Factoring by GCF

Step 1: Find the GCF of all terms

Step 2: Rewrite as GCF times the sum of the quotients of the original terms divided by the GCF

Example: For each of the following factor using the GCF

a) $8x^3 - 4x^2 + 12x$ b) $27a^2b + 3ab - 9ab^2$

c) $18a - 9b$ e) $^1/_7 x^3 + {}^4/_7 x$

Factoring By Grouping

Step 1: Group similar terms and factor out a GCF from each grouping (keep in mind the aim is to get a binomial that is the same out of each grouping(term) – look for a GCF)

Step 2: Factor out the like binomial and write as a product (product of 2 binomials)

Hint: *Trinomials are prime for factoring with the GCF and a polynomial with 4 terms is prime for this method*

Example: In the following problems factor out a GCF from binomials in such a way that you achieve a binomial in each of the resulting terms that can be factored out.

a) $8x + 2 + 3y^2 + 12xy^2$ b) $2zx + 2zy - x - y$

It is important to point out a pattern that we see in the factors of a trinomial such as this:

$$(x + 2)(x + 1)$$

$= x^2$	$+ 3x$	$+ 2$
↑	↑	↑
$x \bullet x$	$2+1$	$2 \bullet 1$
↑	↑	↑
Product of 1st 's	**Sum of 2nd 's**	**Product of 2nd 's**

Because this pattern exists we will use it to factor trinomials of this form.

Factoring Trinomials of the Form $x^2 + bx + c$

Step 1: Start by looking at the constant term (including its sign). Think of all it's possible factors

Step 2: Find two factors that add to give middle term's coefficient

Step 3: Write as $(x \pm 1^{st}\text{ factor})(x \pm 2^{nd}\text{ factor})$;where x is the variable in question & signs depend upon last & middle terms' signs (c is positive both will be the same as middle term, c is negative larger factor gets middle terms' sign)

Step 4: Check by multiplying

Example: Factor completely.

a) $x^2 + xy - 2y^2$

b) $a^2 + 8a + 15$

c) $z^2 - 2z - 15$

d) $x^2 - 7x + 5$

e) $2x^2 + 10x + 12$

f) $x^3 - 5x^2 + 6x$

g) $(2c - d)c^2 - (2c - d)c + 4(2c - d)$

Sometimes we will see some of the special patterns that we talked about in chapter 7, such as:

$$\mathbf{a^2 + 2ab + b^2 = (a + b)^2} \quad \underline{\textbf{or}} \quad \mathbf{a^2 - 2ab + b^2 = (a + b)^2}$$

These are <u>perfect square trinomial.</u> They can be factored in the same way that we've been discussing or they can be factored quite easily by recognizing their pattern.

Factoring a Perfect Square Trinomial

Step 1: The numeric coefficient of the 1st term is a perfect square

i.e. 1,4,9,16,25,36,49,64,81,100,121,169, 196, 225, etc. & 625

Step 2: The last term is a perfect square

Step 3: The numeric coefficient of the 2nd term is twice the product of the 1st and last terms' coefficients' square roots

Step 4: Rewrite as:

$$(\sqrt{1^{st}\text{ term}} + \sqrt{\text{last term}})^2 \quad \underline{\textbf{or}} \quad (\sqrt{1^{st}\text{ term}} - \sqrt{\text{last term}})^2$$

Note: *If the middle term is negative then it's the difference of two perfect squares and if it is positive then it is the sum.*

Note2*: that whenever we see the perfect square trinomial, the last term is always positive, so if the last term is negative don't even try to look for this pattern!!*

Example: Factor completely

a) $4x^2 - 12x + 9$

b) $32x^2 + 80x + 50$

Difference of Two Perfect Squares

Remember the pattern:

$$(a + b)(a - b) = a^2 - b^2$$

Example: $(x - 3)(x + 3) = x^2 - 9$

Now we are going to be "undoing" this pattern.

Factoring the Difference of Two Perfect Squares

Step 1: Look for a difference binomial and check

a) Is there a GCF? If so, factor it out and proceed with b) & c)

b) Is 1st term coefficient is a perfect square? (If no, then stop, problem is complete)

c) Is 2nd term is a perfect square? (If no, then stop, problem is complete)

Step 2: Yes to both b) and c) then factor the difference binomial in the following way

$$(\sqrt{1^{st}\text{ term}} + \sqrt{2^{nd}\text{ term}})(\sqrt{1^{st}\text{ term}} - \sqrt{2^{nd}\text{ term}})$$

Step 3: If there was a GCF don't forget to multiply by that GCF.

Example: Factor completely.

a) $x^2 - y^2$ b) $4x^2 - 81$ c) $z^2 - {}^1/_{16}$

d) $27z^2 - 3y^2$ e) $12x^2 - 18y^2$

Any time an exponent is evenly <u>divisible by 2</u> it is a **perfect square**. If it is a **perfect cube** it is evenly <u>divisible by 3</u>, and so forth (we will need perfect cubes for §8.4). So, in order to factor a perfect square binomial that doesn't have a variable term that is square you need to divide the exponent by 2 and you have taken its square root.

Example: Factor completely.

a) $16x^4 - 81$ b) $9x^6 - y^2$

<u>**Sum and Difference of Two Perfect Cubes**</u>

This is the third pattern in this section. The pattern is much like the pattern of the difference of two perfect squares but this time it is either the sum or the difference of two perfect cubes. At this time it might be appropriate to review the concept of a perfect cube and or finding the cube root of a number. It may also be appropriate to review some perfect cubes: $1^3=1$, $2^3=8$, $3^3=27$, $4^3=64$, $5^3=125$, $6^3 = 216$, $7^3 = 343$, and $10^3=1000$

If you have the <u>**difference of two perfect cubes**</u> then they factor as follows:

$$\mathbf{a^3 - b^3 = (a - b)(a^2 + ab + b^2)}$$

If you have the <u>**sum of two perfect cubes**</u> then they factor as follows:

$$\mathbf{a^3 + b^3 = (a + b)(a^2 - ab + b^2)}$$

<u>**Factoring the Sum/Difference of Two Perfect Cubes**</u>

Step 1: Look for a sum or difference binomial and check

a) Is there a GCF? If so, factor it out and proceed with b) & c)

b) Is 1st term coefficient is a perfect cube? (If no, then stop, problem is complete)

c) Is 2nd term is a perfect cube? (If no, then stop, problem is complete)

Step 2: Yes to both b) and c) then factor the difference binomial in the following way

Where a = cube root of the 1st term and

b = the cube root of the 2^{nd} term

If the binomial is the difference

$$(a - b)(a^2 + ab + b^2)$$

If the binomial is the sum

$$(a + b)(a^2 - ab + b^2)$$

Step 3: If there was a GCF don't forget to multiply by that GCF.

Example: Factor each of the following perfect cube binomials.

a) $125x^3 + 27$

b) $27b^3 - a^3$

c) $24z^3 + 81$

d) $48x^3 - 54y^3$

<u>Factoring Trinomials of Form – $ax^2 + bx + c$</u>

Step 1: Find the factors of a

Step 2: Find the factors of c

Step 3: Find all products of factors of a & c $(a_1x + c_1)(a_2x + c_2)$ where $a_1x \bullet c_2$ and $c_1 \bullet a_2x$ are the products that must add to make b! (This is the hard part!!!) The other choice is $(a_1x + c_2)(a_2x + c_1)$ where $a_1x \bullet c_1$ and $c_2 \bullet a_2x$ must add to make b. And then of course there is the complication of the sign. Pay attention to the sign of b & c still to get your cues and then change your signs accordingly. (But you have to do this for every set of factors. You can narrow down your possibilities by thinking about your middle number and the products of the factors of a & c. If "b" is small, then the sum of the products must be small or the difference must be small and therefore the products will be close together. If "b" is large then the products that sum will be large, etc.)

Step 4: Rewrite as a product.

Step 5: Check by multiplying. (Especially important!)

Example: Factor completely.

a) $2x^2 + 5x + 2$

b) $10x^2 + 9x + 2$

c) $15x^2 - 4x - 4$

d) $-2a^2 - 5a - 2$

e) $21x^2 - 48x - 45$

Zero Factor Property
If A & B are polynomials and if $A = 0$ or $B = 0$, then $A \bullet B = 0$ according to the multiplicative property of zero.

Here is what this means to us:

Example: Find what value of the variable makes A & B zero.
$A = (x + 3)$ & $B = (x - 2)$

Now consider the following polynomial function in standard form:

Example: Find the solution(s) to $(x + 3)(x - 2) = 0$

<u>**Solving Quadratic Equations**</u>
Step 1: Put the equation in standard form
Step 2: Factor the polynomial
Step 3: Set each term that contains a variable equal to zero and solve for the variable
Step 4: Write the solution as: *variable* = **or** *variable* =
Step 5: Check

Sometimes book exercises give you equations where step 1 or steps 1 and 2 have already been done. Don't let this fool you, the steps from there on are the same.

Example: Solve each of the following by applying the zero factor property to give the solution(s).

a) $(x + 2)(x - 1) = 0$

b) $x^2 - 4x = 0$

c) $x^2 - 6x = 16$

d) $x^2 = 4x - 3$

e) $-2 = -27x^2 - 3x$