of symmetry

Example: Graph the following quadratic function from it's vertex form by graphing it's vertex, y-intercept, x-intercept(s) and symmetric points.

$$y = -(x - 1)^2 + 3$$

Recall: For y-intercept; let x = 0 For x-intercept; let y = 0

Symmetric Points are the same distance from line of symmetry in opposite direction



<u>Translations of Functions</u> move a function around in space. Remember the idea of a family of functions? This is what translations do – they create families. We can translate a function in several ways: horizontally (subtracting a constant from the independent), stretching (multiplication of the function by a positive constant), reflecting (multiplying the function by a negative), , vertically (adding a constant to the function). Here is a summary of these translations in function notation.

Stretching:	If y = f(x), then the reflected function is where '**a**' is some positive constant	y = **a**f(x)
Reflection:	If y = f(x), then the reflected function is	y = -f(x)
Horizontal:	If y = f(x), then the translated function is where '**h**' is some constant by which the function is shifted. If '**h**' is positive it is right and if '**h**' is negative it is left. *Note* that it is read from "x – h"	y = f(x – **h**),
Vertical:	If y = f(x), then the reflected function is where '**k**' is some constant by which the function is shifted up or down. If '**k**' is positive it is shifted up and if '**k**' is negative it is shifted down.	y = f(x) + **k**

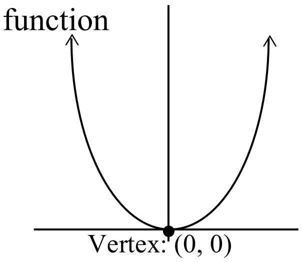
Note: *I have listed the translations in the order that we generally think of moving a function about the coordinate system. We start with the based shape at the zero position and then we consider the movement of that function translation by translation. Note that the horizontal can as easily come first as 3rd, this is generally just the way that advanced texts translate funcitons.*

Example: We will take the following function through a series of translations One of the things that will help a great deal in learning to graph the trig functions is an understanding of translation. I'm going to go over the translation of a quadratic function to assist you in learning how to graph trig functions.

1) Every function has a general form of the equation and a graph centered at the origin.
2) It is from this general form that translations happen. Think of a translation as moving the shape formed from the general equation around in space. We can move the shape up, down, left, right, flip it over or stretch/shrink it. It really gets fun when we do multiple movements!
    a) Stretching/Shrinking → Multiplies the function value (the y-value) by a constant
    b) Reflection (Flipping it over) → Multiplies the function by a negative
    c) Vertical Translation (Moving it up/down) → Adds a constant to the function value (the y-value)
    d) Horizontal Translation (Moving it left/right) → Adds a constant value to the x-value while still outputting the same y-value

Let's go through this with the quadratic function: $y = x^2$

1) First you must know the basics about the general function
   a) This function forms a parabola with its vertex at (0, 0)



   b) The general parabola (being an even function) is symmetric about the y-axis; the line to which the parabola is symmetric is called the line of symmetry
   c) The vertical line through the vertex is called the line of symmetry
      i) Relating to being an even function
      ii) For every f(-x) there is an equivalent f(x)

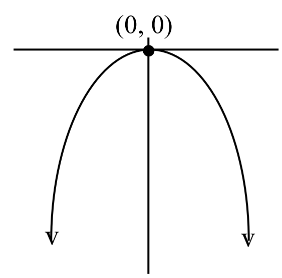
Now, let's look at a typical table of values that we use to graph this function. This will assist in seeing the translations of this function.

x	**$y = x^2$**
0	0
1	1
-1	1
2	4
-2	4
3	9
-3	9

Let's take our first translation to be the **reflection**. This simply multiplies the y-value by a negative.

x	**$y = x^2$**	↓ reflection **$y = -x^2$**
0	0	0
1	1	-1
-1	1	-1
2	4	-4
-2	4	-4
3	9	-9
-3	9	-9

Visual of Reflection

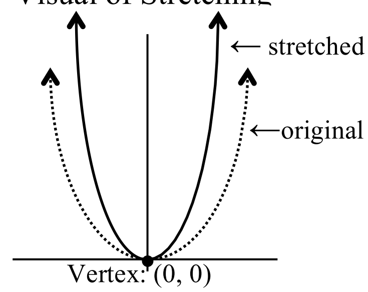


Next, take a **stretching/shrinking** translation. This multiplies the y-value by a negative. Visually it is like "pulling the parabola up by its ends or pushing it down." When the constant is > 1 the parabola is stretched and when it is < 1 but > 0 it is shrunk.

↓ strecthing

x	$y = x^2$	$y = 3x^2$
0	0	0
1	1	3
-1	1	3
2	4	12
-2	4	12
3	9	27
-3	9	27

Visual of Stretching

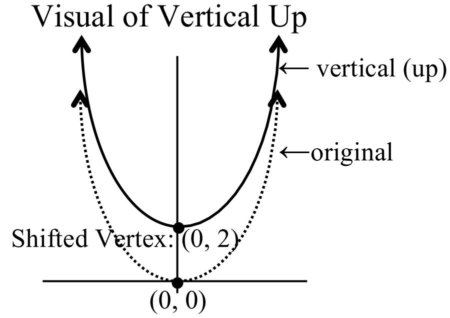


Next, take the **vertical** translation. This adds to the y-value. Visually it moves the parabola up and down the y-axis. When a constant is added to the function, the translation is up and when the constant is subtracted from the translation is down.

↓ vertical

x	$y = x^2$	$y = x^2 + 2$
0	0	2
1	1	3
-1	1	3
2	4	6
-2	4	6
3	9	11
-3	9	11

Visual of Vertical Up



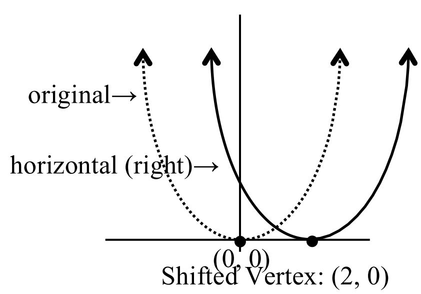
Last, the most difficult translation to deal with in terms of ordered pairs, because it changes the **x** not the y-coordinate. This is the **horizontal** translation. Visually it moves the parabola to the left or right. When a *constant is subtracted* from the x-value, the translation is *right* and when it is added it is to the left (this is the opposite of what you think, and it is due to the form that the equations take).

↓ horizontal

x	x′	$y = (x - 2)^2$
0	2	0
1	3	1
-1	1	1
2	4	4
-2	0	4
3	5	9
-3	-1	9

↑ there's notice no change from $y = x^2$

Visual of Horizontal Right



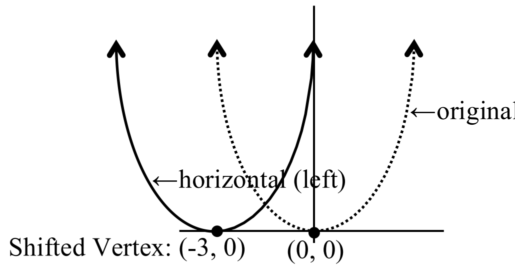
Because this one is the mind bender, I will also include the left shift.

↓ horizontal

x	**x′**	**y = (x + 3)^2**
0	-3	0
1	-2	1
-1	-4	1
2	-1	4
-2	-5	4
3	0	9
-3	-6	9

↑ there's notice no change from y = x^2

Visual of Horizontal Left



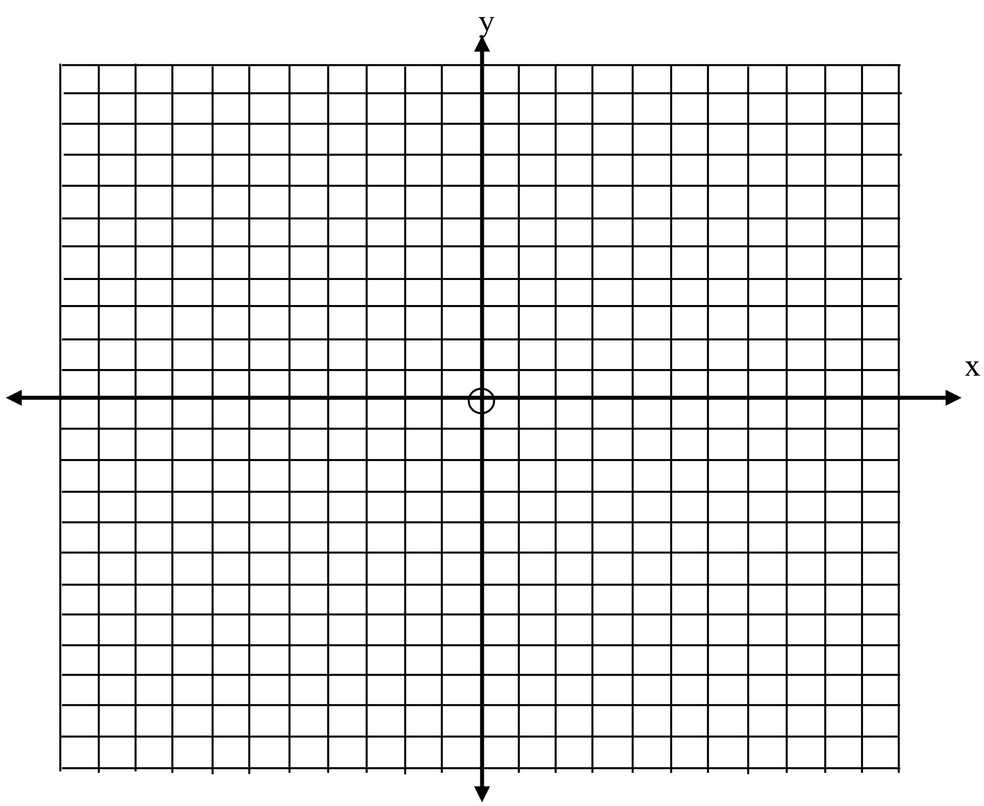
Note: *When we do translations, there is a specific order that we think of them in:, 1) Stretching, 2) Reflection, 3) Horizontal, 4) Vertical. The Horizontal does nothing to f(x), it simply changes the x value by adding the constant "h" (must be in form x – h). The Stretching multiplies the f(x) value by the constant. The Reflection takes the opposite of the stretched f(x) value. The Vertical adds the constant "k" to the stretched, reflected f(x) value. *Since the horizontal can come first because it doesn't actually effect the y-values I've chosen to write it as the first translation it my next example.*

Example: Graph $m(x) = 2(x - 1)^2 + 3$ by using a t-table as follows

x	**$f(x) = x^2$**	**$g(x) = (x - 1)^2$**	**$h(x) = 2(x - 1)^2$**	**m(x)**
-2	4	x	x	x
-1	1	4	8	11
0	0	1	2	5
1	1	0	0	3
2	4	1	2	5
3	9	4	8	11

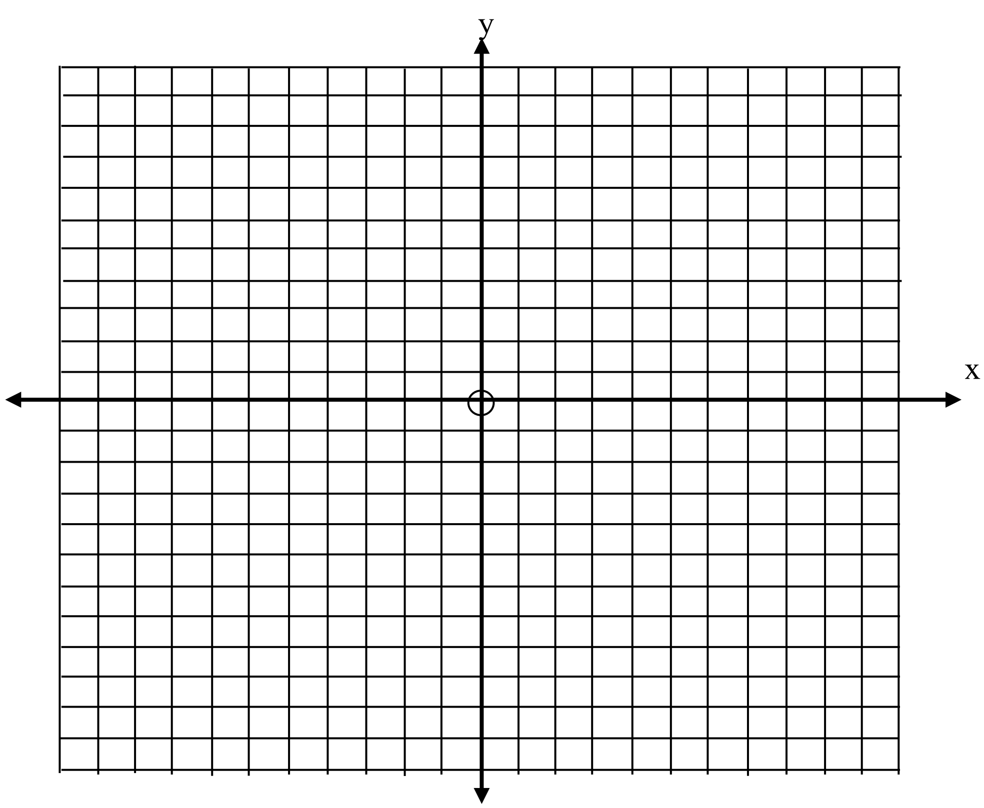
Many times I will describe the translation for g(x) as an **x′ translation** as follows

x	x′	$f(x) = x^2$	$g(x) = (x - 1)^2 \rightarrow f(x')$
-2	-1	4	4
-1	0	1	1
0	1	0	0
1	2	1	1
2	3	4	4



Example: Graph the following quadratic by "thinking through" the translations.

$$y = 2(x + 2)^2 - 1$$



Applications of Quadratic Functions
Minimum & Maximum Values

- Up Facing Minimum at vertex
  - Vertex y-coordinate tells f(x) value at lowest
  - Vertex x-coordinate tells independent value needed to attain lowest
- Down Facing Maximum at vertex
  - Vertex y-coordinate tells f(x) value at highest
  - Vertex x-coordinate tells independent value needed to attain highest

*The following have been modified from: http://www.purplemath.com/modules/quadprob3.htm

***Example:** I wish to fence a rectangular shaped paddock using 500 linear feet of fencing. What are the dimensions (length and width) of the largest (in area) such paddock I can create? What is the area of the paddock?

Step 1: Use the perimeter to write a formula to relate the length and width of the paddock to the number of linear feet

Step 2: Solve the perimeter for the width

Step 3: Write the area formula in function notation using the length as the independent & substitute the width's expression from Step 2.

Step 4: Expand the equation in Step 3 & find the vertex

Step 5: The value of the independent in the vertex is the length that maximizes area. Solve for the width using Step 2's expression.

Step 6: Find the area of the paddock by multiplying length times width.

***Example:** A factory produces little plastic pieces to be sold to other manufacturers. The more pieces produced the cheaper it becomes to produce the pieces. However, the cost will eventually go up as supply outweighs demand. According to the head accountant the cost of producing x-thousand plastic pieces can be modeled using:

$$C(x) = 0.04x^2 - 8.504x + 25302$$

Find the number of plastic pieces in thousands that will minimize the cost of production. How do you know that solving this problem using the vertex will result in a minimum?

Projectile Motion Problems

The x-intercept(s) of a parabola can be used to model when objects that are thrown or launched, return to the ground. I call these **parabolic motion problems**. The *x-intercepts represent the time it takes for the object to reach 'the ground'*. There are always 2, but one is negative and therefore is considered as an extraneous solution. The y-value being equal to zero represents the object's height, which is zero – the ground.

BTW if the x is zero you are getting the height from which the object is thrown, and that is what the constant in the quadratic represents – the height from which the object is thrown or launched. The numeric coefficient of 'x' in these problems represents the speed at which the object is thrown or launched. The numeric coefficient of the x^2 represents the pull of gravity and is therefore always the same number when dealing with feet (-16; in meters, so you'll see -1.9).

Example: A rocket is launched straight up with an initial velocity of 100 feet per second. The height of the rocket at any given time t, h(t), can be described by the following equation. (*Beginning Algebra*, Elayn Martin-Gay, 5th edition p. 409)

$$y = -16x^2 + 100x$$

a) Find the time for the rocket to return to the ground.

b) At what height was the rocket launched?

Example: An object is thrown up-ward off a building that is 80 feet tall. The following quadratic describes the height at time t:

$$h(t) = -16t^2 + 64t + 80.$$

How long before the object hits the ground? How tall is the building?

Modeling using quadratic equations:

By Hand
1) Plot the points on a scattergram
2) Find the one that looks to be the maximum or minimum & note as (h,k)
3) Sketch an approximate parabola using (h, k) as the vertex
4) Choose a 2^{nd} point that is on your sketch
5) Use the vertex form, (h, k) & the 2^{nd} point to solve for "a"
6) Put "a" & (h, k) into the vertex form

Using Calculator
1) Put (x, y)'s into the Stat Editor
2) Use Stat➔Calc➔QuadReg and enter the x & y lists
3) Use the values of the variables a, b & cto write the quadratic in std. form

Most real-life data will not fit exactly on the graph of a particular function. In this case, the points in our scatterplot do not lie perfectly on the parabola. But, we like to find a "function of best fit" to make predictions. To help determine if the "function is a best fit" go to CATALOG, DIAGNOSTIC ON and hit enter twice. This will make sure we get the r^2 value. This value is called a coefficient of determination and indicates the percentage of variability in the dependent explained by the regression function. *The closer r^2 gets to 100% the better the model*. You can find out more about r and r^2 in a statistics class.

You may want to graph both your ordered pairs and your regression equation. To begin this process go to Y=, VARS, Statistics, EQ, select RegEQ, 2^{nd} Y=, enter, ON & enter, choose the first picture & enter, give list containing the independents & enter, give list containing the dependents & enter, ZOOM, down arrow to ZoomStat and enter. This will plot your ordered pairs & the regression function over the top.

Example: Let's do problem #34 from p. 485 together. We will use our both by hand and calculator to create the model.

HW & CW for §9.1 & 9.2
§9.1 p. 484 #2, 5, 8, 11, 18, 19, 22, 25, 30, 31, 33, 36, 43, 47, 51, 54, 59 & 60
§9.2 p. 496 #2, 5, 9, 12, 19, 34, 41, 42, 47, 48, 50, 57, #61-68all, #71