<u>Ch. 12 Rational Functions</u>

§12.4 Simplifying Complex Expressions

Outline
Definition – A fraction with a complex expression in the numerator and/or denominator.
Methods for Simplifying
- *Method 1 – Division of numerator by denominator.*
  - *Best for*
    - *Fraction over Fraction*
    - *Rational Expression Fraction over Rational Expression Fraction*
- *Method 2 – Multiplication of all terms by LCD of all denominators*
  - *Best when there is a sum or difference in numerator and/or denominator*

A <u>complex fraction</u> is a fraction with an expression in the numerator (top) and an expression in the denominator (bottom).

<u>Simplifying Complex Fractions Method #1</u>
Step 1: Solve or simplify the problem in the numerator
Step 2: Solve or simplify the problem in the denominator
Step 3: Divide the numerator by the denominator
Step 4: Reduce

Example: Simplify each of the following using Method #1.

a) $$\frac{\frac{3}{4}}{\frac{2}{3}}$$

b) $$\frac{\frac{6x - 3}{5x^2}}{\frac{2x - 1}{10x}}$$

c) $$\frac{\frac{1}{2}+\frac{2}{3}}{\frac{5}{9}-\frac{5}{6}}$$

d) $$\frac{\frac{1}{5}-\frac{1}{x}}{\frac{7}{10}+\frac{1}{x^2}}$$

Simplifying Complex Fractions Method #2

Step 1: Find the LCD of the numerator and the denominator fractions
Step 2: Multiply numerator and denominator fractions by LCD
Step 3: Simplify resulting fraction

Example: Using Method #2, solve the same problems as before.

a) $$\frac{\frac{3}{4}}{\frac{2}{3}}$$

b) $$\frac{\frac{6x - 3}{5x^2}}{\frac{2x - 1}{10x}}$$

c) $$\frac{\frac{1}{2} + \frac{2}{3}}{\frac{5}{9} - \frac{5}{6}}$$

d) $$\frac{\frac{1}{5} - \frac{1}{x}}{\frac{7}{10} + \frac{1}{x^2}}$$

There are some other problems in this section that you really should take a look at. They involve the Fibonnaci sequence which in later algebra work could come up in relation to finding a square root. We'll do one here just so you see how they work.

Example: Simplify $4 + \frac{5}{4+5}$, $4 + \frac{5}{4 + \frac{5}{4+5}}$, $4 + \frac{5}{4 + \frac{5}{4 + \frac{5}{4+5}}}$,

Your Turn §12.4

1. Simplify using method #1.

a) $\dfrac{3/_{5y}}{15/_{xy}}$

b) $\dfrac{2/_{3}}{1\ 5/_{7}}$

c) $\dfrac{\frac{2x^3}{7y^5}}{\frac{12x^4}{49y^2}}$

2. Use method #2 to simplify the following. Don't forget your factoring skills!

a) $$\frac{1 + {}^{1}/_{a}}{a - {}^{1}/_{a}}$$

b) $$\frac{\frac{1}{x+1} + x}{x + \frac{x}{x-1}}$$

c) $$\frac{\frac{1}{x} + \frac{2}{x^2}}{1 - \frac{1}{x} - \frac{6}{x^2}}$$

§12.5 Solving Rational Equations

Outline

Solving Equations Containing Rational Expression

Similarity to linear equations containing fractions

Extraneous Solutions are solutions that will not solve the equation because they make a denominator Zero

Steps

Find zeros (restrictions)

Find LCD

Multiply every term by LCD (Don't build the higher term!!)

Expand and solve the resulting linear or quadratic equation (Notice it could be either type!!)

Check for extraneous solutions and eliminate those as true solutions

The first appearance of the null set, ∅

Writing a solution using set notation – called the solution set, {x}

The only difference between solving rational expression equations and regular equations is that all solutions must be checked to make sure that it does not make the denominator of the original expression zero. If one of the solutions makes the denominator zero, it is called an extraneous solution and will not be part of the solution set for the equation. As a result, these type of equations can be said to have a solution known as the null set, written ∅. This means that there is no solution. Any valid solution is written as a solution set using set notation, {x}.

Solving a rational expression equation can be liked to solving linear (in 1 variable) or quadratic equations that contain fractions. You must first get rid of the denominators in order to solve the equations with less complexity. This requires using the LCD to remove all the denominator, not to build the higher term!! That is the most common error in solving rational expressions, which only leads to a problem if you do not know anything about proportions!

Concept Example: Solve for the variable. $\frac{1}{2}x + {}^{2}/_{3} = {}^{5}/_{8}$

Solving a Rational Expression Equation

Step 1: Find zeros (restrictions) of the rational expressions (See page 1 & 2 of notes)

Step 2: Find the LCD of the denominators in the equation

Step 3: Multiply all terms by the LCD

Step 4: Solve appropriately (may be either a linear or a quadratic, so be careful!)

Step 5: Eliminate extraneous solutions as possible solutions to the equation and write as a solution set.

Example: Solve each of the following equations for the variable and write the solution set.

a) $\frac{9}{2a - 15} = -3$

b) $\frac{5}{c + 2} + 2 = \frac{c}{c - 2}$

c) $\frac{y}{2y + 2} + \frac{2(y - 8)}{4y + 4} = \frac{2(y - 1) - 1}{y + 1}$

d) $\frac{2m}{m + 4} - \frac{2}{m - 1} = \frac{m - 8}{}$

Your Turn §12.5

1. Solve each of the following equations for the variable and write the solution set.

a) $\dfrac{2(y + 10)}{y^2 - 9} = \dfrac{5y - 2}{y^2 - 3y}$

b) $\dfrac{a + 6}{2} = \dfrac{5a}{a - 1}$

c) $\frac{x}{x - 1} + \frac{1}{2} = \frac{1}{x - 1}$

d) $\frac{1}{2} - \frac{x}{x + 1} = \frac{-1}{x + 1}$

<u>WE DO NOT COVER §12.6</u>

§12.7 Proportions & Similar Triangles

Outline

Proportion

Definition – The equality of 2 ratios.

Terms are the numbers in the numerators and denominators of each ratio.

Numbered from numerator on left to denominator on left to numerator on right and finally denominator on the right.

Extremes are the 1st and 4th terms.

Means are the 2nd and 3rd terms.

Cross Product is the product of the means or extremes.

Cross Multiplying is finding the cross products

Means-Extremes Property – Cross Products are equal in a true proportion.

Comes from equality property of multiplication by the LCD

Used to find an unknown term in a ratio.

Applications

Key is to write the proportion in words & fill in the numbers to form an equation.

x_1 *is to* y_1 *as* x_2 *is to* y_2 *where* x_1, x_2, y_1 *or* y_2 *will be missing.*

A proportion is a mathematical statement that two ratios are equal.

$\frac{2}{3} = \frac{4}{6}$ is a proportion

It is read as: **2 is to 3** as **4 is to 6**

The numbers on the diagonal from left to right, 2 and 6, are called the extremes

The numbers on the diagonal from right to left, 4 and 3, are called the means

If we have a true proportion, then the product of the means equals the product of the extremes. This is called the means-extremes property. It actually comes from the fact that when we multiply both sides of an equation by the same number (in this case the LCD) we get an equivalent equation. It is a very useful property for finding a missing term (one of the four numbers in a proportion 2 – 1st term, 3 – 2nd term, 4 – 3rd term, 6 – 4th term, in the example above) in a proportion.

The products of the means and extremes are also called the cross products and finding the product of the means and extremes is called cross multiplying.

Example: Find the cross products of the following to show that this is a true proportion

$$\frac{27}{72} = \frac{3}{8}$$

I am positive that this concept is not a new one, but instead of coming up with only linear equations to solve for we will also come up with quadratics, and we will have to make sure that we know what type of equation we have before we can solve for the variable.

<u>Solving a Proportion</u>

Step 1: Find the cross products and set them equal.
Step 2: Simplify the expressions on both the left and the right side of the equal sign.
Step 3: Inspect the equation closely

a) Is there a second or more degreed term? If so proceed as a quadratic by moving everything to one side of the equation.
b) Is the highest degreed term of degree one? Proceed as a linear equation by isolating the variable.

Step 4: Solve for the variable

a) For a quadratic, factor and set each factor equal to zero, be sure to check for restrictions and eliminate those as possible solutions.
b) For a linear equation move variables to one side and constants to the opposite and isolate the variable by multiplying by the reciprocal of the numeric coefficient. Still check for restrictions.

Step 5: Write solution(s), being careful to eliminate any that are restrictions!

Example: Solve each of the following proportions

a) $\dfrac{x + 1}{3} = \dfrac{2}{3x}$

b) $\dfrac{3a + 2}{a - 2} = \dfrac{-4}{2a}$

c) $\frac{m - 8}{18} = \frac{m}{2}$

Truly, the most useful thing about ratios and proportions are their usefulness in word problems. We can solve distance problems, rate of pay problems, gas mileage problems, unit price problems, equal concentration mixture problems (not like those in §4.4), etc. using this method.

The key is to set up equal ratios of one thing to another and write it out in words 1st.

Example: Using proportions, solve each of the following problems.

a) If Alexis gets 25 mpg in her car how many miles can she drive on 12 gallons of gas?

b) Joe gets a check for $230 for working 5 hours on Tuesday, if he works 7 hours on Wednesday, how much should he expect his check to be?

d) A car travels 231 miles in 3 hours. If it continues at the same speed, how long would you expect it to take in order to travel 385 miles?

Your Turn §7.7

1. Solve each of the following proportions.

a) $\frac{a}{5} = \frac{15}{25}$

b) $\frac{z}{7} = \frac{3}{5}$

c) $\frac{m + 1}{6} = \frac{m}{9}$

d) $\frac{t - 1}{6} = \frac{5}{t}$

e) $\frac{z + 7}{z} = \frac{6}{z}$

2. Solve each of the following problems using a proportion.

a) If one inch on a map represents 25 miles in the real world, how far apart are two cities that are 2.5 inches apart on the map?

b) If a 14 g serving of margarine contains 50 calories how many calories are in a 35 g serving?

c) A player scores 5 goals in the first 3 games of the season. In every season there are 21 games, how many goals would one expect the player to make in these 21 games?

§12.8 Variation

Outline

Direct Variation

A number (y) is equal to another (x) times a constant (K).

Inverse Variation

A number (y) is equal to the reciprocal of another (1/x) times a constant (K).

Problem Solving

Using direct & inverse variation

Direct variation means that two numbers vary proportionally by a constant. Another way of saying this is that one is a constant multiple of the other.

$y = Kx$, where y and $x \in \Re$ and K is a constant

Example: When y varies directly with x, find the constant of proportionality, K.

a) $y = 10$ and $x = -2$

b) $y = 7$ and $x = -3$

The constant of proportionality is always the same for a given relationship and this can be used to solve for unknowns, using a proportion. Since K is always equal to y/x when x and y vary proportionally and the constants of proportionality are always equal for a given relationship, we can set up a proportion and solve it based upon this unique constant, K.

Example: Given that y varies directly with x, solve the following.

a) $x = 5$ and $y = -2$, find y when $x = 20$.

b) $y = -8$ and $x = 12$, find x when $y = -24$.

Inverse variation means that two numbers vary inversely by a constant. Another way of saying this is that one is a constant multiple of the other's reciprocal.

$y = {}^{K}/_{x}$, where y and x $\in \Re$ and K is a constant

Example: When y varies inversely with x, find the constant of proportionality, K.

a) $y = 10$ and $x = -2$

b) $y = 7$ and $x = -3$

The constant of proportionality is always the same for a given relationship and this can be used to solve for unknowns, using a proportion. Since K is always equal to x•y when x and y vary inversely and the constants of proportionality are always equal for a given relationship, we can set up a proportion and solve it based upon this unique constant, K.

Example: Given that y varies inversely with x, solve the following.

a) $x = 5$ and $y = -2$, find y when $x = 20$.

b) $y = -8$ and $x = 12$, find x when $y = -24$.

There are many things in our real world that vary directly and indirectly. For instance, distance and time vary directly and when you consider the formula $d = r \bullet t$, and time and rate vary inversely if you look at this equation rewritten as $t = d/r$. There are many other relationships that occur in our physical world that can be described like this and many others that we can concoct to show this, but I will focus on just this one to show some applications of the idea.

Example: Distance and time vary directly, when rate is constant. If a car travels 250 miles in 3 hours, how many hours will it take to travel 700 miles?

Example: The intensity of a light sources varies inversely with the square of the distance from the source. The intensity of a light is 36 foot-candles at 3 feet, find the intensity at 6 feet.