§9.7 Solving Systems of Linear Eq. in 3 Var & Finding Quadratic F(n)

Linear Equation in Three Variables
For A, B, C & D $\in \mathbb{R}$ where $A = B = C = D \neq 0$

$$Ax + By + Cz = D$$

will have solutions of the form (x, y, z)

The **solution** to a system of three equations, in three unknowns (a third order system) is called an **ordered triple**. An ordered triple is just like an ordered pair, but in coincides with a system that has 3 dimensions. We usually use the axes x, y & z in a three dimensional coordinate system, where the x & y are the same axes that we are familiar with and the z comes out of the paper at us. Just like the ordered pair, an ordered triple is always written in alphabetical order in parentheses: (x, y, z). The solution to a system of 3 equations in 3 unknowns is just like the solution to a system in 2 unknowns. To check to see if we have a solution the solution must work for all equations. We can also have dependent, consistent systems and independent inconsistent systems, but the scenarios are a little more complex. I will refer you to p. 541 of Lehmann's text for some visual aids.

Example: Is (0, 5, 7) a solution to:

$$\begin{aligned} x + y + z &= 12 \\ x + y &= 5 \\ x + y - z &= 2 \end{aligned}$$

Solving a 3rd Order System Using Addition (Elimination)
Step 1: Select 2 equations and eliminate a single variable using addition/elimination
Step 2: Select a different pair of equations and eliminate the same variable as in Step 1
Step 3: Step 1 & Step 2 will result in 2 equations without one of the 3 variables – Use elimination or substitution on these 2 equations to solve for the 2 remaining variables.
Step 4: Finally use the 2 values found in Step 3 and substitute into any equation to solve for the 3rd and final value.
Step 5: Write the answer as an ordered triple

Example:

$$\begin{aligned} x + y - z &= -3 \\ x + z &= 2 \\ 2x - y + 2z &= 3 \end{aligned}$$

Example:

$$\begin{aligned} -x + 3y + z &= 0 \\ -2x + 4y - z &= 0 \\ 3x - y + 2z &= 0 \end{aligned}$$

Example:

$$\begin{aligned} 2x + y + 2z &= 1 \\ x - 2y - z &= 0 \\ 3x - y + z &= 2 \end{aligned}$$

Note: *One of the steps yields a false statement and this how we know that there is no solution to this problem.*

Example:

$$\begin{aligned} -\tfrac{1}{4}x + \tfrac{1}{2}y - \tfrac{1}{2}z &= -2 \\ \tfrac{1}{2}x + \tfrac{1}{3}y - \tfrac{1}{4}z &= 2 \\ \tfrac{1}{2}x - \tfrac{1}{2}y + \tfrac{1}{4}z &= 1 \end{aligned}$$

Example:

$$\begin{aligned} 3x - 4y + z &= 4 \\ x + 2y + z &= 4 \\ -6x + 8y - 2z &= -8 \end{aligned}$$

Note: *One of the steps yields a true statement and this how we know that there are infinite solutions to this problem. Unlike 2 equations and 2 unknowns however, there are 2 scenarios under which this can happen. The first is that all 3 planes are parallel and the other is when all three intersect in a line.*

An Application of Third Order Systems

Can be used to find a model for a parabola when 3 points on the parabola are known/given.

1) Substitute (x, y) from each ordered pair into $y = ax^2 + bx + c$
2) Solve the resulting 3rd order system
3) Use the results of a, b & c to write a quadratic model in the form of $y = ax^2 + bx + c$

Example: Find the equation of the parabola that passes through the 3 points
(#18 p. 545 Intermediate Algebra, 1st Edition, Jay Lehmann)

(1,1), (2, 5), (3, 15)

Class Exercises & Homework

§9.7 p. 544 #1, 5, 14, 15, 22, 27, 51, 52, #53-58all