# §9.5 & 9.6 Solving Quadratics by Completing the Square & Quadratic Formula

## Completing the Square

Remember the perfect square trinomial? $a^2 + 2ab + b^2$? Well, if you work from the middle term's coefficient, by taking half of it and squaring that number, when $a^2$ is a variable with a coefficient of one, what you will get is $b^2$. Let's try it and see.

**Example:** $x^2 + 6x + 9$

Take $^1/_2$ the middle term: $^1/_2(6) = 3$
Square that number: $(3)^2 = 9$
That is the constant; the $b^2$

This was how you were probably taught to check for a PST (perfect square trinomial). Finding a perfect square trinomial made our factoring job much easier since then we knew that the trinomial factored as a binomial squared with the form $((\sqrt{a})x \pm \sqrt{c})^{2-}$

We will use the same facts to **<u>create our own perfect square trinomials</u>**! Why? Well, perfect square trinomials can be re-written as a binomial squared and then we can use the square root property to solve an un-factorable trinomial.

**<u>Steps to Creating a Perfect Square Trinomial</u>**

1) Remove the leading coefficient by dividing every term by it
2) Move the constant to the other side of the equation
3) Take $^1/_2$ the middle terms coefficient & then square it
4) Add the number from step 3 to both sides of the equation
5) Rewrite the perfect square trinomial as a binomial squared
   You don't have to factor, just use the variable and the ($^1/_2$middle term's coefficient) with the middle term's sign to form a binomial.
6) Use the square root property to finish the problem

**Example:** Solve by completing the square.

a) $x^2 + 2x - 4 = 0$

b) $2x^2 + 3x - 20 = 0$

## The Quadratic Formula

For $ax^2 + bx + c = 0$ $\qquad$ $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

***Note:*** *It is not just the radical expression that is divided by 2a, it is the entire* $-b \pm \sqrt{b^2 - 4ac}$. *I mention this because it is a common error.*

Prior to this class, you may not have had quadratic formula problems that ended up with negative numbers under the radical or radical expressions that could be simplified. **Watch for this simplification!!**

**Example:** Solve using the quadratic formula. Make sure to simplify to a + bi form.

a) $x^2 - 2x - 3 = 0$ $\qquad$ *b) $2x^2 - 2x - 3 = 0$

c) $3x^2 - 2x + 2 = 0$

*If we wish to apply the solution in part b) to a graphing scenario, we will wish an approximation using decimals. Your book uses 2 decimals, and to plot by hand I typically approximate with one decimal since my plots are seldom accurate to 100ths.

**The Discriminant**:

$$\mathbf{b^2 - 4ac}$$

***Note**: This is the number under the radical in the quadratic formula

If the discriminant is zero then the quadratic formula gives only one solution, meaning there is one x-intercept, (since there is no ± value), if the discriminant is positive then you will get two real roots, and thus 2 x-intercepts (either rational or irrational depending upon whether $b^2 - 4ac$ is a perfect square or not), and if the discriminant is negative then the roots are not real numbers and meaning that there are no intercepts since complex numbers can't be graphed on the rectangular coordinate system.

**Summary of Discriminant Value Meanings**

$(b^2 - 4ac) = 0$ then 1 x-intercept exists

$(b^2 - 4ac) > 0$ then 2 x-intercepts exist

If $(b^2 - 4ac)$ is a perfect square then the roots are rational

If $(b^2 - 4ac)$ is not a perfect square then the roots are irrational

$(b^2 - 4ac) < 0$ then no x-intercepts exist

**Example:** For the following indicate how many intercepts exist and then find the intercepts, if they exist.

a) $y = x^2 - 2x - 15$

b) $y = 2z^2 - 7z - 4$

c) $y = 8x^2 - x + 2$

Not only can the discriminant be used to tell us the number of x-intercepts, which is when y = 0, but it can also be used to determine how many solutions a parabola has at any indicated height (y = #).

**Example:** How many points lie on the parabola $f(x) = 2x^2 - 3x - 2$ at a height of $y = -1$? $y = {}^7/_8$? $y = 2$?

## Why so many methods to solve a quadratic? How to choose?

**Factoring**: If a quadratic can be factored do so because it is far less prone to error & it is quicker.

**Example**: $2x^2 - 3x - 2 = 0$

**Square Root Property**: Can only be used if there is not a first-degree term. It is very easy and fast to use.

**Example**: $4(x - 1)^2 = 8$

**Completing the Square**: Great for finding the vertex form of a parabola. This is a good choice for an application problem. It is not the easiest method.

**Example:** Put in vertex form

$$f(x) = x^2 - 4x + 11$$

**Quadratic Formula**: Any quadratic can be solved using the formula. It is prone to computational error. It can become a crutch when factoring & square root property would actually have been faster.

**Example:** $(x + 1)(x - 3) = 2x - 5$

**Class Exercises & Homework**

§9.5 p. 525 #2, 11, 15, 20, 25, 30, 35, 44, 56, 62, 64, 72, 77, 78, 96 & 97

§9.6 p. 536 #9, 18, 19, 24, 29, 30, 37, 67, 70, 72, 79, 83, 107 & 110