§10.3 Graphing Exponential Functions
§10.4 Finding Eq. of Exp. F(n)
§10.5 Using Exponential Functions to Model Data

An **Exponential Function**
is any positive $\Re$ not equal to one raised to some $\Re$ valued exponent.

$\mathbf{f(x) = ab^x}$ where a,b > 0 & b ≠ 1

a & b are parameters: a is initial amount & b is growth rate

The **domain** of the function, the values for which x can equal, are all ℝ so **(-∞,∞)**.
Why? Because an exponent can be any real number. And the exponent is the independent variable.

The **range** of the function is all positive ℝ so **(0, ∞).**
Why? Because any number raised to some power will always be a positive number.

The quantity **a** represents the **initial quantity** (the vertical intercept)
Why? Because when x = 0 that makes “a^t” equal to one

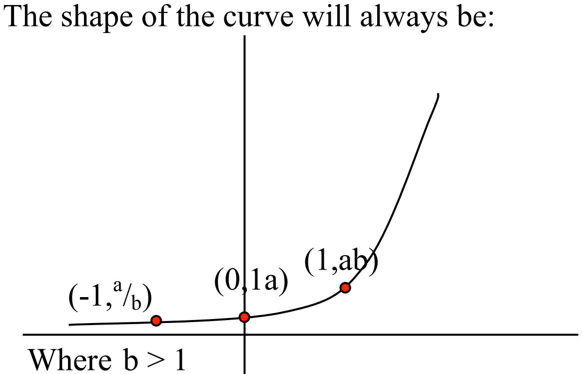
For **exponential growth** b > 1, giving the increasing function you see below on the left
b represents a **positive rate of change**

For **exponential decay** 0 < b < 1, giving the decreasing function you see below on the right
b represents a **negative rate of change or a decay**

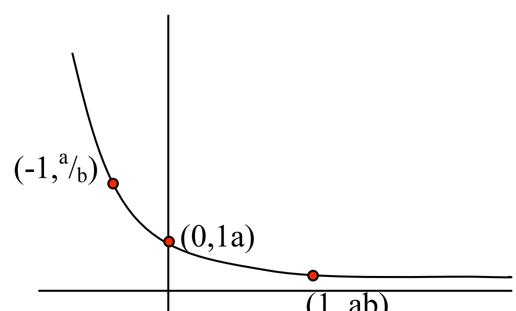
For all exponential functions, the graph will pass through the points:

$(-1, {}^{a}/_{b})$ *Why?* That’s the reciprocal!
$(0,a)$ *Why?* Anything to zero power is 1
$(1, ab)$ *Why?* Anything to 1st power is itself.

The shape of the curve will always be:



Where b > 1
Increasing Function



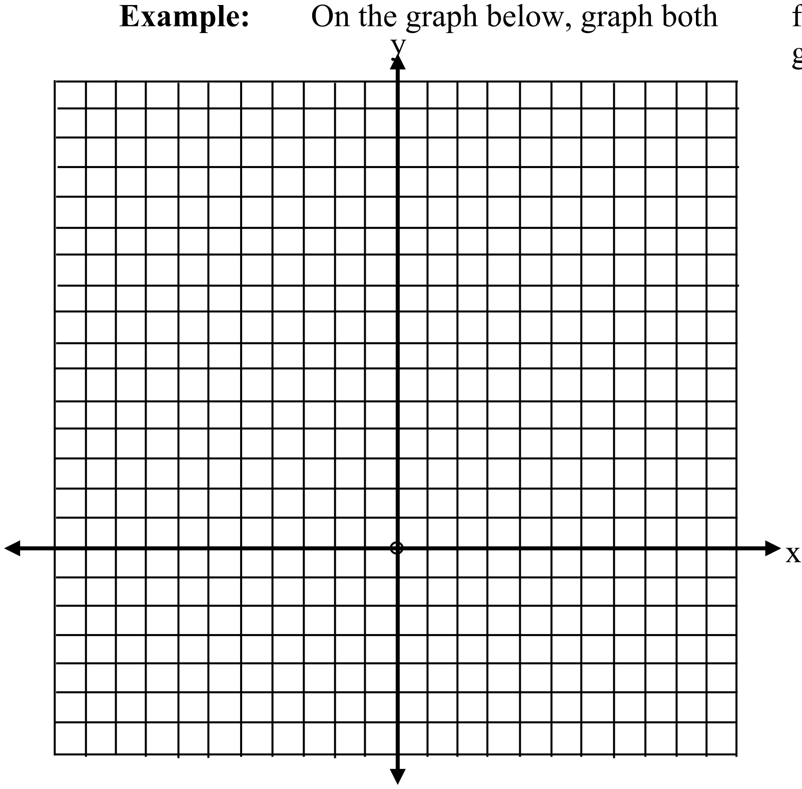
Where 0< b < 1
Decreasing Function

Just some notes:

1) An exponential graph will always approach the x-axis (horizontal asymptote) but will never touch the axis (asymptote).
2) The graph will grow larger and larger if the number "b" is greater than 1. This is like population. Think of 2 people, they have children, then those have children and so on, this is an exponential function, and you should be able to see that it grows to an infinite number.
3) If the "b" is between one and zero (fractional; decimal less than one but greater than zero) the shape is the same but starts at infinity and decreases to approach the x-axis.
   Note: *If we have a^{-x}, we are looking at the reciprocal of a to the x, so it will look like the graph of the function when $0 < a < 1$, when a is greater than 1, and when a is between 0 and 1 the graph will look like the graph when $a > 1$.*
4) The y-intercept of the graph is where $x = 0$ which means that $b^0 = 1$ and the value of the function will be "a". **The y-intercept is "a".**
5) When $a < 0$ the function is reflected across the x-axis. This is why we demand that $a > 0$

Exponential functions are all over in the world. Some of the most prominent areas are economics, biology, archeology, and some cross these boundaries into our everyday lives. One of the most prevalent cases is that of compounded interest. We would like to believe (based upon all interest problems that we have computed) that interest is simple interest, but it is generally compound, which increases at an exponential rate. Even the value of our vehicles once we drive them off the lot can be determined by an exponential function!

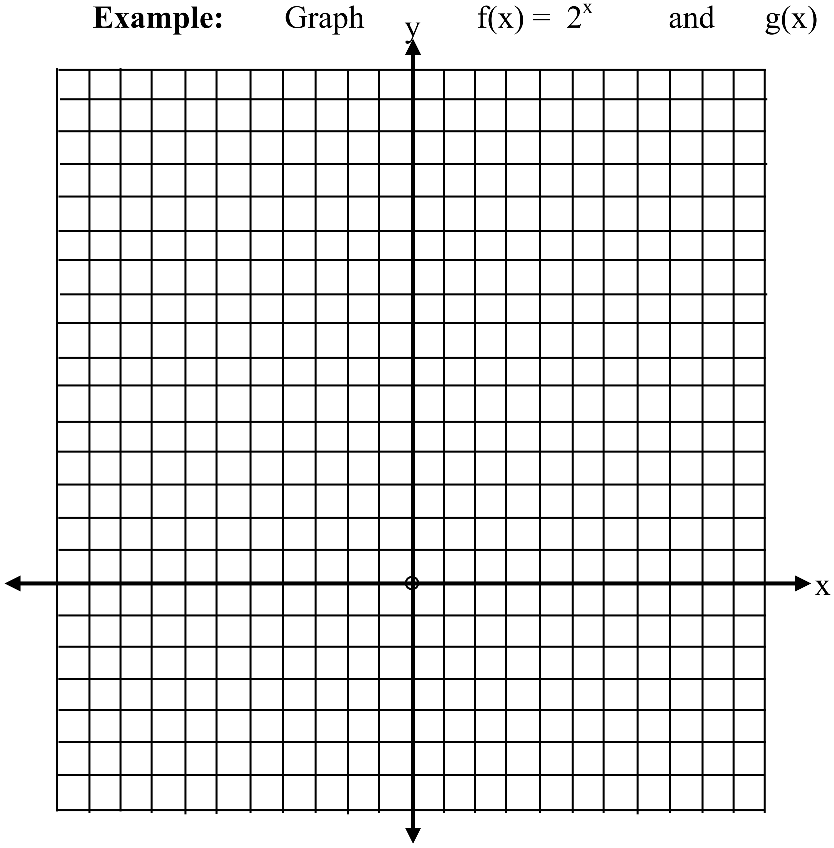
Example: On the graph below, graph both $f(x) = 4^x$ and
$g(x) = ({}^1/_4)^x$ (Also $f(x) = 4^{-x}$)



Remember our translations for quadratic equations? Exponential functions can be translated too. A vertical translation moves the horizontal asymptote from y = 0 (the x-axis) to whatever the constant added to the functions is.

$f(x) = ab^x + D$ graphs with $(0, a + D)$, $(1, ab + D)$ & $(-1, {}^{a}/_{b} + D)$

Example: Graph $f(x) = 2^x$ and $g(x) = 2^x + 3$



Note: *Notice how the horizontal asymptote for the graph of g(x), went from being the x-axis to a horizontal line 3 units higher, the line y = 3.*

Recall also that we talked about the horizontal translations of exponential functions, these are translations that move the x-coordinate. Lehmann doesn't discuss this type of translation, but I'm going to include this example in my notes for my own purposes.

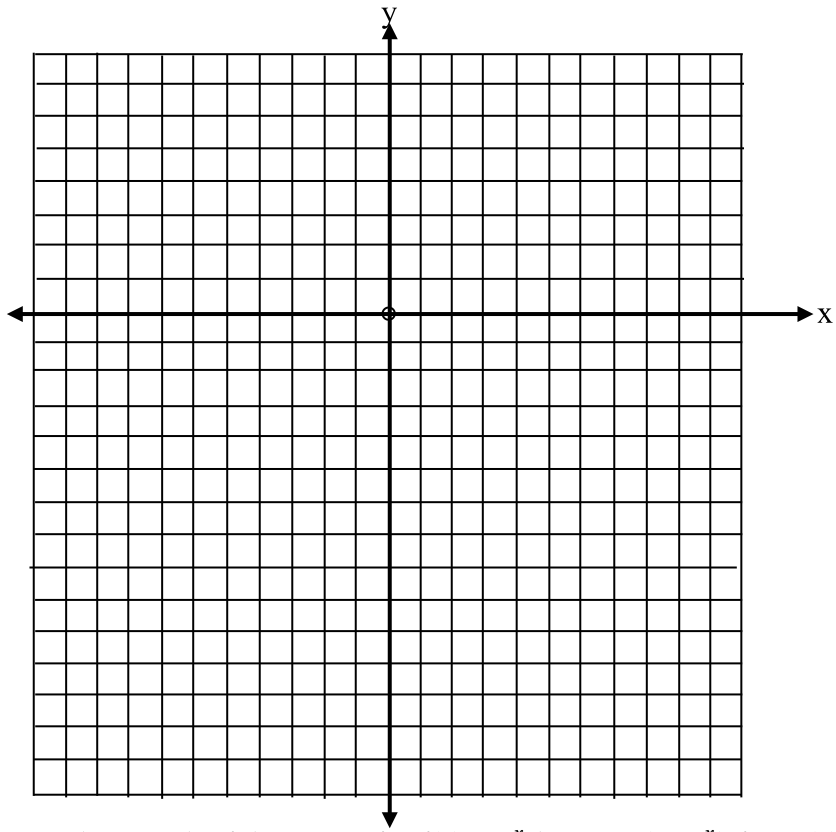
$f(x) = ab^{x-C}$ graphs with $(0 + C, a)$, $(1 + C, ab)$ & $(-1 + C, {}^{a}/_{b})$

Example: Graph $h(x) = 2^{x-3}$ on the graph above.

Note: *The y-intercept is no longer the point (0, 1) [since a =1, the point (0, a) is (0, 1)]. It is now the point* $(0, b^{-C})$*, which is* $(0, {}^{1}/_{8})$ *in this problem. To really see this translation, find h(3), h(4) & h(2) for* $f(x) = 2^x$ *and compare the values to f(0) and compare to h(3), f(1) and compare to h(4), and f(-1) and compare to h(2).*

Let’s now consider the **reflection** of the exponential function. This will be when “a” is less than zero.

Example: Graph $m(x) = -2^x$



Note: *Notice how each of the points for $f(x) = 2^x$ became $(x, -2^x)$ for $m(x)$. Compare $f(-1)$ to $m(-1)$, $f(0)$ to $m(0)$, and $f(1)$ to $m(1)$.*

Last in this section we **model using exponential functions**.

To **recognize** an exponential function from values, look for **ratios** of f(x) values that are for **constant intervals** of x

Example: Consecutive f(x) values are 40000, 42400, 44944, 47640.64

42400/40000=1.06

44944/42400=1.06

47640.64/44944=1.06

Which indicates that the growth factor is 1.06; a **growth factor** is (1 + r)

Where “r” is the constant **rate of growth** 6%

To **create** an exponential equation

1) Find the **constant ratio (growth factor)**
2) Raise the constant ratio (growth factor) **to the power of x**
3) **Multiply** the constant ratio to the x^{th} power **by** the **initial** value

Example: Create an equation for the above scenario, knowing that the initial value was 40000.

Now, let's see if we have the basics of the exponential function.

Example: The following functions give the amount of substance present at time *t*. In each, give i) the amount present initially, ii) state whether the function represents exponential growth or decay, iii) give the growth factor and iv) give the constant percent of growth or decay rate.*(#2 p.43, *Applied Calculus, Hughes-Hallet, et al, 4th ed.*)

a) $A = 100(1.07)^t$ b) $A = 3500(0.98)^t$

Example: A town has a population of 1000 people at t = 0. In each of the following cases, write a formula for the population, P, of the town as a function of year *t*.*(#4 p. 44, *Applied Calculus, Hughes-Hallet, et al, 4th ed.*)

a) The population increases by 50 people a year

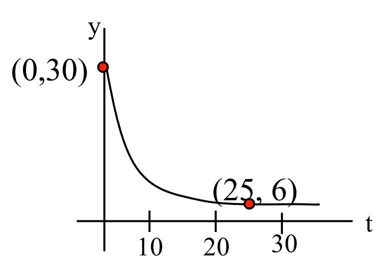
b) The population increases by 5% per year

Note: *One of these is linear.*

When we don't know the values for consecutive independents, we can use the general function $f(x) = ab^x$ to model as we did with a quadratic. Here is the process:

1) "a" is the y-intercept if the model appears to have a horizontal asymptote y=0
2) use a second ordered pair (x, f(x)) to plug-in to: $f(x) = ab^x$
3) solve for the growth/decay rate by dividing f(x) by "a" and taking the x^{th} root of that value

Example: Find a possible formula for the function. *(#18 p. 45, *Applied Calculus, Hughes-Hallet, et al, 4th ed.*)



Example: For the data in the table showing annual soybean production in millions of tons, answer the following questions.

*(#26 p.45, *Applied Calculus, Hughes-Hallet, et al, 4th ed.*)

Year	2000	2001	2002	2003	2004	2005
Production	161.0	170.3	180.2	190.7	201.8	213.5

a) Does this table correspond to a linear or exponential function? How can you tell?

b) By hand find a formula for the world soybean production in millions of tons, as a function of time, *t*, since 2000.

c) What is the annual percent increase in soybean production?

d) Use the exponential regression feature on your calculator to find a model for this data. Compare the model from b) & d) to the data using your calculator's graphing function.

Let's close with a discussion about **half-life** and **doubling time**. Both these terms are related to the time (independent variable) it will take to achieve *half or double the original population* value. An increasing function ($b > 1$) will not be able to achieve a half-life. A decreasing function ($0 < b < 1$) will not be able to achieve a doubling time.

Half-Life — we can use $b = {}^1/_2$ when x is divided by half-life
only possible if $0 < b < 1$
exponential decay

***Note:** later we will use ratio of P/P_0 is equal to ${}^1/_2$ or simply ${}^1/_2 = b^x$

Doubling Time — we can use $b = 2$ when x is divided by doubling time
only possible if $b > 1$
exponential growth

***Note:** later we will use ratio of P/P_0 is equal to 2 or simply $2 = b^x$

Example: A storage tank contains a radio-active element that has a half-life of 5730 years. The function, f(t), represents the percentage of this radio-active element remaining t years since the element was placed in the storage tank. Find an equation for f(t). (Adapted from #47 p. 636, *Intermediate Algebra*, Jay Lehmann, 1st Edition)

***Note**: The function is a percentage so the initial value is 100%.

Example: Total sales, in a large company, have doubled every year since 2000. The total sales in 2007 were \$17 million. Write an equation for the total sales, S(t), t years since 2000. (Adapted from #46 p. 636, *Intermediate Algebra*, Jay Lehmann, 1st Edition)

What happens if we want to model an exponential function but we don't know the initial value?

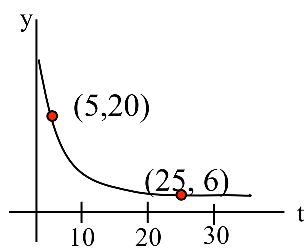
1) use two ordered pairs to find the following ratio and solve for b.

$$\frac{y_2}{y_1} = \frac{b^{x2}}{b^{x1}} \Rightarrow \frac{y_2}{y_1} = b^{x2-x1} \Rightarrow b = (x_2 - x_1)^{th} \text{ root of } ({}^{y2}/_{y1})$$

2) use a one of the ordered pairs (x, f(x)) to plug-in to: $f(x) = ab^x$

3) solve for the growth/decay rate by dividing f(x) by "a" and taking the x^{th} root of that value

Example: Find a possible formula for the function. *(Adapted from#18 p. 45, *Applied Calculus, Hughes-Hallet, et al, 4th ed.*)



***Note:** Use 3 decimals for your base and two for your initial value.

§10.3 p. 604 #1, 6, 11, 12, 13, 15, 20, 24, 27, 39, 44, 49, 52, 56, 58, 62, 63, 81, 82, 90 & 91
§10.4 p. 613 #4, 9, 12, 16, 20, 30, 36, 39, 43, 53, 56, 64, 65, 69, 72, 79 & 80
§10.5 p. 623 #1, 2, 5, 6, 7, 11, 12, 13, 14, 15, 16, 17, 23, 29, 36 & 40