§5.7 Solving Linear Inequalities

Objectives
- Inequality Symbols
- Graphing Inequalities – both simple & compound
- Understand a solution set for an inequality
- Solving & Graphing a Simple Linear Inequality
- Solving & Graphing a Compound Linear Inequality
- Interval Notation (Not in Lehmann's text)
- Applications

Review of Inequality Symbols

$<$ This is a less than symbol. It is a strict inequality (meaning the endpoint isn't included)

$>$ This is a greater than symbol. It is also a strict inequality.

$\geq$ This is a greater than or equal to symbol. The **endpoint** is included. An endpoint is the numeric value that begins or ends an simple linear inequality.

$\leq$ This is a less than or equal to symbol. The endpoint is included.

The "OLD way" of graphing simple inequalities (contain only one inequality)

$<$ -- Graph with an **open circle** and a line to the left, if in **standard form**

$>$ -- Graph with an **open circle** and a line to the right, if in **standard form**

$\geq$ -- Graph with a **closed/solid circle** and line to the right, if in **standard form**

$\leq$ -- Graph with a **closed/solid circle** and a line to the left, if in **standard form**

Summary of the "NEW way" – Lehmann doesn't use this method but I will

$<$ --) and a line to the left

$>$ -- (and a line to the right

$\geq$ -- [and a line to the right

$\leq$ --] and a line to the left

Let's practice **standard form** 1^{st}.
1) Write the variable of the left
2) Write the number on the right.
3) Rewrite the inequality so that the arrow points to the "same thing" it did originally.

Example 37: Put the following in standard form

a) $5 \leq x$ b) $-2 \geq y$ c) $5002 > m$

Now some graphing.

To graph a **simple linear inequality in standard form**

1) Locate and notate (use open/closed circle or parenthesis/bracket) the endpoint on the number line—remember that the endpoint is the number
2) Draw a line in the direction indicated by the inequality (only if in standard form though!)

Example 38: Graph each of the following (on a number line)

a) $x > 3$ b) $2 > y$

c) $-2 \le v$ d) $t \le 5$

Parts b and c in the example are examples of inequalities written in nonstandard form. To read an inequality written in nonstandard form, we must read right to left, instead of our usual left to right. Thus it is easiest to always put inequalities in standard form before reading and trying to graph them.

Graphing of <u>compound inequalities</u> (contains 2 inequalities):

A compound inequality is read as an "and"

Example 39: $15 < x < 20$ is read as:

From the middle toward the left *x is greater than 15*

and

From the middle toward the right *x is less than 20*

Usually when you are **first taught to graph these compound inequalities** you are shown to:

1) Graph each *part* (middle left and middle right) above the number line
2) Graph on the number line the parts that overlap

When a compound linear inequality is in **standard form**

1) The smallest number is on the left
2) The largest number is on the right
3) There are less than or less than or equal to symbols used

Example 40: Decide if the compound inequality is in standard form. If it is not put it in standard form.

a) $7 \ge x > -9$ b) $-2 < x \le 5$

When a **compound linear inequality is in standard form** they are very easy to graph because all that must be done is:

1) Locate and notate (use open/closed circle or parenthesis/bracket) the left endpoint on the number line
2) Locate and notate (use open/closed circle or parenthesis/bracket) the right endpoint on the number line
3) Draw a line between the left and right endpoints

Example 41: Graph the following compound inequalities

a) $8 \leq x < 9$

b) $-2 < x \leq 5$

c) $3 \geq x > -4$

d) $5 \leq x \leq -2$

The following notation is not included in Lehmann's text at this time, but you will be responsible for this notation as it is an extension of the method of graphing that I just taught you.

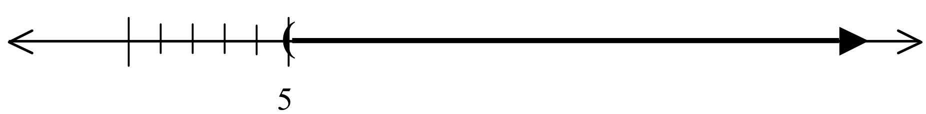
Up to this point you have been taught roster form and set builder notation. We will learn the last method here. It is called **interval notation**. It is essentially the shortcut of writing what you just saw on the number line. It tells about the inclusion or exclusion of left endpoint (**Endpoints** are the beginning or end of the solution set) and the right endpoint and slams them together with a comma in the middle. When graphing simple inequalities, we can get a solution set that travels out to negative or positive infinity $(-\infty, \infty)$. Infinity is an elusive point since you can never reach it, and therefore in interval notation we always use a parenthesis around infinity.

Summary

Endpoint included	[or]
Endpoint not included	(or)
Negative Infinity	$(-\infty$
Positive Infinity	$\infty)$

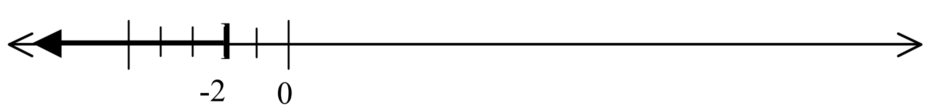
Visually Relating to the Number Graph of an Inequality

$x > 5$ Will be graphed with an parenthesis (open circle) on 5 and a line to the right with an arrow on the end showing that it continues on to infinity.



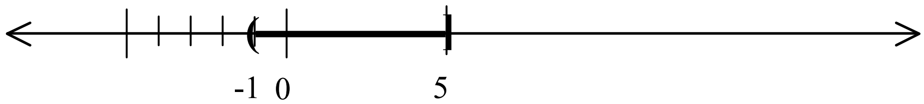
Interval Notation $(5, \infty)$

$-2 \geq x$ Will be graphed with an bracket (closed circle) on 5 and a line to the right with an arrow on the end showing that it continues on to infinity.



Interval Notation $(-\infty, -2]$

$-1 < x \le 5$ Will be graphed with an parenthesis (open circle) on -1 and a bracket (closed circle) on 5 and a line in between.



Interval Notation (-1, 5]

Now, go back to the examples that we graphed and write their interval notation. You will find it a lot easier at first if you have the graph right in front of you.

<u>Solving Linear Inequalities</u>

If all we must do to solve a <u>linear inequality</u> is to add or subtract then they are solved in the exact same manner as a linear equation. Recall the steps to solving a linear equation.

1) Simplify the left and right sides
    a) Distribute
    b) Clear if necessary
    c) Combine like terms
2) Addition Property to move variables to one side
3) Addition Property to move constants to opposite side as variables
4) Multiplication Property to remove the numeric coefficient
    *It is this step that can cause a difficulty if we mult/divide by a negative then the **<u>sense of the inequality must reverse</u>**

Example 41: Solve, graph and give the interval notation for following the linear inequalities

a) $x + 2 > 7$

b) $2x + 7 < 9 + x$

c) $2x - 15 + x \le 15 + 2(x - 1)$

If we must multiply or divide by a positive number to get the solution, then the steps for finding the solution are still the same!

Example 42: Solve, graph and give the interval notation for following the linear inequality

$5x - 15 + 2 \leq 15 + 3x$

The problem occurs when I must multiply or divide by <u>a negative number</u>! Whenever I multiply or divide by a negative number, the <u>sense of the inequality</u> must reverse.

Example 43: Solve, graph and give the interval notation for following the linear inequalities

a) $\frac{3}{4}x - 1 \geq 2x + \frac{1}{4}$ b) $-2x + 5 > 9$

To see why it is true that the sense of the inequality must reverse, let's rework these problems and keep the numeric coefficient of the variable positive. Notice that you must read the inequality in nonstandard form (right to left) and the answer is the same as above.

Example 44: Solve, graph and give the interval notation for following the linear inequalities

a) $\frac{3}{4}x - 1 \geq 2x + \frac{1}{4}$ b) $-2x + 5 > 9$

<u>Solving Compound Linear Inequalities (Lehmann calls these 3 part inequalities)</u>

Step 1: See the problem as having 3 parts – a left, a middle and a right.

Step 2: Simplify the three parts if necessary

Step 3: Remove all constants from the middle by adding their opposite to all three part, the left and right sides of the inequality and the middle

Step 4: Remove the numeric coefficient of the variable term (middle) by multiplying each part by the reciprocal (don't forget to reverse the sense of both inequalities if your numeric coefficient is negative!)

Step 5: Check your answer

Example 45: Solve, graph and give the interval notation for following the linear inequalities

a) $2 \le x - 1 < 5$ b) $2 \le {}^{3}/_{4}x - 1 \le 2\frac{1}{4}$

Don't forget to switch the sense of the inequality if you must multiply or divide by a negative number!

Example 47: Solve, graph and give the interval notation for following the linear inequality

$$6 > -x + 3 \ge 4$$

<u>**Review of Clearing Equations of Fractions**</u>

This is a process that uses the multiplication property of equality to multiply every term by a constant. The constant that we wish to use is the LCD of the fraction/decimal and when the fraction/decimal is multiplied by the LCD, the denominator cancels and by multiplying out (using the associative property) there will no longer be any fractions/decimals in the equation.

<u>**For a Fraction**</u>

1) Make sure all distributive properties are taken care of so there are only individual terms

2) Find LCD of all terms

3) Multiply each term by LCD (symbolically only)

a) Even whole numbers get multiplied

4) Cancel where necessary

5) Multiply out

Example 25: Clear the equation of fractions. Do not solve.

a) ${}^{3}/_{2}\,x - 2 > {}^{3}/_{4}\,x + 1$ b) ${}^{1}/_{3}\,(y - 5) < \frac{1}{4}$

c) ${}^{1}/_{3}(a + {}^{7}/_{3}) - 2a \le 1$ d) ${}^{4}/_{5}\,a - 3a \ge {}^{1}/_{5}(a + {}^{2}/_{5})$

For a Decimal

1) Make sure all distributive properties are taken care of so there are only individual terms

2) Count the largest number of decimal places in any term
(this is really finding LCD for factors of 10)

3) Multiply each term a factor of 10 with the number of zeros found in step 2) (symbolically only)

a) Even whole numbers get multiplied

4) Move the decimal to the right the same number of places as number of zeros in 3)'s factor of 10

Example 26: Clear the equation of decimals. Do not solve.

a) $0.5x + 0.25 < 1.2$ b) $0.25(x - 0.1) \geq 0.5x + 0.75$

Recall that:

Mathematical "OR" means union (all members of sets together)
Mathematical "AND" means intersection (overlap of members)

Example: Graph the following and then give the solution set in interval notation

a) $c \leq 2$ or $c > -3$

b) $d > 0$ and $d \leq 5$

c) $2k + 5 > -1$ and $7 - 3k \leq 7$

d) $2a + 3 \leq 7$ and $-3a + 4 \leq -17$