§10.1 Integer Exponents

Definition of zero exponent if $a \neq 0$

$a^0 =$

Anything to the zero power is 1. Be careful because we still have to be certain what the base is before we rush into the answer!

Example: Simplify each by removing the negative exponent.

a) 2^0 b) $(^1/_2)^0$ c) $-2x^0$

d) $-1^0 - 1^0$ e) $2x^0 - y^0$ d) $(7 + x)^0$

Product Rule for Exponents

$a^r a^s =$

Example: Use Product Rule of Exponents to simplify each of the following. Write the answer in exponential form.

a) $x^2 x^3$ b) $(-7a^2 b)(5ab)$ c) $3^2 \bullet 3^7$ d) $y^2 \bullet y^3 \bullet y^7 \bullet y^8$

Example: Simplify using the product rule $y^{2m+5}\ y^{m-2}$

Definition of a negative exponent (Shorthand for take the reciprocal of the base)

$a^{-r} =$

Anytime you have a negative exponent you are just seeing short hand for take the reciprocal. When a negative exponent is used the negative in the exponent reciprocates the base and the numeric portion tells you how many times to use the base as a factor. **<u>A negative exponent has nothing to do with the sign of the answer.</u>**

Example: Simplify each by removing the negative exponent.

a) 2^{-1} b) $(^1/_2)^{-1}$

When a negative <u>exponent</u> has a <u>numeric portion that isn't one</u>, start by taking the reciprocal of the base and then doing the exponent (using it as a base the number of times indicated by the exponent).

Example: Simplify each by removing the negative exponent.

a) 2^{-2} b) $(^1/_2)^{-3}$

Don't let negative exponents in these bother you. Copy the like bases, subtract (numerator minus denominator exponents) the exponents and then deal with any negative exponents. If you end up with a negative exponent it just says that the base isn't where it belongs – if it's a whole number take the reciprocal and if it is in the denominator of a fraction taking the reciprocal moves it to the numerator. Let's just practice that for a moment.

Example: Simplify each by removing the negative exponent.

a) a^{-2} b) $\dfrac{1}{b^{-3}}$

The Quotient Rule of Exponents

$$\frac{a^r}{a^s} =$$

Example: Simplify each. Don't leave any negative exponents.

a) $\dfrac{a^{12}}{a^7}$ b) $\dfrac{2b^5}{b^3}$ c) $\dfrac{x^8}{x^2}$

d) $\dfrac{2b}{16\,b^3}$ e) $\dfrac{(a+b)^{15}}{(a+b)^7}$ f) $\dfrac{2a^3b^2}{4ab^3}$

Now, let's practice the quotient rule with negative exponents. You have 2 choices in doing these problems: 1) Use the quotient rule on integers (can be tricky if your integer subtraction is not what it should be) or 2) Remove the negative exponents and use product/quotient rules as needed.

Example: Simplify each. Don't leave any negative exponents.

a) $\dfrac{a^8}{a^{10}}$ b) $\dfrac{2}{b^{-3}}$ c) $\dfrac{x^{-8}}{x^{-2}}$

d) $\dfrac{2\,b^{-3}}{4}$ e) $\dfrac{(a+b)^5}{(a+b)^{-7}}$ f) $\dfrac{2a^3b^{-2}}{ab^{-3}}$

Example: Simplify using the quotient rule $\dfrac{15x^{7n-5}}{}$

$3x^{2n-3}$

Power Rule of Exponents

$(a^r)^s$ =

<u>or</u>

$(ab)^s$ =

<u>or</u>

$(^a/_b)^r$ =

Example: Use Power Rule of Exponents to simplify each of the following. Write the answer in exponential form.

a) $(a^3)^2$ b) $(10xy^4)^2$ c) $(-7^5)^2$ d) $(^{-3a}/_{5b})^3$

Note: *A negatvie number to an even power is positive and a negative number to an odd power is negative.*

Now that we have discussed all the rules for exponents, all we have left is to put them together. Let's practice with some examples that use the power rule, the product rule and also the quotient rule. Use the power rule 1^{st}, then the product rule and finally the quotient rule. Deal with the negative exponents last.

<u>Some notes about what **SIMPLIFIED** means</u>

1) No parentheses are involved
2) In a monomial, there is only one constant & each variable appears as a base at most once
3) No numeric base still contains an exponent
4) Numeric fractions are in lowest terms
5) Every exponent is positive

Example: Use the properties of exponents and definitions to simplify each of the following. Write in exponential form.

a) $(x^2 y)^2 (xy^2)^4$ b) $(-8)^3 (-8)^5$ c) $(^3/_2)^3 \bullet 3^4$

d) $\left[\dfrac{9x^2y^5}{15xy^8}\right]^2$ e) $\dfrac{(2xy^2)^3}{4x^2y}$ f) $\left[\dfrac{-5x^2y^3}{4x^{-1}}\right]^2$

g) $\left[\dfrac{9x^2y^5}{15xy^8}\right]^{-2}$ h) $\left[\dfrac{-5x^2y^3}{4x^{-1}}\right]^{-3}$ i) $\dfrac{(2xy^3)^{-2}}{xy^2}$

Your Turn

Example: Simplify each. Don't leave any negative exponents.

a) $\dfrac{(ab)^8}{a^{10}}$ b) $\dfrac{(-2xy)^3}{3x^{-3}\,y}$ c) $\left[\dfrac{x^{-8}}{x^{-2}}\right]^{-3}$

d) $\dfrac{(5xy^2)^2 (4x^{-1}y^{-3})^{-3}}{5x^{-3}(2x^3y^{-5})^{-2}}$ e) $x^{-3} \bullet x^{-5} \bullet x^7$

f) $\dfrac{2a^3b^{-2}}{ab^{-3}}$

Our next topic is an application of exponents that makes writing very large and very small numbers much easier, especially in application. Think of scientific notation as what you see every day in written expression of large numbers – like in a newspaper article. You might see 5,000, 000 written a 5 million. This is the same thing that scientific notation does, but it uses multiplication by factors of ten, which are decimal point movers.

Scientific Notation

When we use 10 as a factor 2 times, the product is 100.

$10^2 = 10 \times 10 = 100$ second power of 10

When we use 10 as a factor 3 times, the product is 1000.

$10^3 = 10 \times 10 \times 10 = 1000$ third power of 10.

When we use 10 as a factor 4 times, the product is 10,000.

$10^4 = 10 \times 10 \times 10 \times 10 = 10{,}000$ fourth power of 10.

From this, we can see that the number of zeros in each product equals the number of times 10 is used as a factor. The number is called a power of 10. Thus, the number

100,000,000

has eight 0's and must be the eighth power of 10. This is the product we get if 10 is used as a factor eight times!

Recall earlier that we learned that when multiplying any number by **powers of ten** that we **move the decimal to the right the same number of times as the number of zeros in the power of ten!**

Example : $1.45 \times 1000 = 1{,}450$

Recall also that we learned that when dividing any number by **powers of ten** that we **move the decimal to the left the same number of times as the number of zeros in the power of ten!**

Example : $5.4792 \div 100 = 0.054792$

Because we now have a special way to write powers of 10 we can write the above two examples in a special way – it is called **scientific notation** .

Example : $1.45 \times 10^3 = 1{,}450$ (since $10^3 = 1000$)

Example: $5.4792 \times 10^{-2} = 0.054792$ (since $10^2 = 100$ and $[10^2]^{-1} = \frac{1}{100}$ which means divided by 100)

All of the above form the basis for changing a number from **scientific notation** to **standard form**. Since ***multiplying*** a number by a factor of 10 simply ***moves the decimal to the right*** the number of times indicated by the # of zeros, that's what we do! If the exponent is negative, this indicates ***division*** by that factor of 10 so we would ***move the decimal to the left*** the number of times indicated by the exponent.

Example : Change 7.193×10^5 to standard form

1) Move Decimal to the Right ________ times.
2) Giving us the number …

Example : Change 6.259×10^{-3} to standard form.

1) Move Decimal Left ____ times
2) Giving us the number …

Example: Write each of the following to standard form.

a) -7.9301×10^{-3} b) 8.00001×10^5 c) 2.9050×10^{-5} d) -9.999×10^6

<u>Writing a Number in Correct Scientific Notation:</u>

Step 1: Write the number so that it is a number ≥ 1 but < 10 (decimals can and will be used)

Step 2: ***Multiply*** this number by 10^x (x is a whole number) to tell your reader where the decimal point is really located. The **x** tells your reader how many zeros you took away! (If the number was 1 or greater, then the **x** will be positive, telling your reader that you moved the decimal to the right to get back to the original number, otherwise the **x** will be negative telling the reader to move the decimal left to get back to the original number.)

Example : Change 17,400 to scientific notation.

Example : Write 0.00007200 in scientific notation

Your Turn : Change each of the following to scientific notation

a) 8,450 b) 104,050,001 c) 34

d) 0.00902 e) 0.00007200 f) 0.92728

Note: *When a number is written correctly in scientific notation, there is only one number to the left of the decimal. Scientific notation is always written as follows:* $a \times 10^x$, *where a is* ≥ 1 *and* <10 *and x is an integer.*

Finally, we need to talk about Exponential Functions and how to simplify those.

An **<u>exponential function</u>** is: $f(x) = ab^x$ where $b > 0$ and $b \neq 1$ & $a \neq 0$, $x \in \mathbb{R}$

We need to make sure that:

1) if "b" is a constant that we rewrite the function as a constant function
   Ex. $f(x) = 4^x$ find $f(-1)$ ➔ $f(-1) = 4^{-1} = {}^1/_4 = 0.25$
2) if "b" exponent is a monomial with a numeric coefficient that we use the reverse of the power rule to rewrite the base
   Ex. $f(x) = 4^x$ find $f(2a)$ ➔ $f(2a) = 4^{2a} = (4^2)^a = 16^a$
3) if "b" exponent is an binomial we use the product rule in reverse to create a product
   Ex. $f(x) = 4^x$ find $f(2 + a)$ ➔ $f(2 + a) = 4^2 \bullet 4^a = 16(4)^a$

Your Turn: For $f(x) = 2(3)^x$ find the following

a) $f(-2)$ b) $f(2a)$ c) $f(a + 3)$

<u>§10.2 Rational Exponents</u>

We have already discussed rational exponents and how they work during our discussion of radicals in chapter 9. You may recall that the index is the denominator of the fractional exponent and any exponent on the radicand is the numerator of the fraction. This allowed us to much more easily evaluate radical expressions containing variables and even large numbers. The trick was to re-write numbers using their prime factorization in exponential form. We will always assume that variables are non-negative.

$$\sqrt[n]{x^m} = x^{m/n}$$

and to evaluate problems that are written in rational exponents you will need to know:

$$b^{m/n} = (b^{1/n})^m = (b^m)^{1/n}$$

You must also be able to take a radical expression written using exponents and put it into standard form as a radical expression. This is simply knowing what the exponent represents. Recall numerator is the radicand's exponent and the denominator is the index.

Example: a) $(-216)^{1/3}$ b) $-81^{3/2}$ c) $343^{-2/3}$

Your Turn: Evaluate if possible. If the expression isn't $\Re$, so state.

a) $(-125)^{1/3}$ b) $(-100)^{1/2}$

c) $(-27)^{-4/3}$ d) $16^{-1/2} + 25^{-1/2}$

All of the exponent rules that we have learned apply to these newly expressed radical expressions. Let's review:

Multiply like bases – Add exponents

$$x^{1/2} \bullet x^{1/3} = x^{1/2 + 1/3}$$

Divide like bases – Subtract denominator exponent from numerator exponent

$$\frac{x^{1/2}}{x^{1/3}} = x^{1/2 - 1/3}$$

Negative Exponent – Take the reciprocal of the base

$$x^{-1/2} = \frac{1}{x^{1/2}}$$

Power Rules – Multiply the exponents

$$(x^{1/2})^{1/3} = x^{1/2 \bullet 1/3}$$

$$(x^2 y^4)^{1/3} = x^{2 \bullet 1/3}\ y^{4 \bullet 1/3}$$

$$\left(\frac{x^2}{y^4}\right)^{1/3} = \frac{x^{2 \bullet 1/3}}{y^{4 \bullet 1/3}}$$

Example: Write in exponential form without negative exponents.

a) $a^{1/3} \bullet a^{1/3}$

b) $x^{1/4} \bullet x^{1/3}$

c) $(x^{-1/3})^{2/3}$

d) $\dfrac{x^{3/4}}{x^{1/2}}$

e) $\left[\dfrac{a^{1/5}}{a^{2/3}}\right]^{-1/2}$

Class Exercises & Homework
§10.1 p. 589 #1, 4, 6, 9, 13, 16, 24, 29, 44, 49, 51, 56, 57, 60, 65, 68, #72-74even, #73-77odd, #84, 86, 90, 94, 95, 96, 104, 108, 111, 112 & 126
§10.2 p. 597 #3, 6, 10, 14, 20, 22, 23, 24, 26, 28, 29, 36, 38, 44, 48, 55, 58, 64, 68, 70 & 71