§11.1 Inverse Functions

We've already discussed functions and that in order to be a function, for every value in the domain of a relation there must be a unique value in the range. We discussed the fact that we could find which relations were functions by seeing their graph and determining whether or not it passed the vertical line test (this shows when there is more than one value in the range for each value in the domain). In this section, we need to talk about another restriction on functions. This is whether a function is **one-to-one** (abbreviated **1:1**). Being **one-to-one** means that each value in the range has only one value in domain. Graphically, this means that if a **horizontal line** is drawn through the graph that it does not cross the graph more than one time. Just as when we looked at ordered pairs to determine whether a relation is a function, we can also look for repeats to find whether a function is oneif we look at the y-values and if there are any repeats then they need to go to the same x-value or the function is not one-to-one.

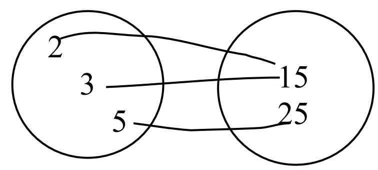
Example: Which of the following are 1:1 functions?

a)

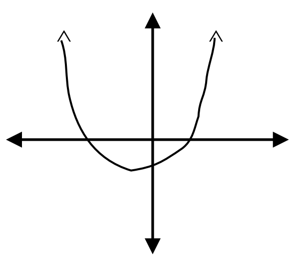
x	1	2	3
f(x)	2.3	2.8	3.2

*(#11 p. 5 *Applied Calculus*, Hughes-Hallett et al, 4th Edition, Wiley, 2010)

b)



c)



d) $y = 10x - x^2$

* (#8 p. 5 *Applied Calculus*, Hughes-Hallett et al, 4th Edition, Wiley, 2010)

d) A patient experiencing rapid heart is administered a drug which causes the patient's heart rate to plunge dramatically and in time, as the drug wears off, the patient's heart rate begins to slowly rise.

One-to-one functions are important in our next concept. In order for a function to have an **inverse** it must be one-to-one and a function. The inverse can be found by exchanging the independent and dependent variables (domain, range).

<u>**Finding the Inverse for 1:1 Function**</u>

1) For a set of ordered pairs:

   Exchange the x & y values

2) For a formula:

   Exchange the independent and the dependent variables

   Solve for the dependent

   Replace the dependent with the $f^{-1}(x)$ notation

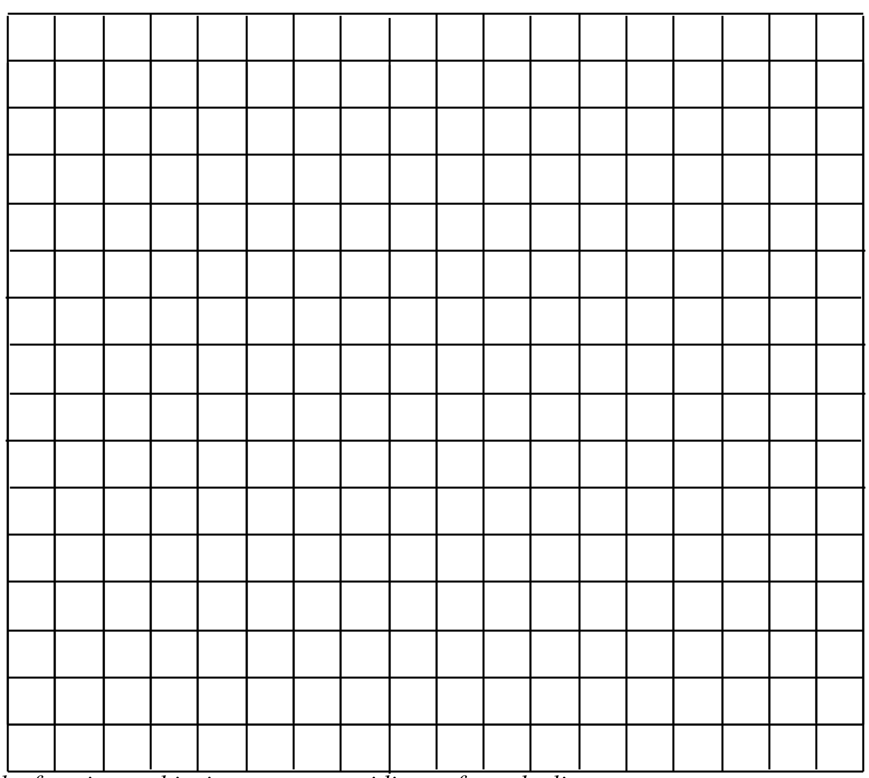
3) Graphically:

   Reflect the graph across the $y = x$ line

   Note: *This is really just the same as plotting (y, x) instead of (x, y)*

Example: $\{(1,2),(2,3),(3,4),(4,5)\}$ is a one-to-one function
What is its inverse?

Example: Find the inverse function for $f(x) = \sqrt{x - 3}$, for $x \geq 3$ then graph both functions on the same axis. Draw in the line $y = x$ for reference.



Note: *The points on the function and its inverse are equidistant from the line y = x.*

Example: For the following 1:1 function, find the indicated values

x	1	2	3
f(x)	2.3	2.8	3.2

*(Adapted from #11 p. 5 *Applied Calculus*, Hughes-Hallett et al, 4th Edition, Wiley, 2010)

a) $f(2)$ b) $f^{-1}(2.3)$ c) $f^{-1}(?) = 3$

***Hint**: Remember for $f(x)$ we have (x, y) and for $f^{-1}(x)$ we have (y, x)*

Example: For the following function, without actually finding the inverse function, find the indicated values.

$f(x) = 10x - x^2$ $x \leq 5$

a) $f(2)$ b) $f^{-1}(0)$ c) $f^{-1}(?) = 2$

***Curious Note:** Why is it that this function that we said was not 1:1 in an earlier example is now considered 1:1? This is something that we want to take note of for future need! We can always make a function 1:1 with careful manipulation. Why?*

The following concept is not noted or covered by Lehmann at this time. I feel it is a good idea to show this as a means of checking whether two functions are inverses of one another and it makes another exercise that practices multiple skills that we are responsible for knowing.

Example: For the function $f(x) = \frac{1}{2}x - 5$, give its inverse, then show that $f^{-1}[f(x)]$ and $f[f^{-1}(x)]$ both equal x.

***Note:** If a function and its inverse are composed, the result should be x. If it is not, then you have either not composed them correctly or the functions are not inverses of one another.*

Class Exercises & Homework

§11.1 p. 645 #1, 6, 8, 11, 15, 36, 37, 40, 41, 62, 78, 90 & 93

§11.1 p. 645 #2, 5, 12, 14, 24, 35, 38, 39, 47, 51, 61, 74 & 77