§5.2 Functions

Objectives

- Know the meanings of relation, domain, range and function
    - Finding for sets of ordered pairs & graphs
- Identifying functions using a vertical line test
- Definition of a linear function
- Rule of Fours

A **relation** is any set of ordered pairs. A **function** is a relation for which every value of the **independent variable** (the values that can be inputted; the t's; used to call the x's) has one and only one value for the **dependent variable** (the values that are output, dependent upon those input; the Q's; used to call the y's). All the possible values of the independent variable form the **domain** and the values given by the dependent variable form the **range**. *Think of a function as a machine and once a value is input it becomes something else, thus you can never input the same thing twice and have it come out differently*. This does not mean that you can't input different things and have them come out the same, however! That is another discussion for a later time (that's called a function being one-to-one).

4 Ways to Represent Functions (Rule of 4's):

1) Description in words

Example: The average population of a city from the turn of the 20th century to present day.

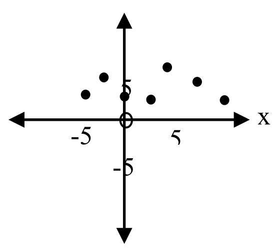
2) Tables

Example:

x	y
2	5
1	2
-3	-10

3) Graphs

Example:



4) Formulas

Example: $y = 2x + 1$

Is a the set of ordered pairs a *function*?

1) For every value in the domain is there only one value in the range?
    a) Looking at ordered pairs – If no x's repeat then it's a function (a map can be used to see this too. Domain on left & range on right, If any domain value has lines to more than one range value, then not a function.)
    b) Looking at a graph – **Vertical line test** (if any vertical line intersects the graph in more than one place the relation is not a function)

c) Mathematical Model needs to consider the domain & range values or draw a picture– If input of any x will give different y's, then not a function (probably a graph is still best!)

d) From a description – Try to model using a set of ordered pairs, a graph or a model to decide if it is a function.

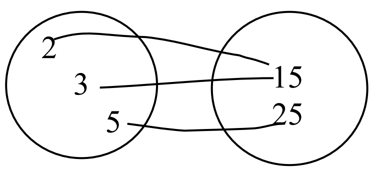
There are many ways to show a function. We can describe the function in words, draw a graph, list the domain and range values using set notation such as roster form or we can make a table of values, or we can use a mathematical model (an equation) to describe the function.

x	1	2	3
f(x)	2.3	2.8	3.2

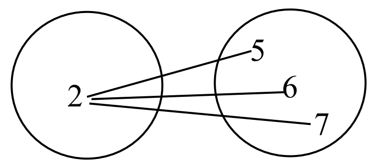
*(#11 p. 5 *Applied Calculus*, Hughes-Hallett et al, 4th Edition, Wiley, 2010)

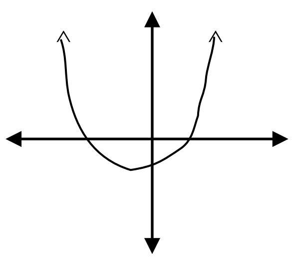
These are tables.
Left is f(n)
Rt. Is not an f(n)

x	1	1	2
f(x)	2.3	2.8	3.2

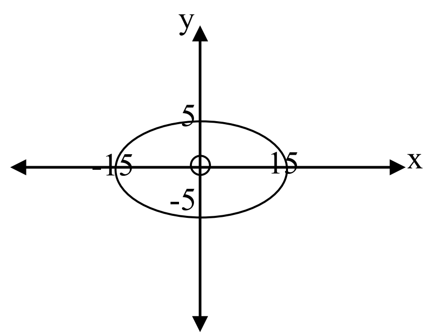


These are maps.
Left is f(n)
Rt. Is not an f(n)





These are graphs.
Left is f(n)
Rt. Is not an f(n)



$y = 10x - x^2$

(#8 p. 5 *Applied Calculus*, Hughes-Hallett et al, 4th Edition, Wiley, 2010)

These are models.
Left is f(n)
Rt. Is not an f(n)

$y^2 + x^2 = 9$

Although I won't give you an **example** that isn't a function, here an example in words that is a function. We'll investigate it by sketching a graph.

IS: A patient experiencing rapid heart rate is administered a drug which causes the patient's heart rate to plunge dramatically and as the drug wears off, the patient's heart rate begins to slowly rise.

§5.3 Function Notation

Objectives

- Function notation
- Using function notation
  - To Evaluate
  - Describing a linear model
- Finding inputs & outputs of a model

Function notation is just a way of describing the dependent variable as a function of the independent. It is written using any letter, usually f or g and in parentheses the independent variable. This notation replaces the dependent variable – y.

f**(x)** **Read as f of x** or f with x

The notation means evaluate the equation at the value given within the parentheses. It is exactly like saying "y="!

Here are some pointers:

- f(3) = 5 is the same as writing (3, 5) it means that x = 3 and y = 5
- Finding f(3) is saying Let x = 3 and evaluate the expression which is the right side of the function f(x) = x + 2

Example 1: Evaluate f(x) = 2x + 5 at

a) f(2)

b) $f(-{}^{1}/_{2})$

Example 2: For $h(g) = {}^{1}/_{g}$ find

a) $h({}^{1}/_{2})$

b) h(0)

***Note:** *The domain of this function does not contain zero, because zero makes the function undefined. We find the domain of functions based on values that make the function undefinded.*

When using function notation in everyday life the letter that represents the function should relate to the dependent variable's value, just as the independent variable should relate to its value.

Example 3: The perimeter of a rectangle is P = 2l + 2w. If it is known that the length must be 10 feet, then the perimeter is a function of width.

a) Write this function using function notation

b) Find the perimeter given the width is 2 ft. Write this using function notation.

<u>Finding Output Values</u>

Step 1: Understand that the x value is within the parentheses & f(x) is asking for the y value – the dependent value.

Step 2:
- Input the x value to a model and solve for dependent – the f(x)
- Find the x-value in the table and look at it's corresponding dependent value.
- Find the x-value on a graph follow the vertical line to the graph and then follow across to the y-axis to find the dependent associated with the input – independent

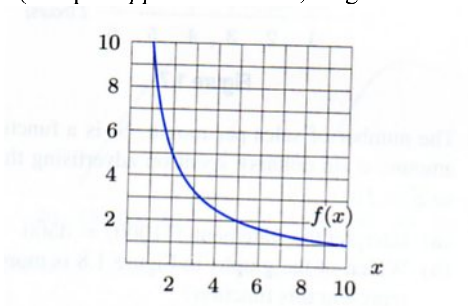
Example 4: Find the value of f(5)

a) $f(x) = 2x + 3$

b)

x	1	3	5	7
f(x)	9	12	15	18

c) *(#10 p.5 *Applied Calculus*, Hughes-Hallett et al, 4th Edition, Wiley, 2010)



<u>Finding Input Values</u>

Step 1: Understand that the f(x) value is the output & f(x) is asking for the x-value that gives the dependent value, f(x).

Step 2:
- Input the f(x) value to a model and solve for the independent – the x-value
- Find the f(x)-value in the table and look at it's corresponding independent value.
- Find the f(x)-value on a graph follow the horizontal line to the graph and then follow up/down to the x-axis to find the independent associated with the output

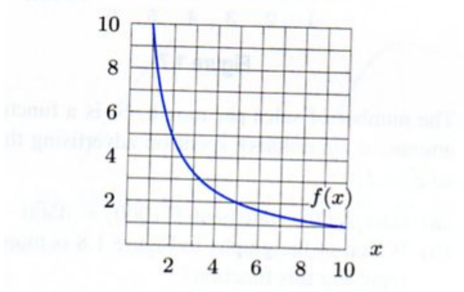
Example 5: Find the value of f(x) = 9

a) f(x) = 2x + 3

b)

x	1	3	5	7
f(x)	9	12	15	18

c) *(#10 p.5 *Applied Calculus*, Hughes-Hallett et al, 4th Edition, Wiley, 2010)



Function notation can also be used to solve equations.

We can use function notation to represent slope-intercept form of a line and to find our x and y-intercepts.

Slope-Intercept Form w/ Function Notation

f(x) = mx + b

m = slope
b = y-intercept
x is independent variable (represents the input; domain values)
f(x) is the dependent variable (represents the output; range values)

Finding Y-Intercept w/ Function Notation
Find f(0) is finding the y-intercept

Finding the X-Intercept w/ Function Notation
Find f(x) = 0 is finding the x-intercept

Example 6: For f(x) = 2x + 5

a) Show how to find the x & y-intercept using function notation

b) Find each and given them as ordered pairs.

Class Exercises & Homework
§ 5.2 p. 249 # 2, 4, 6, 8, 10, 14, 16, 20, 22, 34, 40 & 51
§ 5.3 p. 256 # 2, 4, 6, 16, 20, 21, 43, 44, 46, 47, 53, 60, 65, 66, 74, 76, 78 & 80